ある Cell polarization model の数学的解析について

０．研究の背景

１．細胞極性モデル

２．極限方程式について

３．数値計算による大域的分岐図と安定性の結果

４．大域的分岐シートとその性質

５．大域的分岐シート作成のアイデア

６．大域的分岐シートの性質の数学的証明

研究の背景

Y. Mori, A. Jilkine and L. Edelstein-Keshet,
Asymptotic and bifurcation analysis of wave-pinning in a reaction-diffusion model for cell polarization
SIAM J, Appl. Math 71(2011), 1401-27.

モデルの提示
非線形項を双安定な
３次式に置き換え,
極限方程式（D→∞）
の数値計算による
分岐構造の研究

定常極限方程式の分岐構造の
数学的研究

K. Kuto and T. Tsujikawa,
Bifurcation structure of steady-states for bistable equations with nonlocal constraint
DCDS Supplement 2013, 467-476.

分岐点および特異極限の近傍での
解形状と分岐曲線の
概要に関する数学的結果

森竜樹，久藤衡介，辻川亨，四ツ谷晶二，
日本数学会2013 年度秋季総合分科会
函数方程式論分科会アブストラクト.

全ての解の表示式を求め,
数値的に,
二次分岐以降も含む,
大域的分岐構造を解明した

↑ 変数変換　Cahn-Hilliard方程式

S. Kosugi, Y. Morita and S. Yotsutani,
Stationary solutions to the one-dimensional Cahn-Hilliard equation: Proof by the complete elliptic integrals,
Discrete and Continuous, Dynamical Systems 19(2007), 609-629.

研究の目的

- 細胞運動モデルの極限方程式の，表示式を用いて
  1次分岐のみならず2次分岐以上等のすべてを含む，
  大域的分岐構造を数学的に証明したい.

- 極限方程式の分岐構造を元に拡散係数が十分大のときの
  細菌運動モデルの定常解の極限形状を数学的に証明したい.

- 各分岐曲線の安定性を数学的に証明したい.

細胞極性モデル

Y. Mori, A. Jilkine and L. Edelstein-Keshet,
Asymptotic and bifurcation analysis of wave-pinning in a reaction-diffusion model for cell polarization
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非線形項を双安定な３次式に置き換え,

極限方程式（D→∞）の数値計算による分岐構造の研究

Fig.1 in Y. Mori, A. Jilkine and L. Edelstein-Keshet,
Biophys. J., 94 (2008), pp. 3684-97.

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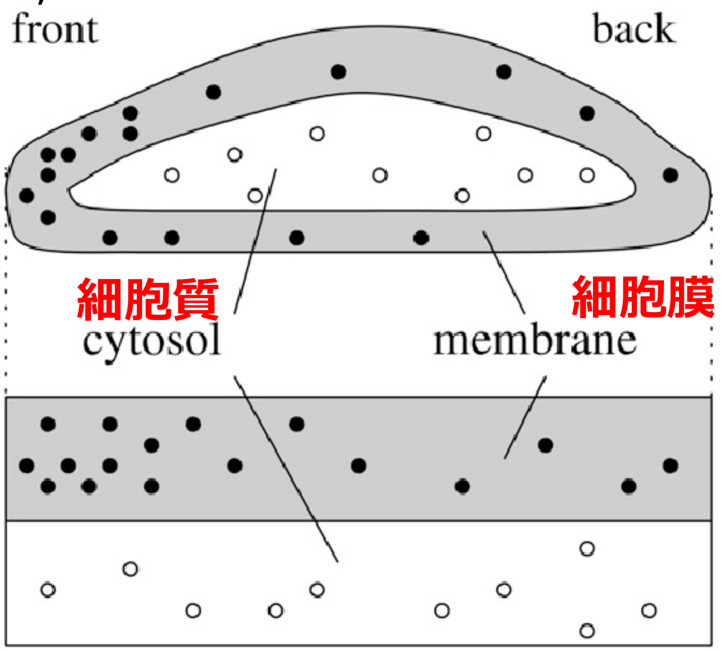

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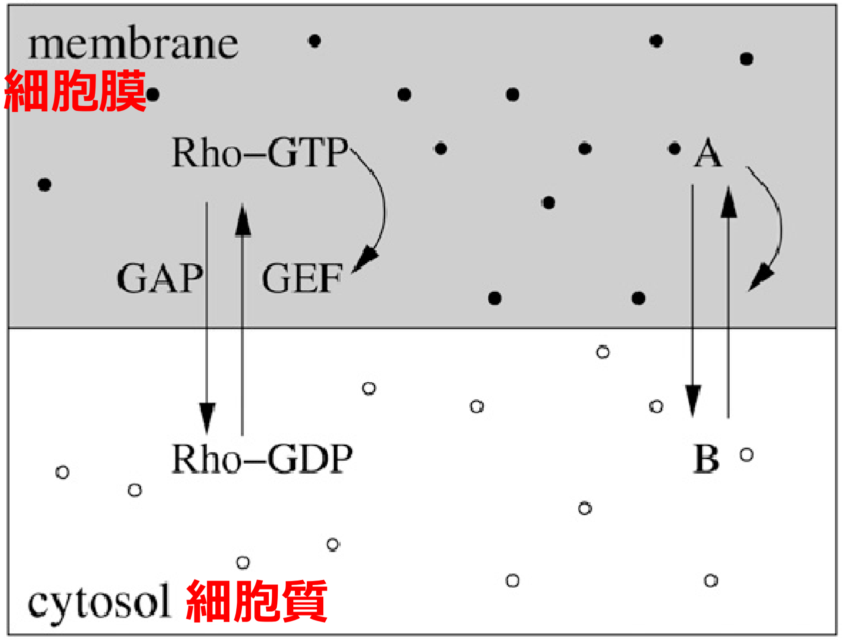

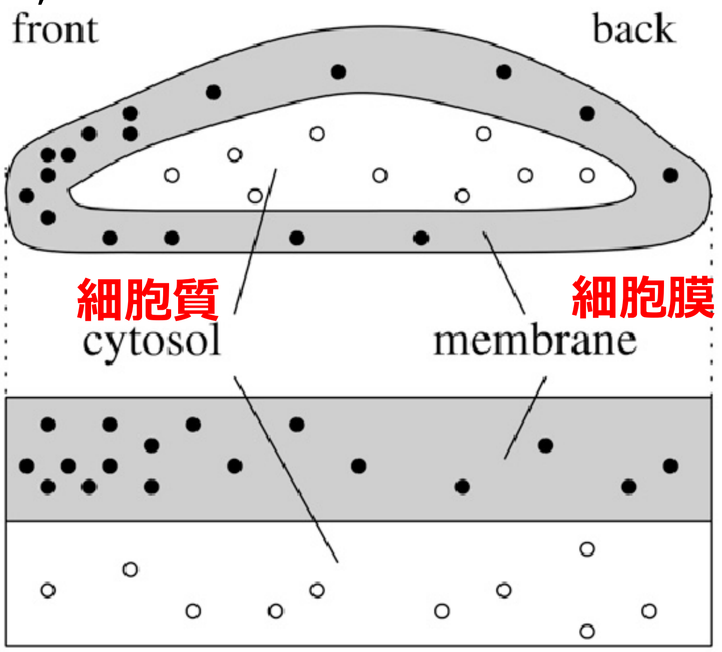

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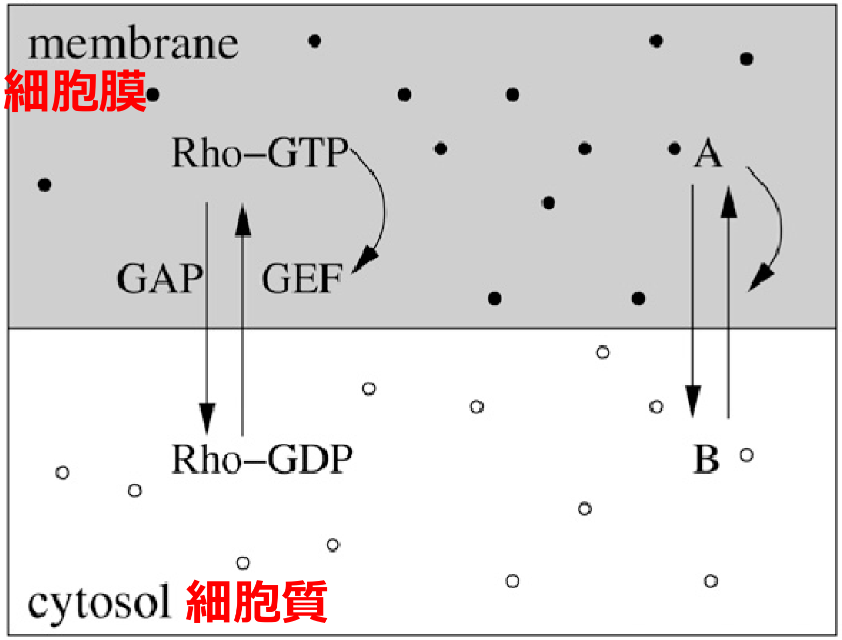

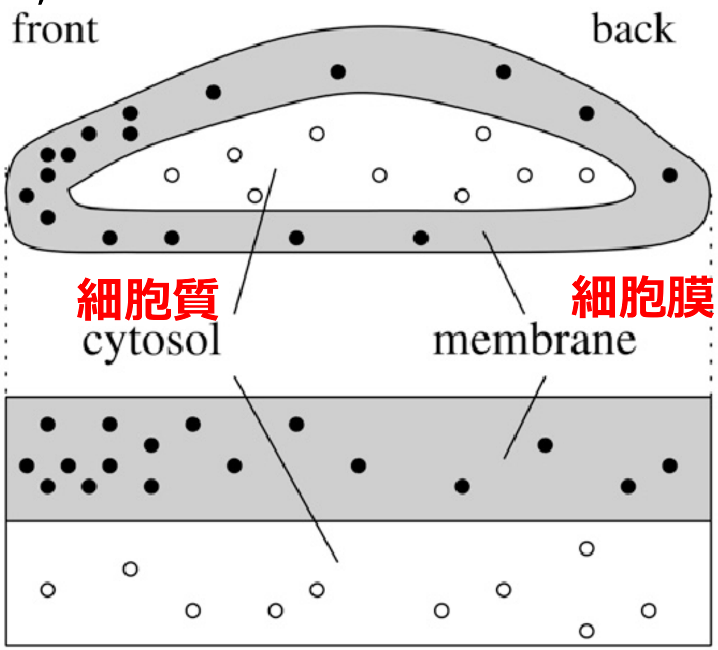

A： 活性たんぱく質 (Rho-GTP, ●)

B：不活性たんぱく質 (Rho-GDP, ○)

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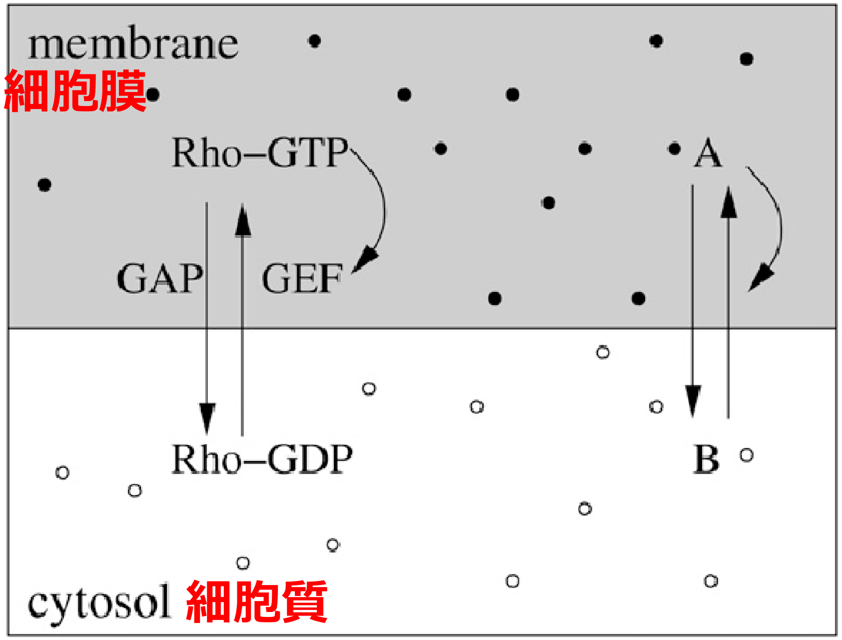

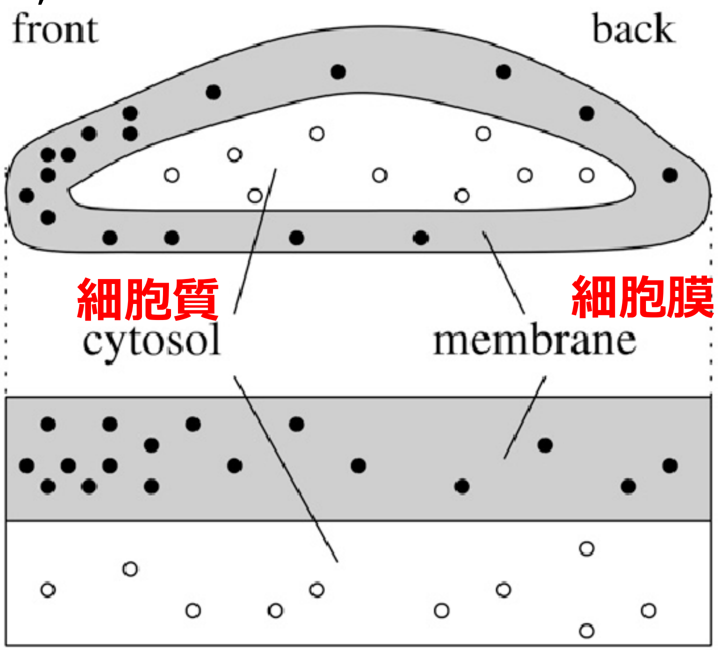

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GAP：不活性化
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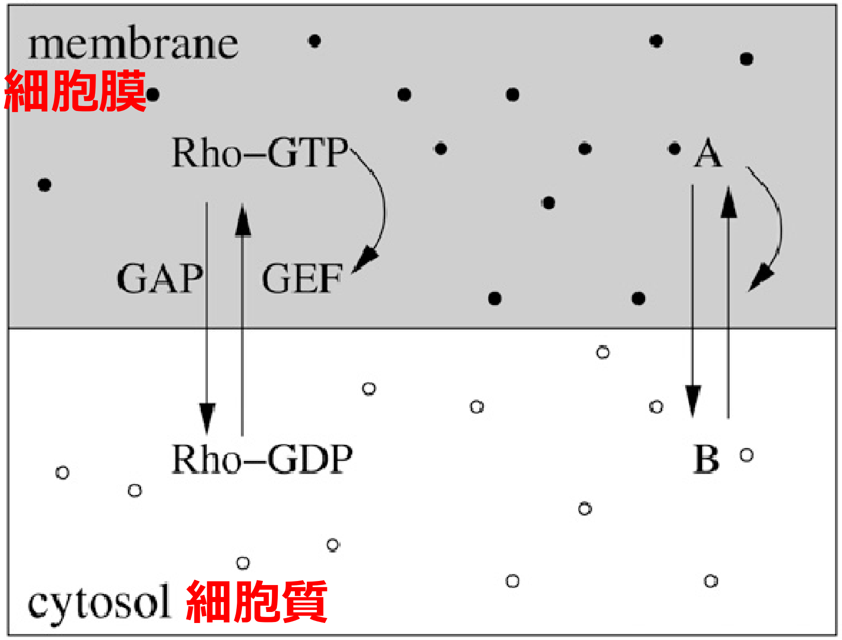

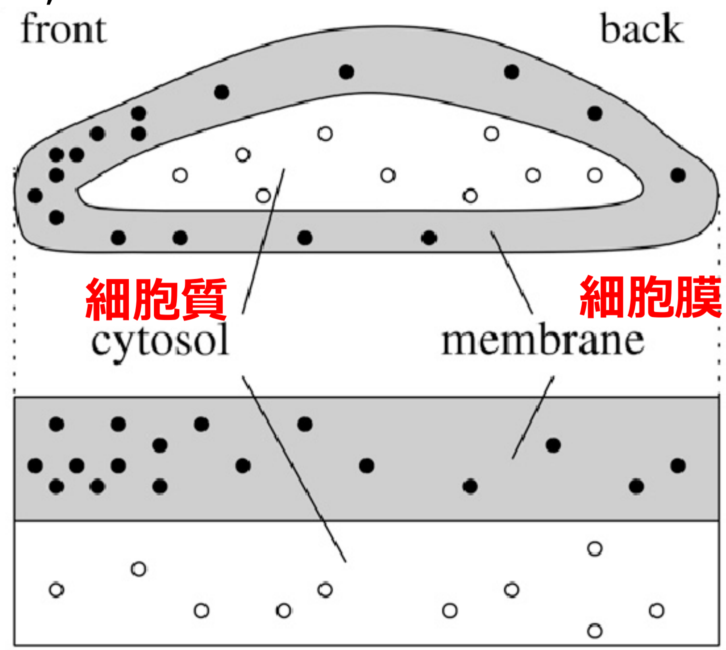

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不活性たんぱく質の拡散係数は,活性たんぱく質よりも,
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不活性たんぱく質の濃度は,ほぼ一様と見られる.

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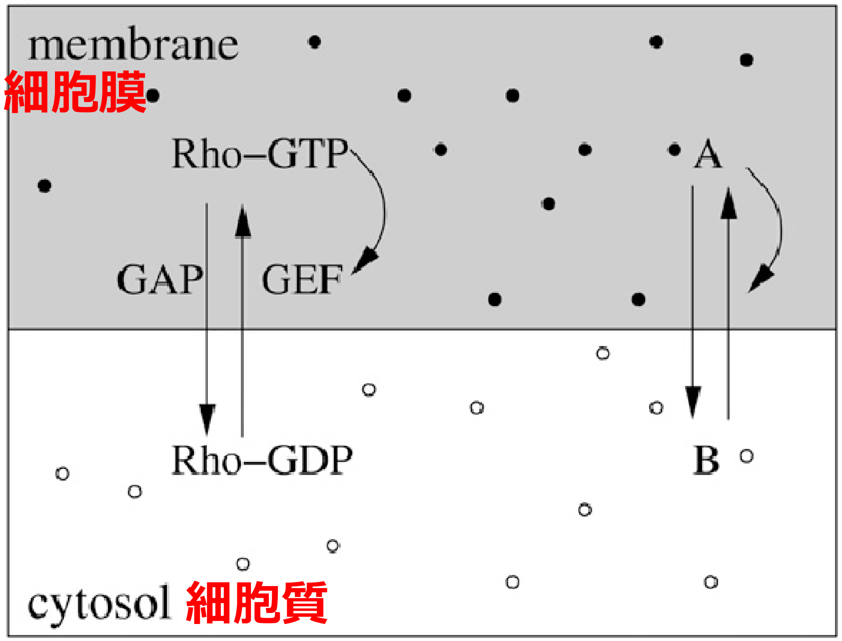
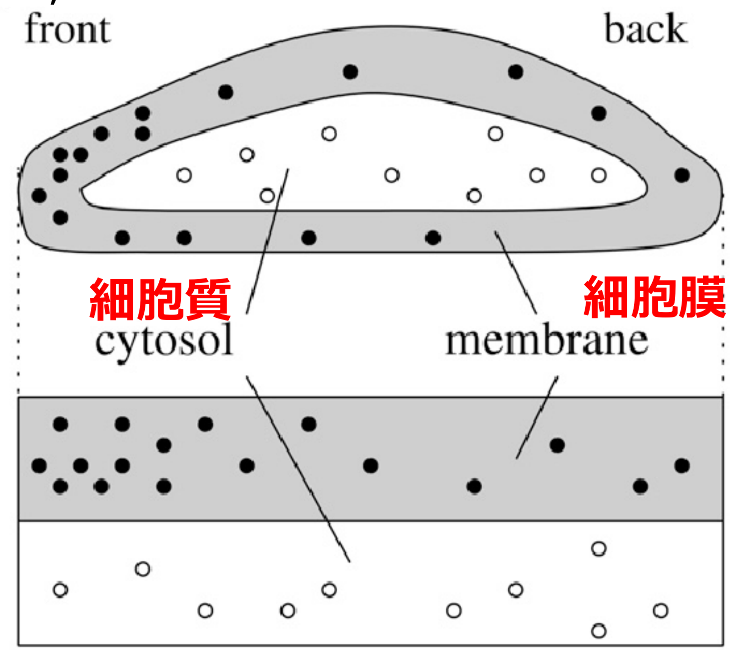
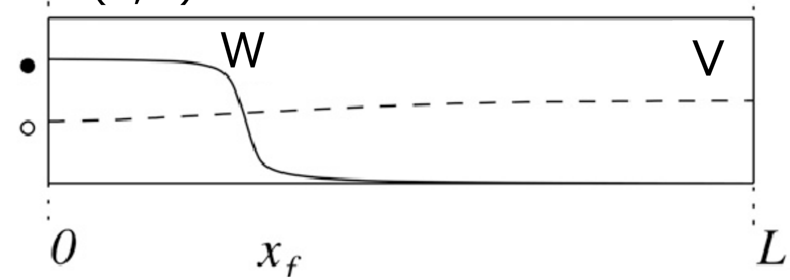




W(x, t)： 活性たんぱく質の濃度
V(x, t)：不活性たんぱく質の濃度



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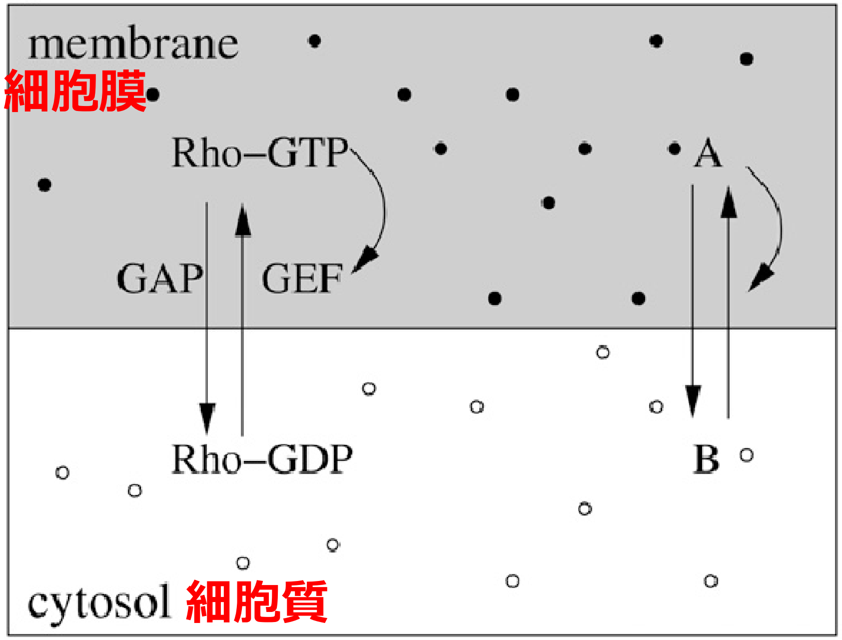

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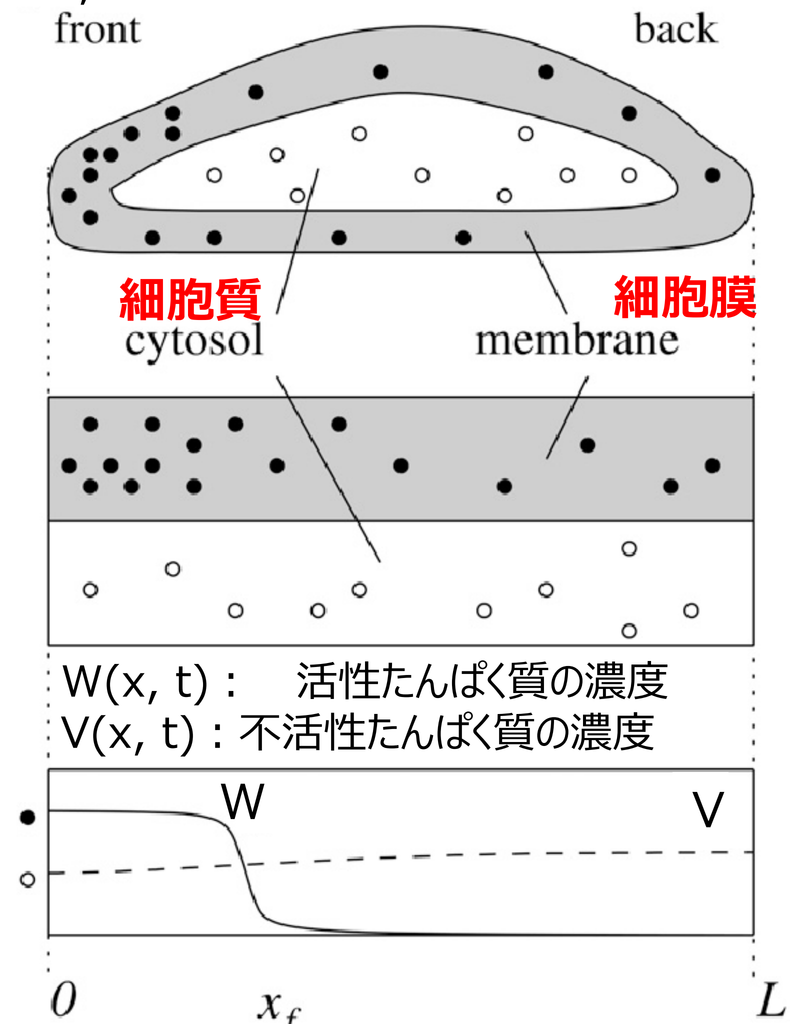

極性は活性たんぱく質の濃度の偏りに対応.

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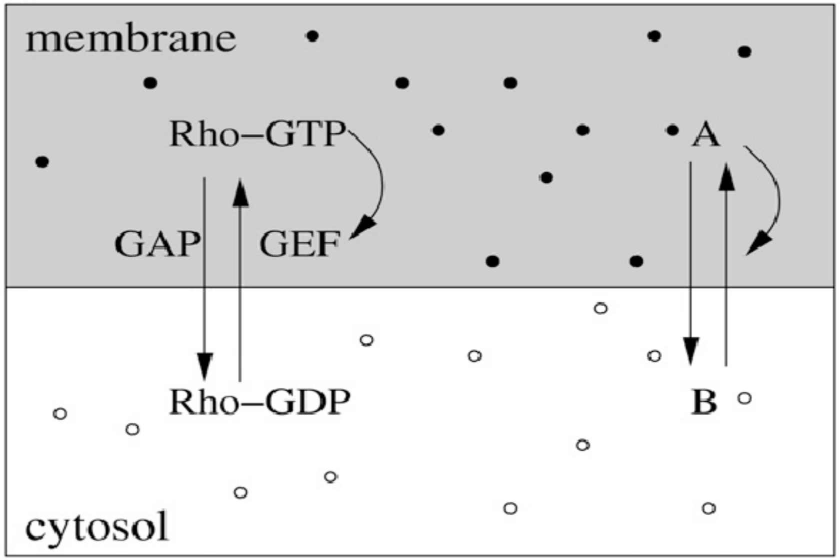
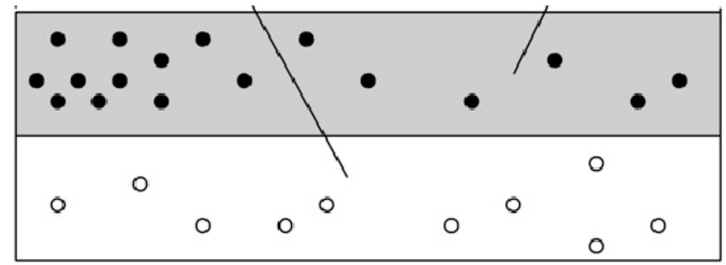
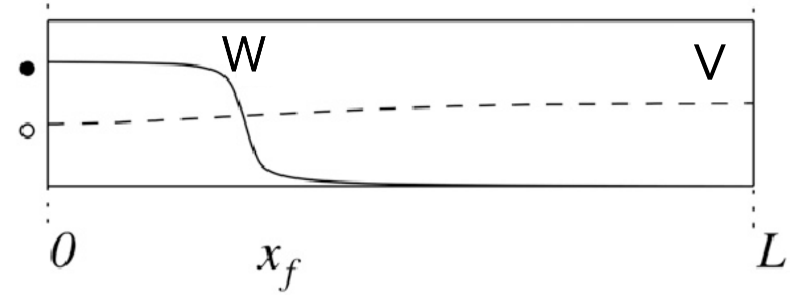




Wave-pinning for cell polarization



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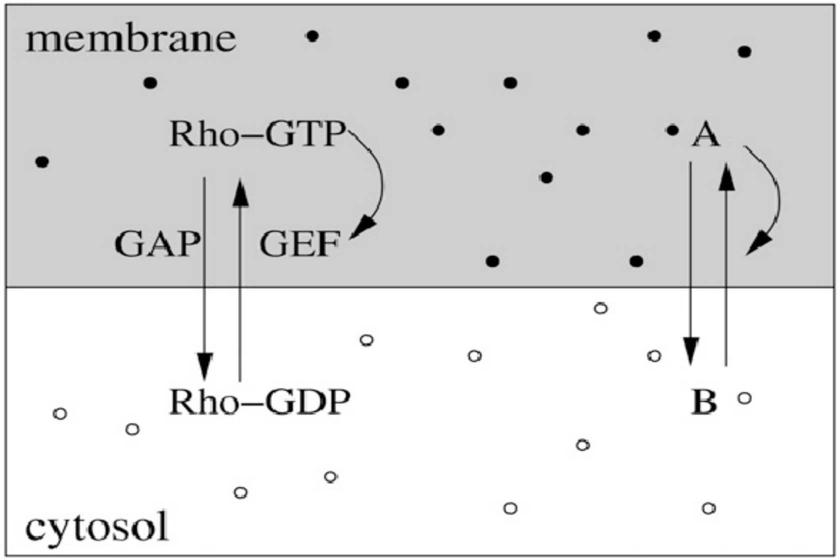

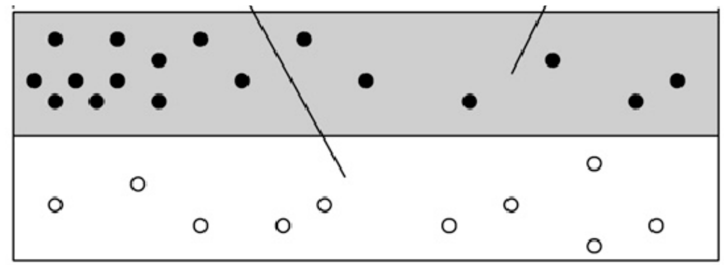

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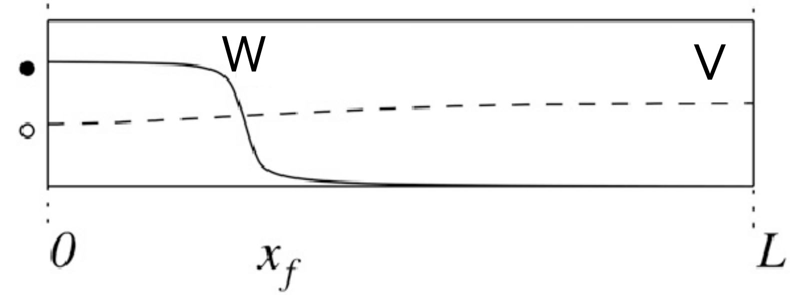

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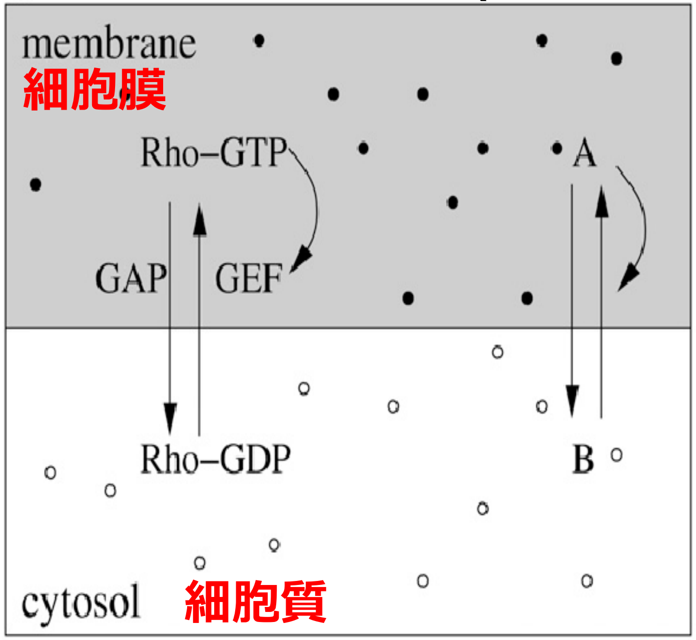

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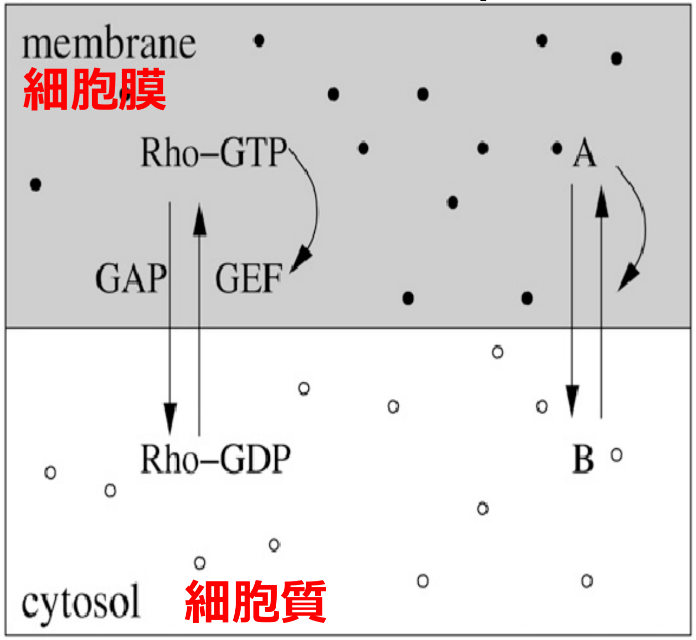

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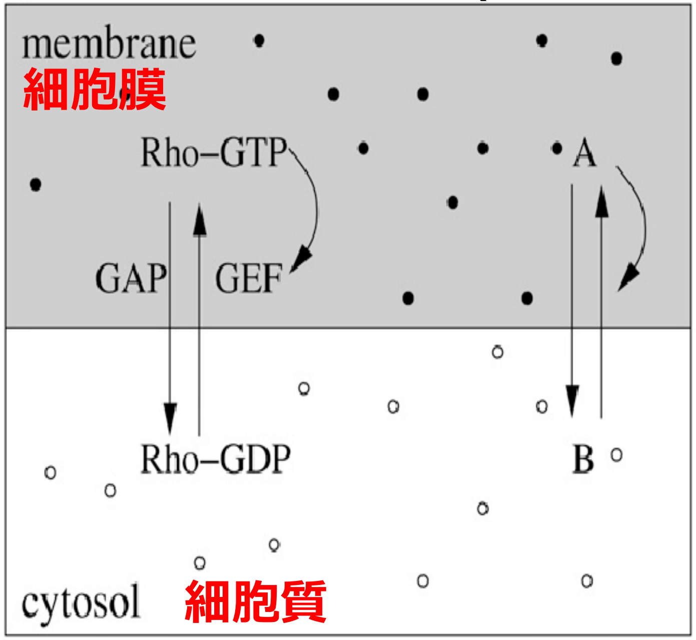

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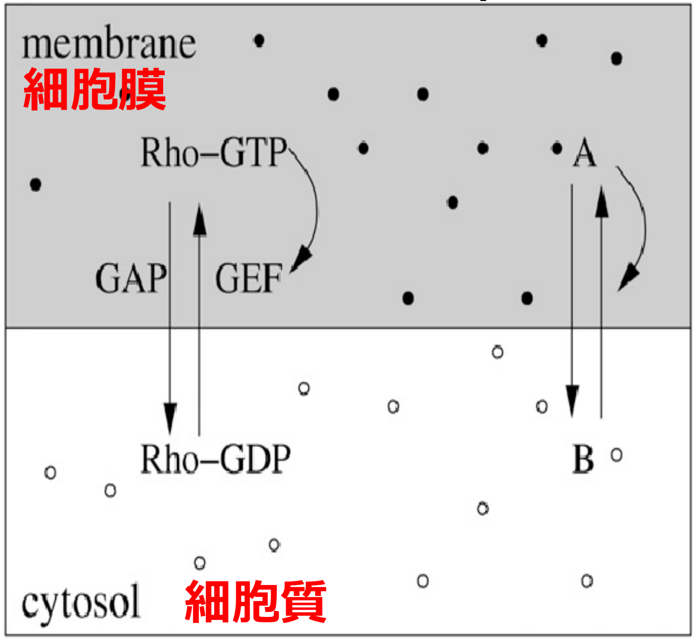

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$$f(W,V)$$

＝(活性化速度)・V － (不活性化速度)・W

$$= \eta\left(\delta + \frac{\gamma W^2}{m^2 + W^2}\right)V - \eta W \qquad \eta,\ \gamma,\ m > 0, \quad \delta \geq 0$$

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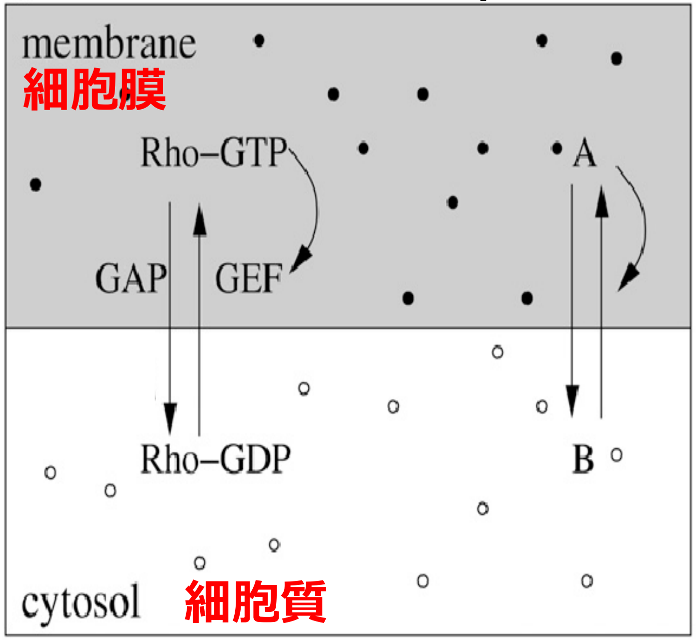

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(第１項について V → W) GEFs で活性化される速度.
生物学的には十分に解読されていないが、
Vが多ければ多いほどVがWに変化するというプラスのフィードバックがある
ということが経験的に知られている.
活性化速度はプラスのフィードバックがあるとしてヒル関数と仮定する.

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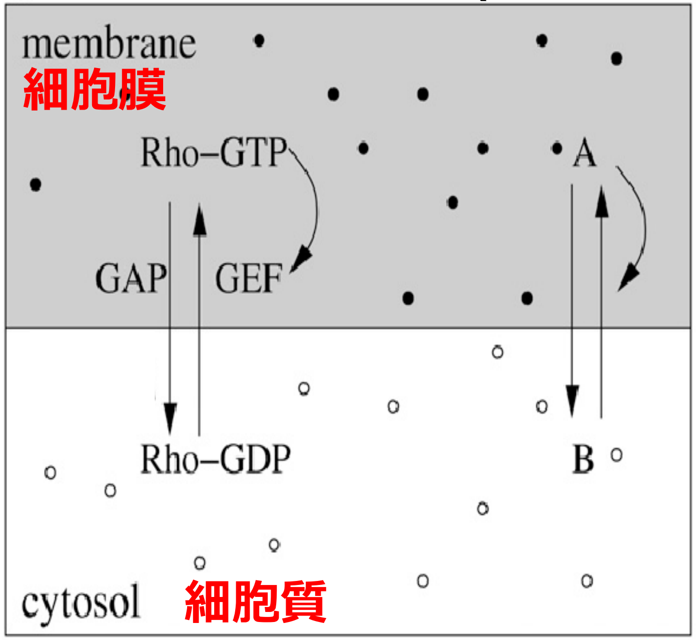

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(第２項について, W → V) GAPsで不活性化される速度は単純に定数とする.

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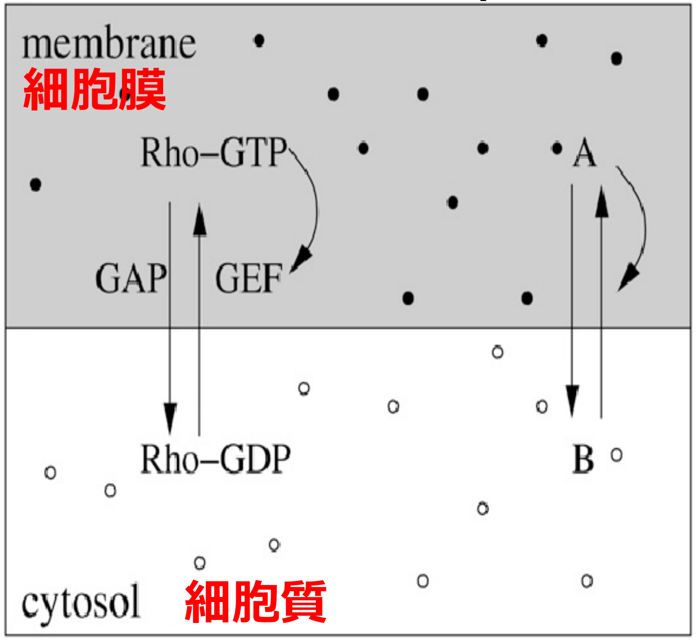

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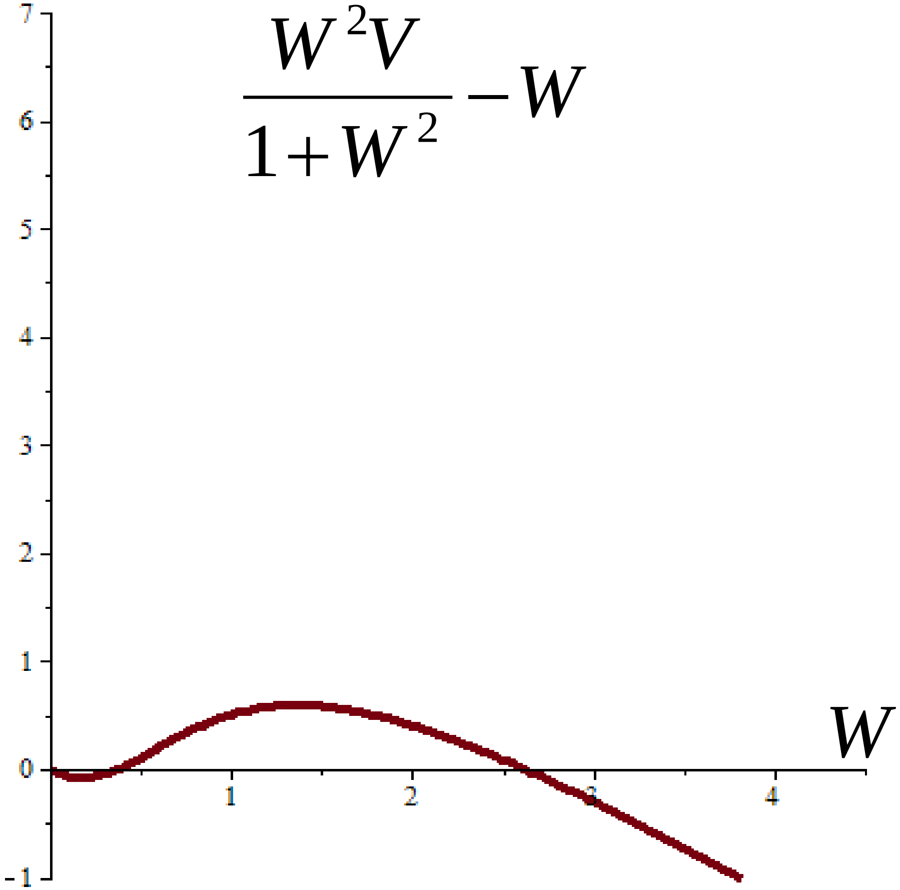

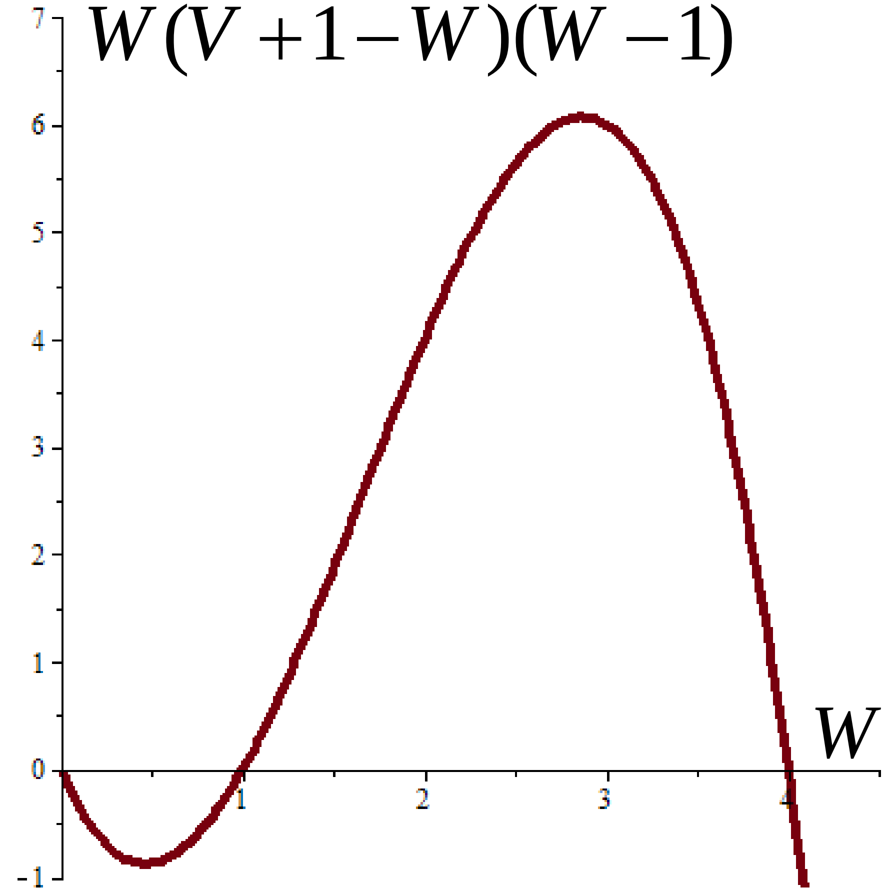

$\frac{W^2V}{1+W^2}-W$ ：双安定

$W(V+1-W)(W-1)$ ……数学的考察が行いやすい、多項式

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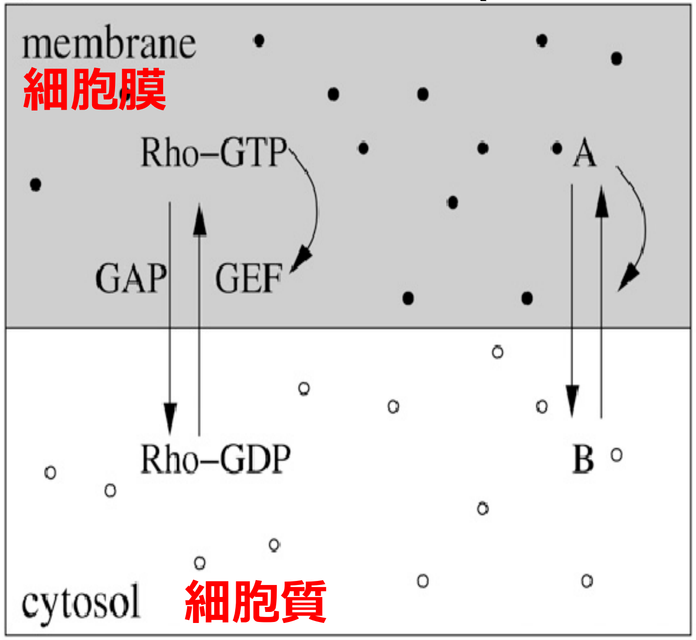

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数学的な解析のため
非線形項を置き換えた.

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細胞極性モデル

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質量保存

$$
\frac{d}{dt}\left\{\int_0^1 \big(W(x,t)+V(x,t)\big)\mathrm{dx}\right\} = 0
$$

$$
\therefore\ \int_0^1 \big(W(x,t)+V(x,t)\big)\mathrm{dx} = \int_0^1 \big(W_0(x)+V_0(x)\big)\mathrm{dx} = \mathrm{m}.
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$D \to \infty$の時間発展問題の極限方程式

$$
(TLP)\begin{cases}
\varepsilon \dfrac{\partial}{\partial t} W(x,t) = \varepsilon^2 \Delta W(x,t) + W(x,t)\left(\tilde{V}(t) + 1 - W(x,t)\right)\left(W(x,t) - 1\right), \\
\varepsilon \dfrac{d}{dt}\tilde{V}(t) = -\int_0^1 W(x,t)\left(\tilde{V}(t) + 1 - W(x,t)\right)\left(W(x,t) - 1\right)\mathrm{dx}, \\
W_x(0,\ t) = W_x(1,\ t) = 0, \\
W(x,0) = W_0(x),\ \tilde{V}(0) = \tilde{V}_0.
\end{cases}
$$

$$
(CP)\begin{cases}
\varepsilon W_t = \varepsilon^2 \Delta W + W(V+1-W)(W-1), & (1)\\
\varepsilon V_t = D\Delta V - W(V+1-W)(W-1), & (2)\\
W_x(0,t) = W_x(1,t) = V_x(0,t) = V_x(1,t) = 0, & (3)\\
W(x,0) = W_0(x),\ V(x,0) = V_0(x). & (4)
\end{cases}
$$

(2)式の両辺をxで積分する.

$$
\varepsilon \frac{\partial}{\partial t}\int_0^1 V(x,t)\,\mathrm{dx} = D\int_0^1 \Delta V(x,t)\,\mathrm{dx} - \int_0^1 W(x,t)\bigl(V(x,t)+1-W(x,t)\bigr)\bigl(W(x,t)-1\bigr)\mathrm{dx}
$$

$$
\varepsilon \frac{\partial}{\partial t}\int_0^1 V(x,t)\,\mathrm{dx} = \qquad - \int_0^1 W(x,t)\bigl(V(x,t)+1-W(x,t)\bigr)\bigl(W(x,t)-1\bigr)\mathrm{dx}
$$

$D\to\infty \quad V(x,t)\to\tilde{V}(t)$ とする.

$$
\varepsilon \frac{d}{dt}\tilde{V}(t) = -\int_0^1 W(x,t)\bigl(\tilde{V}(t)+1-W(x,t)\bigr)\bigl(W(x,t)-1\bigr)\mathrm{dx}
$$

このとき(1)式は

$$
\varepsilon \frac{\partial}{\partial t}W(x,t) = \varepsilon^2 \Delta W(x,t) + W(x,t)\bigl(\tilde{V}(t)+1-W(x,t)\bigr)\bigl(W(x,t)-1\bigr)
$$

以上より $D\to\infty$ の時間発展問題を得る.

時間発展問題の極限方程式（ $D \to \infty$ ）

$$
(TLP)\begin{cases}
\varepsilon \dfrac{\partial}{\partial t} W(x,t) = \varepsilon^2 \Delta W(x,t) + W(x,t)\left(\tilde{V}(t) + 1 - W(x,t)\right)\left(W(x,t) - 1\right), \\
\int_0^1 W(x,t)\,\mathrm{dx} + \tilde{V}(t) = \int_0^1 W_0(x)\,\mathrm{dx} + \tilde{V}_0 = \mathrm{m}, \\
W_x(0,\ t) = W_x(1,\ t) = 0, \\
W(x,0) = W_0(x),\ \ \tilde{V}(0) = \tilde{V}_0.
\end{cases}
$$

定常問題

$$
(SP)\begin{cases}
\varepsilon^2 \Delta W + W(V+1-W)(W-1) = 0, & x \in (0,1), \\
D\Delta V - W(V+1-W)(W-1) = 0, & x \in (0,1), \\
W_x(0) = W_x(1) = V_x(0) = V_x(1) = 0, & \\
W(x) > 0,\ V(x) > 0, & \\
\int_0^1 \big(W(x) + V(x)\big)\mathrm{dx} \left(= \int_0^1 \big(W_0(x) + V_0(x)\big)\mathrm{dx} \right) = m. &
\end{cases}
$$

定常極限方程式（ $D \to \infty$ ）

$$
(SLP)\begin{cases}
\varepsilon^2 \Delta W + W(\tilde{V}+1-W)(W-1) = 0, & x \in (0,1), \\
W_x(0) = W_x(1) = 0, & \\
W(x) > 0, & \tilde{V} > 0, \\
\int_0^1 W(x)\,\mathrm{dx} + \tilde{V} \left(= \int_0^1 W_0(x)\,\mathrm{dx} + \tilde{V}_0 \right) = m. &
\end{cases}
$$

定常問題

$$
(SP)\begin{cases}
\varepsilon^2 \Delta W + W(V+1-W)(W-1) = 0, & x \in (0,1), \\
D\Delta V - W(V+1-W)(W-1) = 0, & x \in (0,1), \\
W_x(0) = W_x(1) = V_x(0) = V_x(1) = 0, \\
W(x) > 0,\ V(x) > 0, \\
\int_0^1 \bigl(W(x)+V(x)\bigr)\mathrm{dx} \left(= \int_0^1 \bigl(W_0(x)+V_0(x)\bigr)\mathrm{dx} \right) = m.
\end{cases}
$$

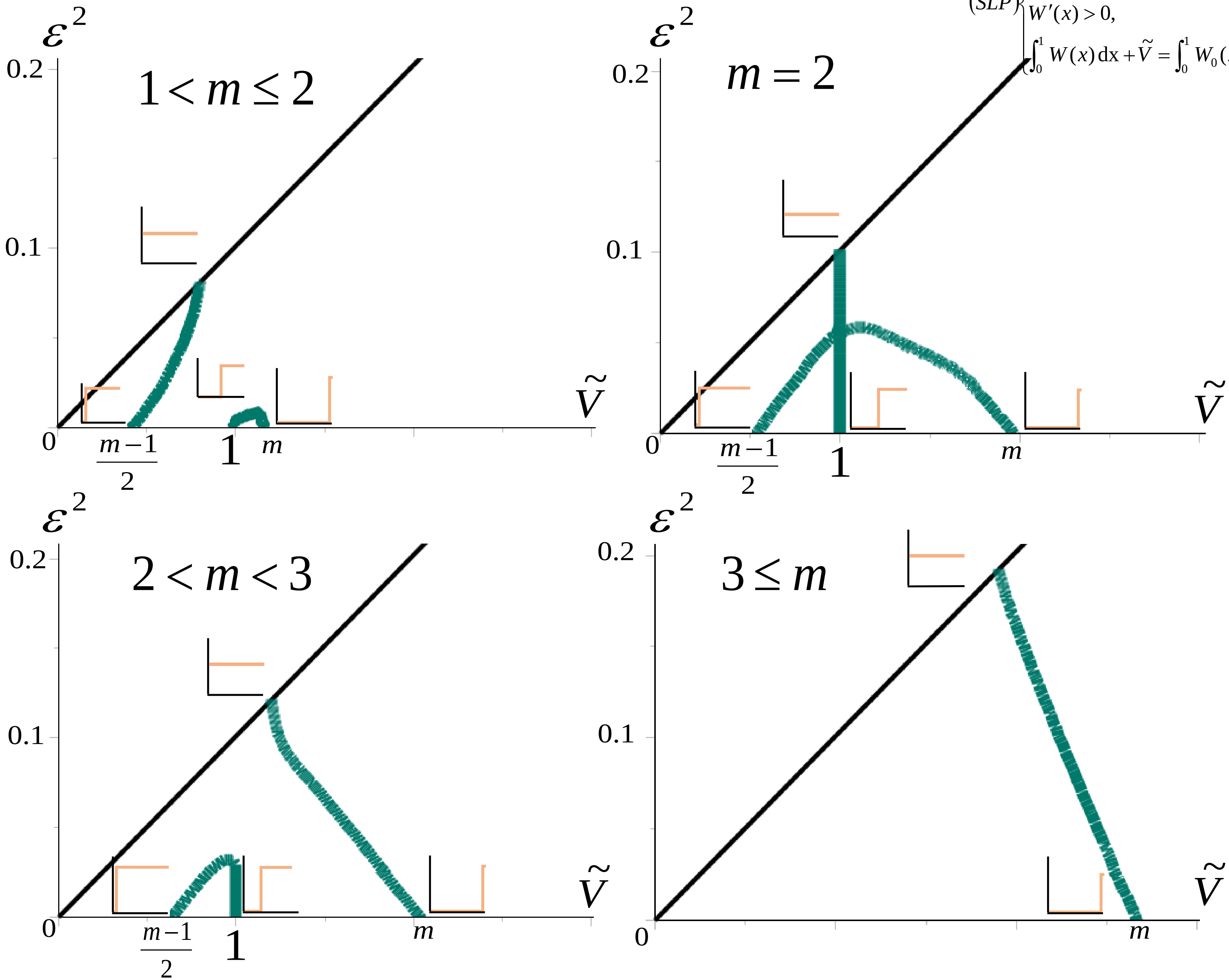
定常極限方程式（ $D \to \infty$ ）

$$
(SLP)\begin{cases}
\varepsilon^2 \Delta W + W(\tilde{V}+1-W)(W-1) = 0, & x \in (0,1), \\
W_x(0) = W_x(1) = 0, \\
W(x) > 0, \quad W'(x) > 0, \quad \tilde{V} > 0, \\
\int_0^1 W(x)\,\mathrm{dx} + \tilde{V} \left(= \int_0^1 W_0(x)\,\mathrm{dx} + \tilde{V}_0 \right) = m.
\end{cases}
$$

簡単のため単調増加
なものを考える

(SLP)の分岐曲線

$$(SLP)\begin{cases} \varepsilon^2 \Delta W + W(\tilde{V} + 1 - W)(W-1) = 0, \\ W_x(0) = W_x(1) = 0, \\ W'(x) > 0, \\ \int_0^1 W(x)\,\mathrm{dx} + \tilde{V} = \int_0^1 W_0(x)\,\mathrm{dx} + \tilde{V}_0 = m. \end{cases}$$



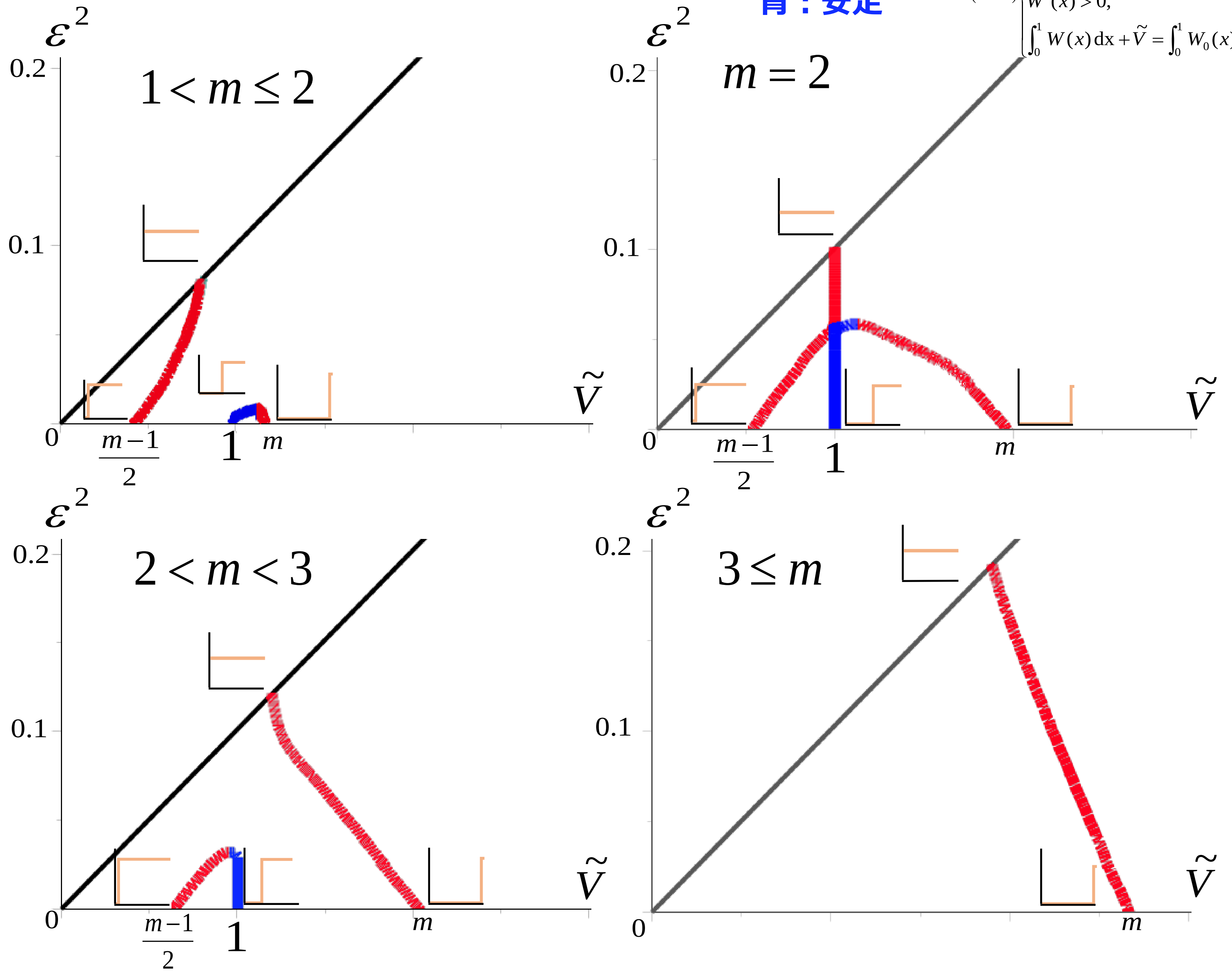
(SLP)の安定性

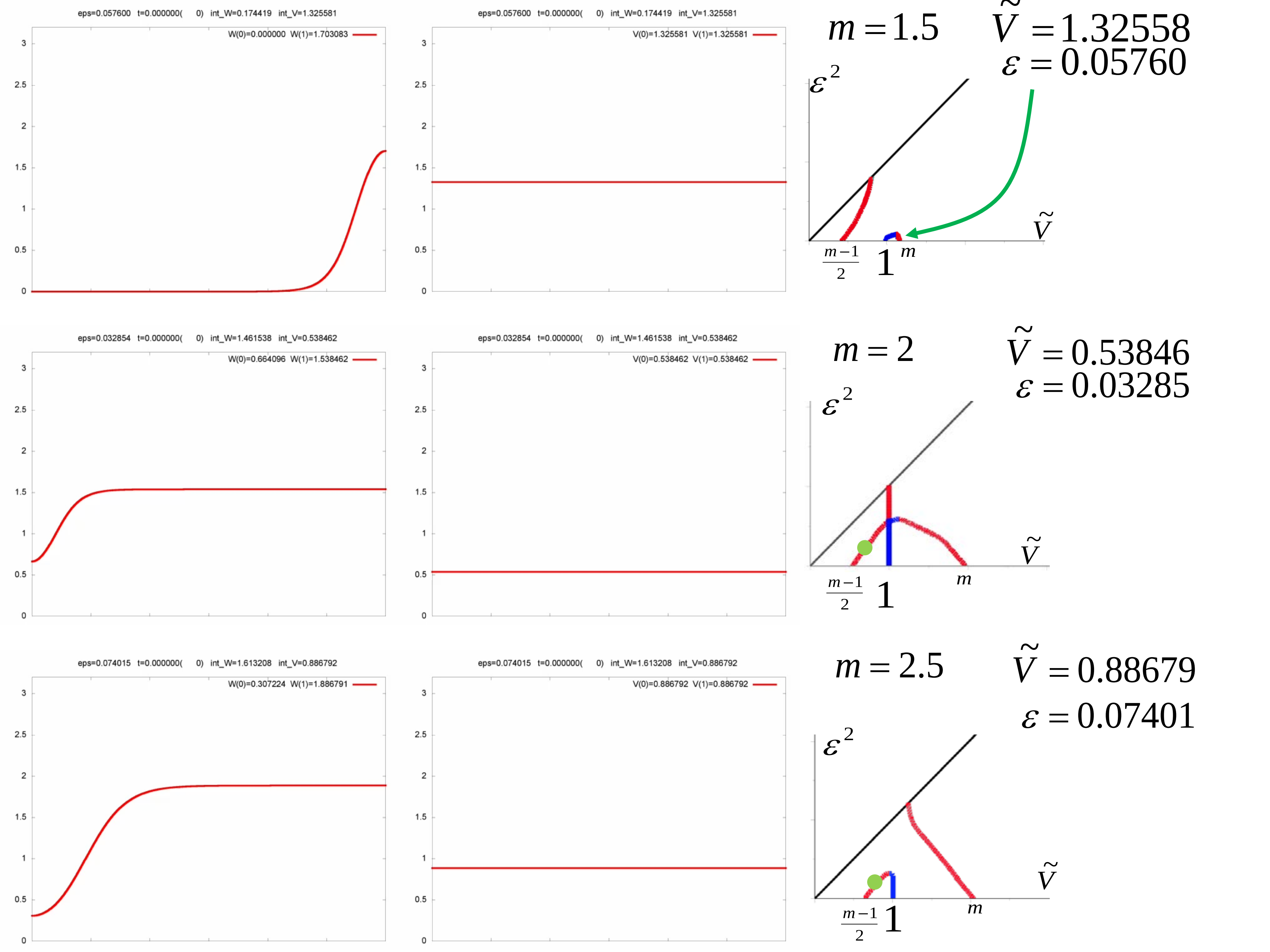
数値計算結果

赤：不安定

青：安定

$$(SLP)\begin{cases} \varepsilon^2 \Delta W + W(\tilde{V} + 1 - W)(W - 1) = 0, \\ W_x(0) = W_x(1) = 0, \\ W'(x) > 0, \\ \int_0^1 W(x)\,\mathrm{d}x + \tilde{V} = \int_0^1 W_0(x)\,\mathrm{d}x + \tilde{V}_0 = m. \end{cases}$$



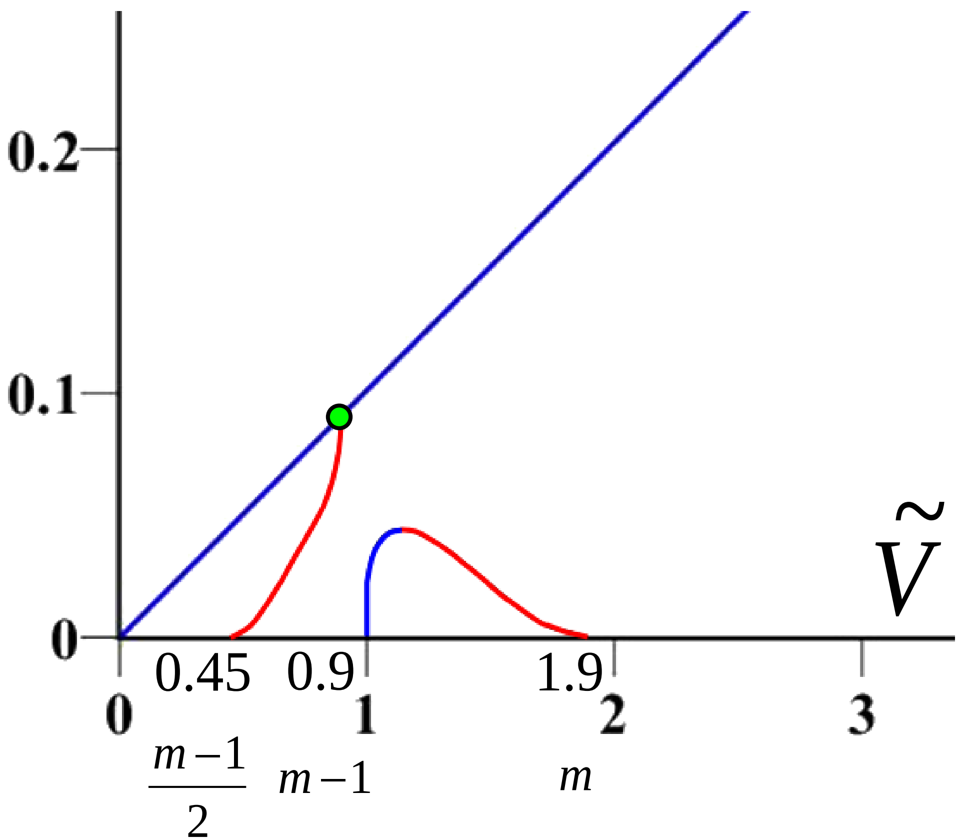

eps=0.057600 t=0.000000(0) int_W=0.174419 int_V=1.325581
W(0)=0.000000 W(1)=1.703083
eps=0.057600 t=0.000000(0) int_W=0.174419 int_V=1.325581
V(0)=1.325581 V(1)=1.325581
$m = 1.5$
$\tilde{V} = 1.32558$
$\varepsilon = 0.05760$
ε^2
$\tilde{V}$
$\frac{m-1}{2}$ 1 m
eps=0.032854 t=0.000000(0) int_W=1.461538 int_V=0.538462
W(0)=0.664096 W(1)=1.538462
eps=0.032854 t=0.000000(0) int_W=1.461538 int_V=0.538462
V(0)=0.538462 V(1)=0.538462
$m = 2$
$\tilde{V} = 0.53846$
$\varepsilon = 0.03285$
ε^2
$\tilde{V}$
$\frac{m-1}{2}$ 1 m
eps=0.074015 t=0.000000(0) int_W=1.613208 int_V=0.886792
W(0)=0.307224 W(1)=1.886791
eps=0.074015 t=0.000000(0) int_W=1.613208 int_V=0.886792
V(0)=0.886792 V(1)=0.886792
$m = 2.5$
$\tilde{V} = 0.88679$
$\varepsilon = 0.07401$
ε^2
$\tilde{V}$
$\frac{m-1}{2}$ 1 m

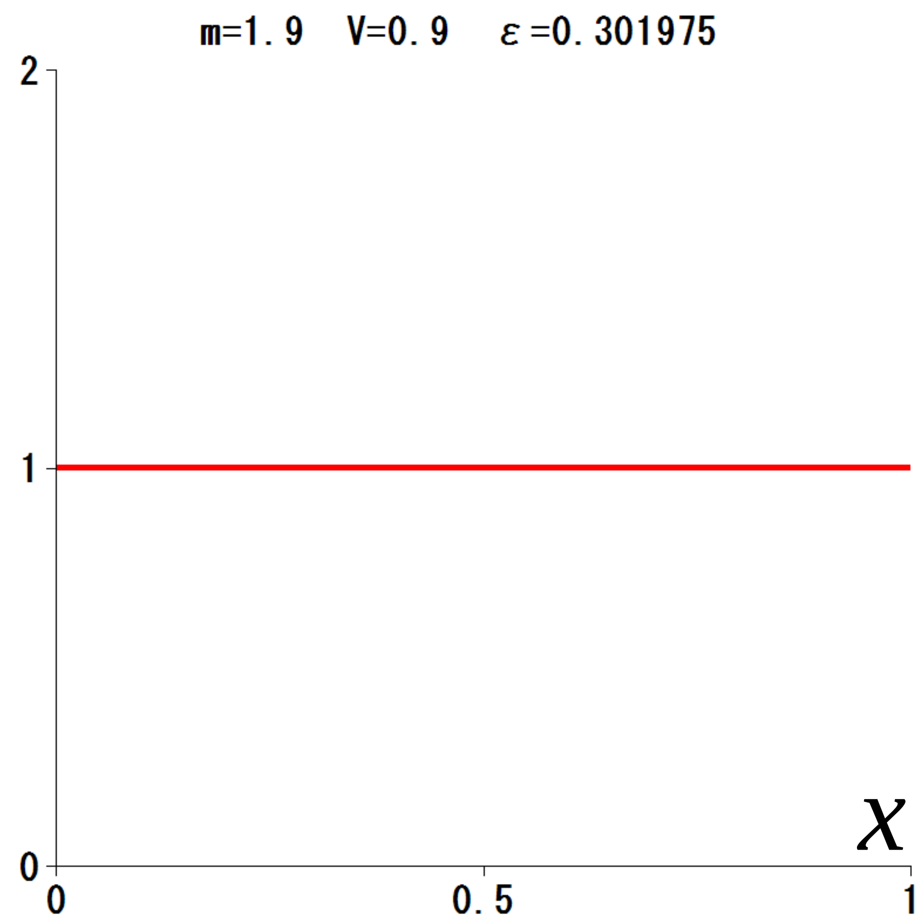
$$(SLP)\begin{cases} \varepsilon^2 W_{xx} + W(W-1)(\tilde{V}+1-W) = 0, \\ W_x(0) = W_x(1) = 0, \\ W'(x) > 0, \\ \int_0^1 W(x)\,\mathrm{dx} + \tilde{V} = m. \end{cases}$$

不安定

$\tilde{V} \to 0.45,\ \varepsilon^2 \to 0$ の形状

$$m = 1.9$$





$$(SLP)\begin{cases}\varepsilon^2 W_{xx} + W(W-1)(\tilde{V}+1-W) = 0, \\ W_x(0) = W_x(1) = 0, \\ W'(x) > 0, \\ \int_0^1 W(x)\,\mathrm{d}x + \tilde{V} = m.\end{cases}$$

不安定

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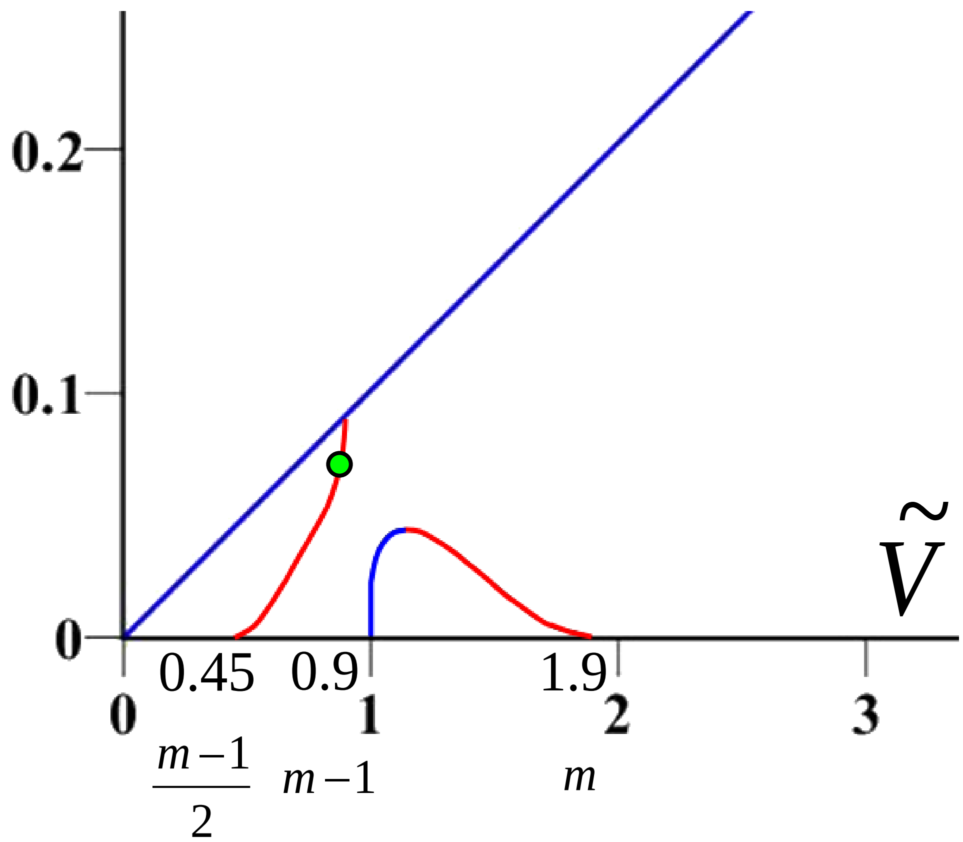
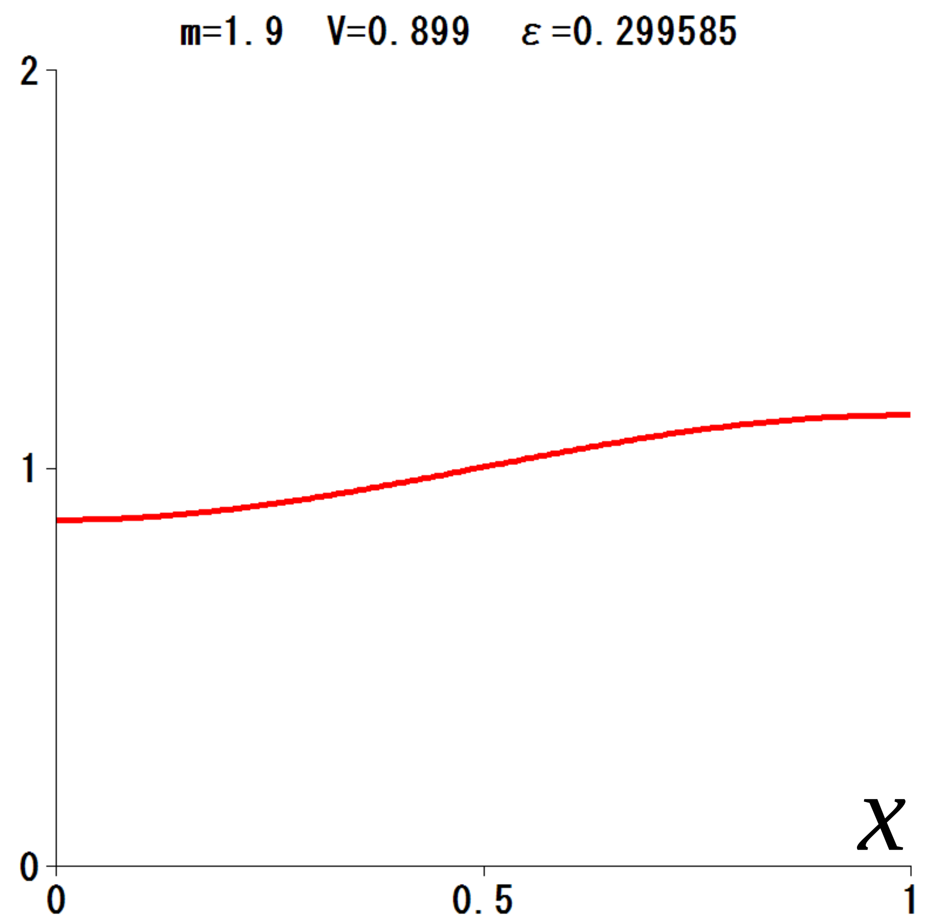
$m = 1.9$





$$(SLP)\begin{cases}\varepsilon^2 W_{xx} + W(W-1)(\tilde{V}+1-W) = 0, \\ W_x(0) = W_x(1) = 0, \\ W'(x) > 0, \\ \int_0^1 W(x)\,\mathrm{d}x + \tilde{V} = m.\end{cases}$$

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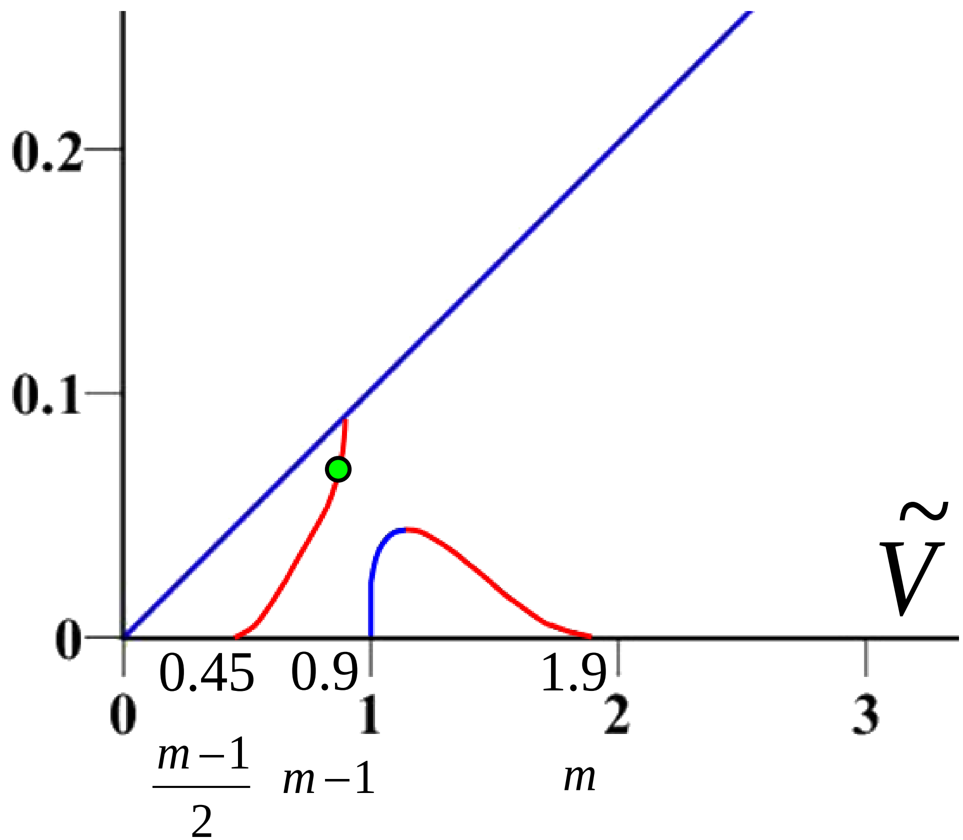

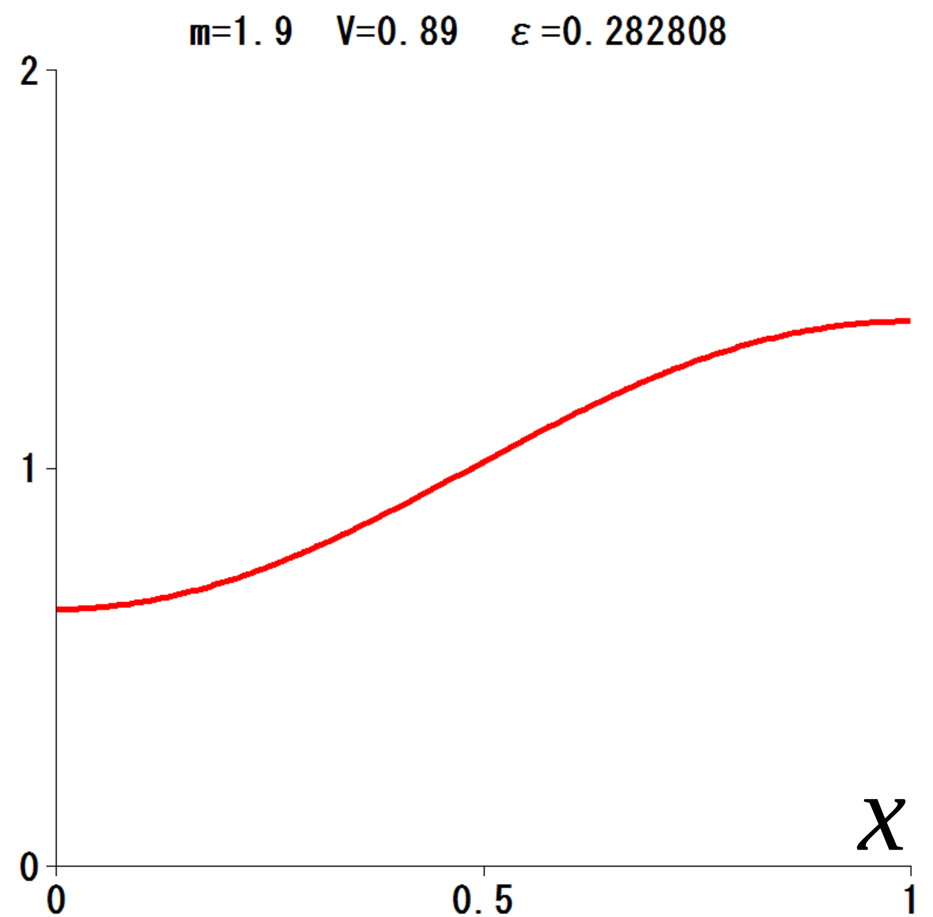

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不安定

$\tilde{V} \to 0.45,\ \varepsilon^2 \to 0$ の形状

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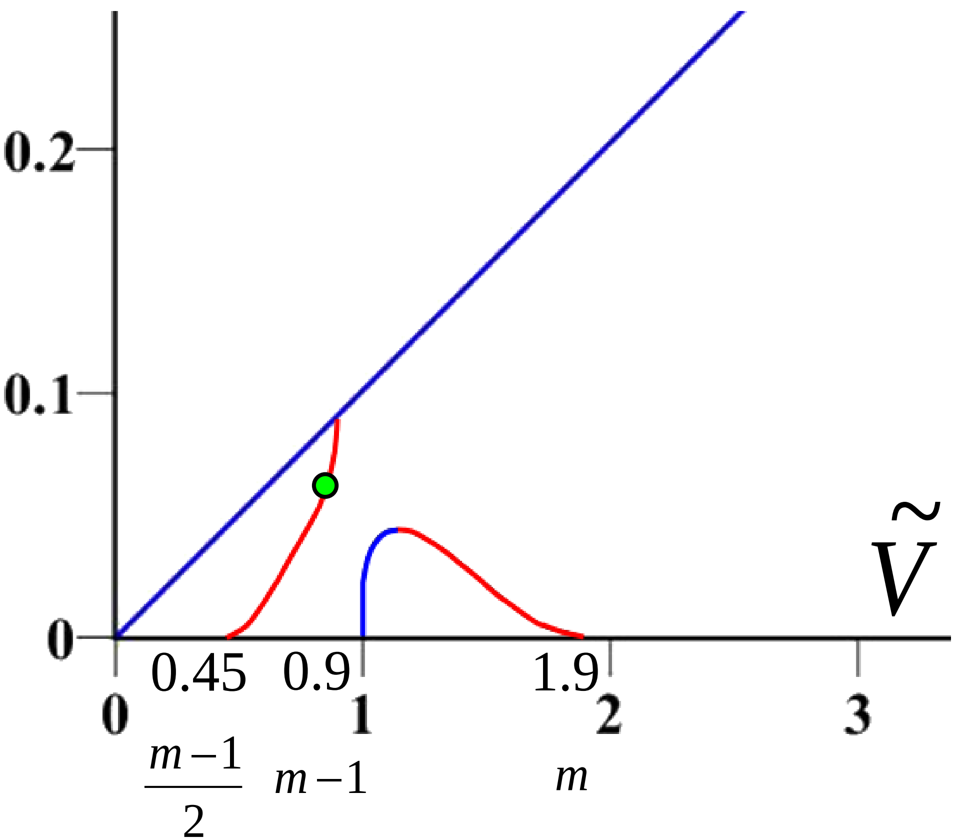

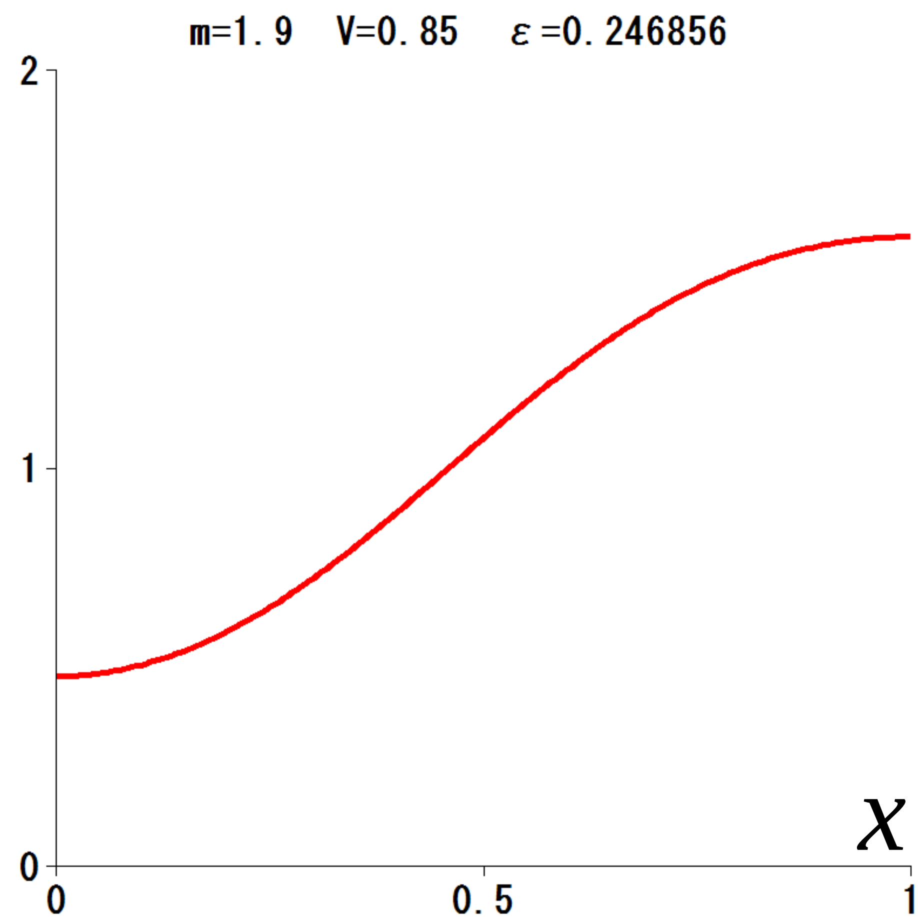

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不安定

$\tilde{V} \to 0.45,\ \varepsilon^2 \to 0$ の形状

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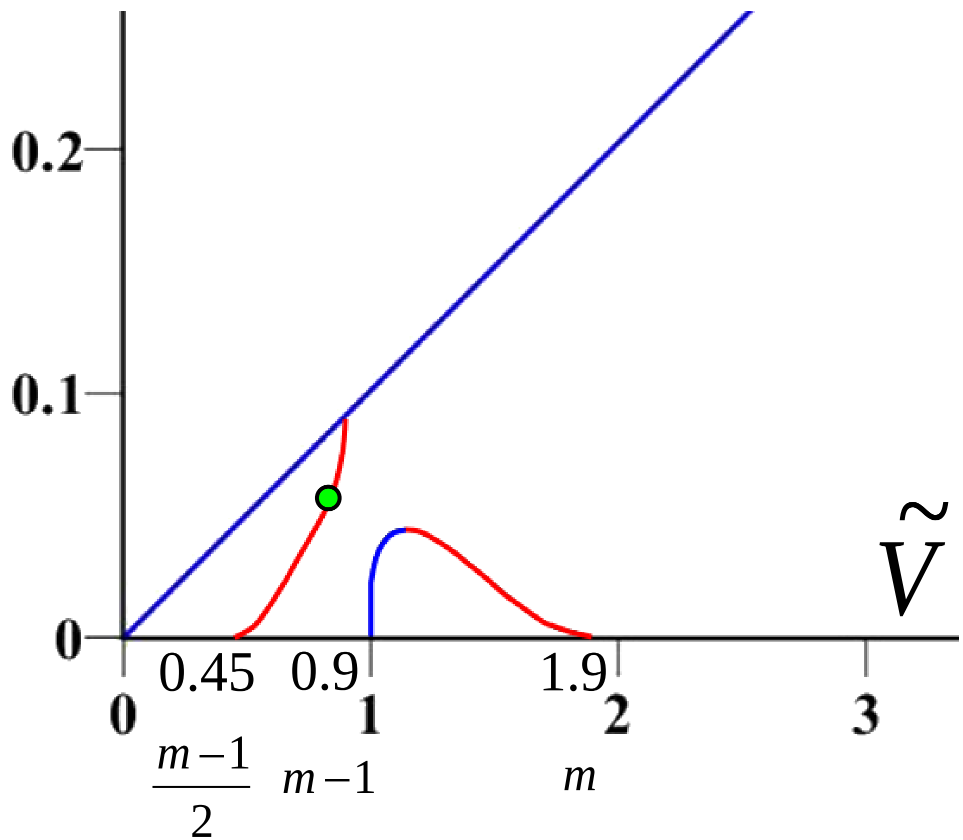

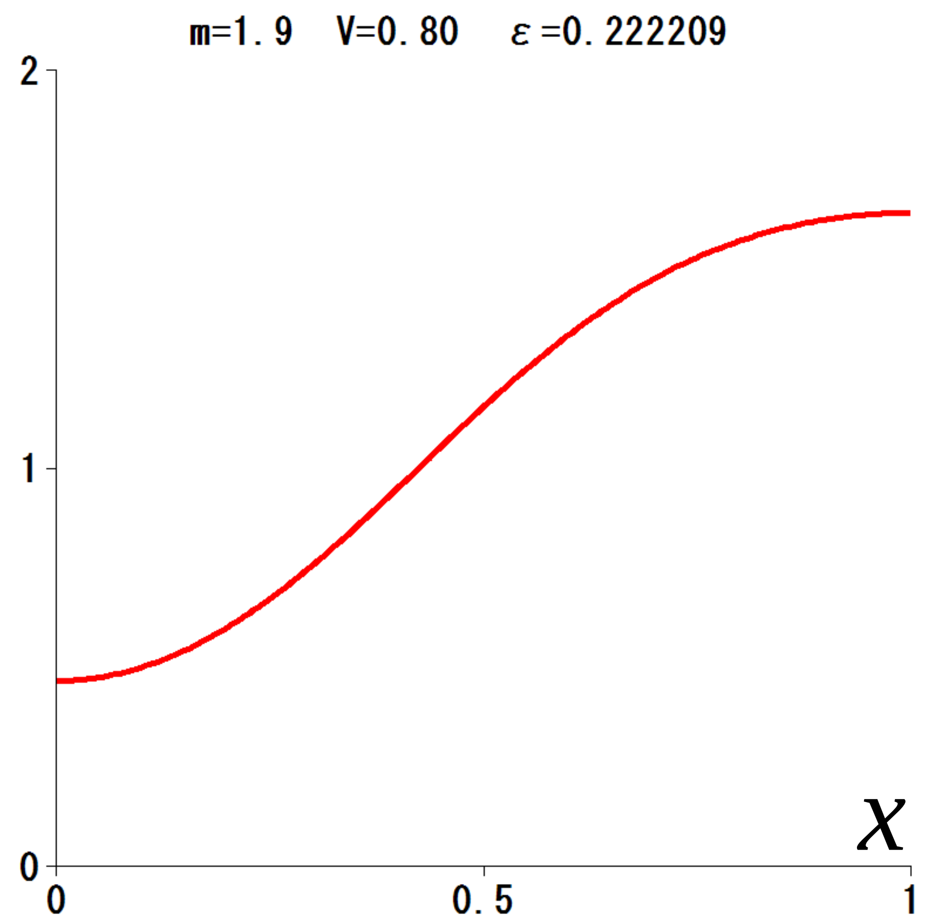

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不安定

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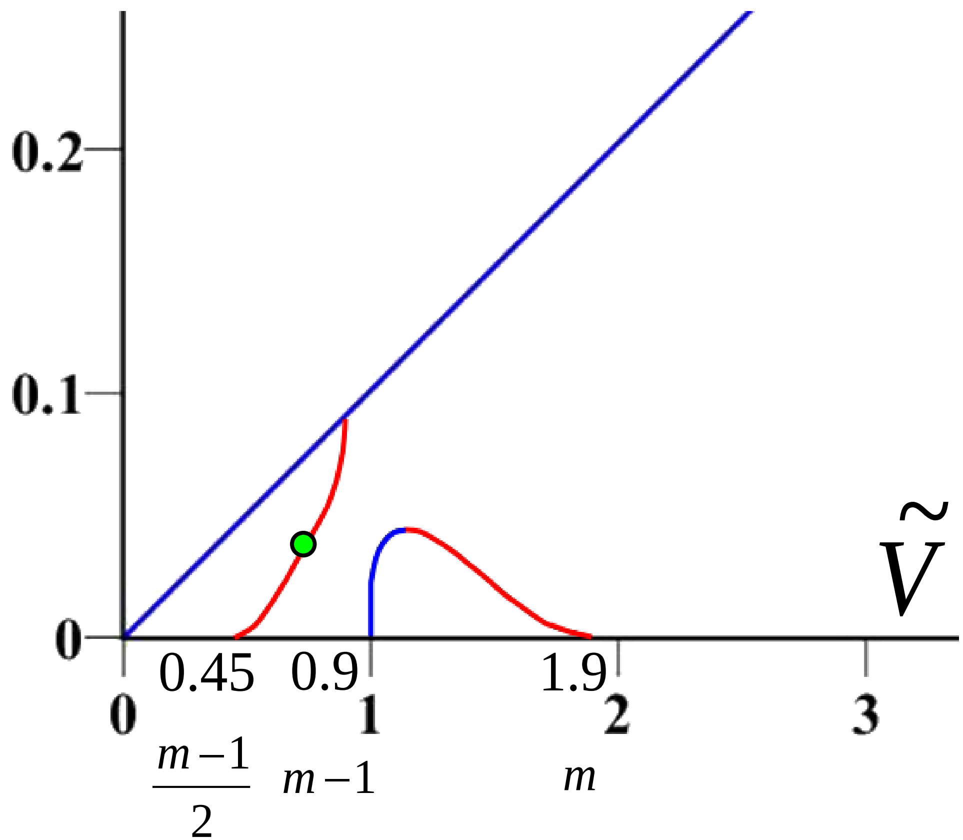

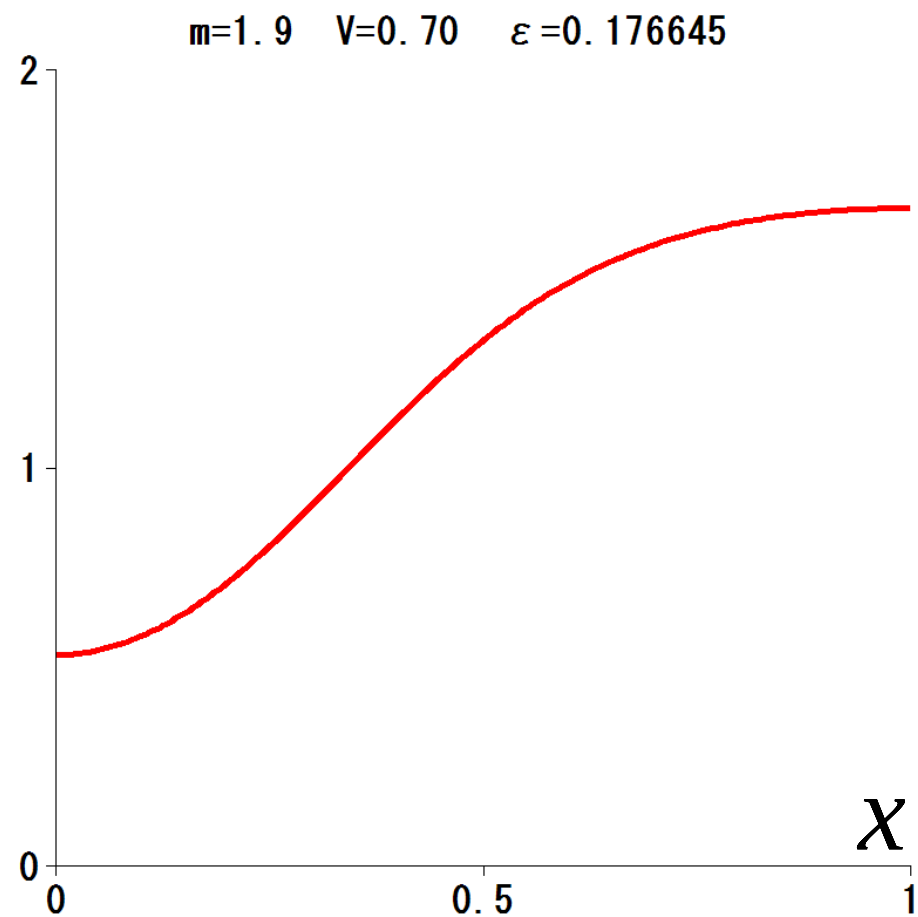

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不安定

$\tilde{V} \to 0.45,\ \varepsilon^2 \to 0$ の形状

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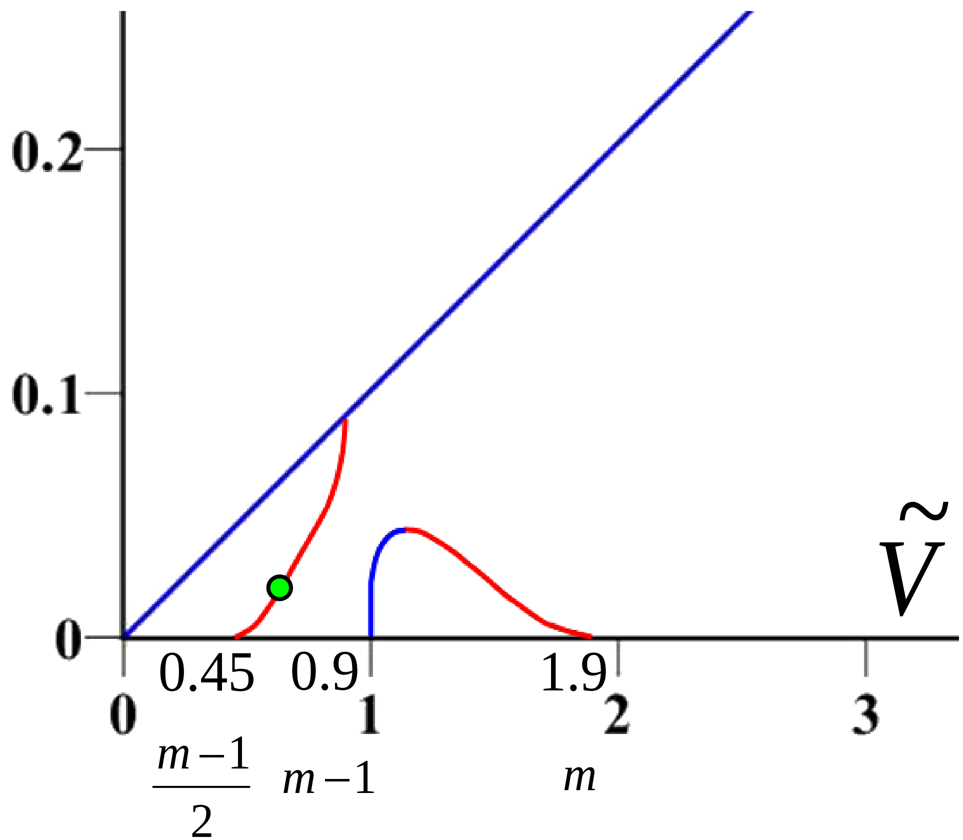

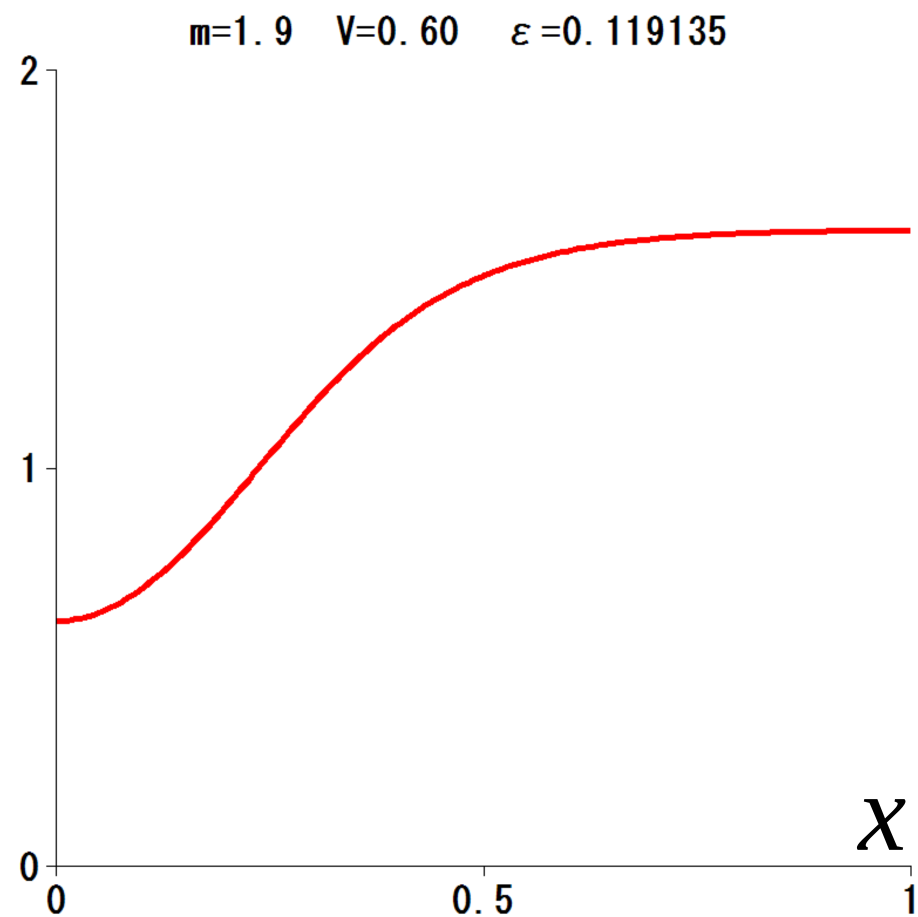

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不安定

$\tilde{V} \to 0.45,\ \varepsilon^2 \to 0$ の形状

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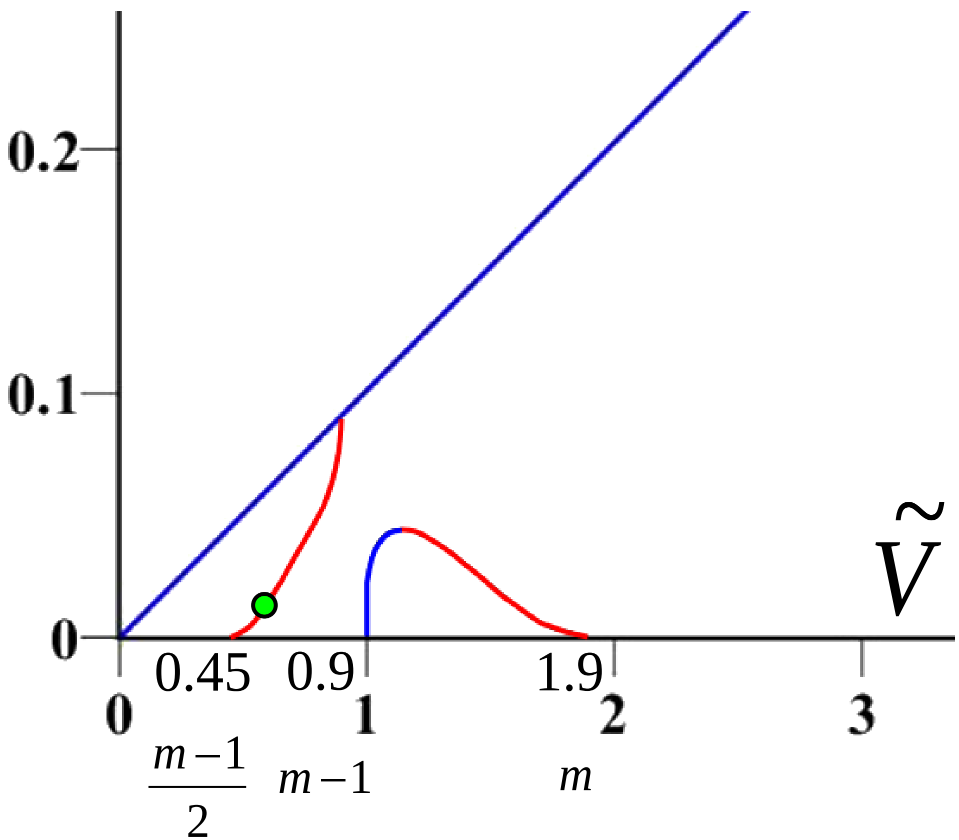

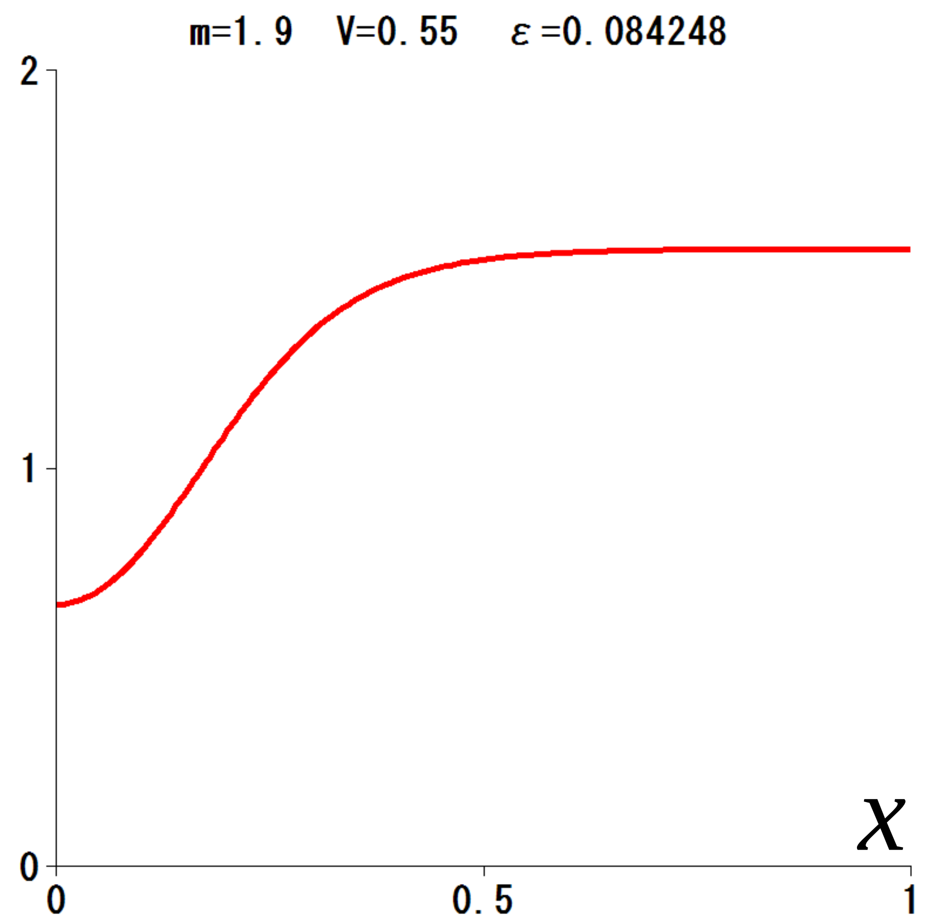

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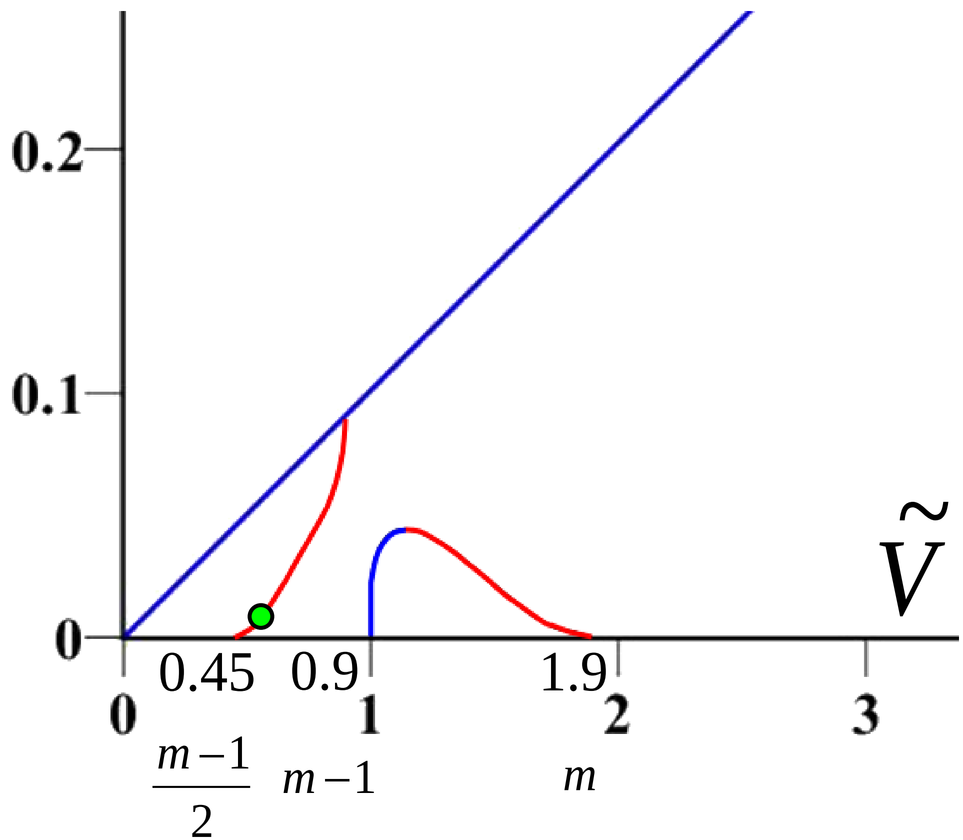
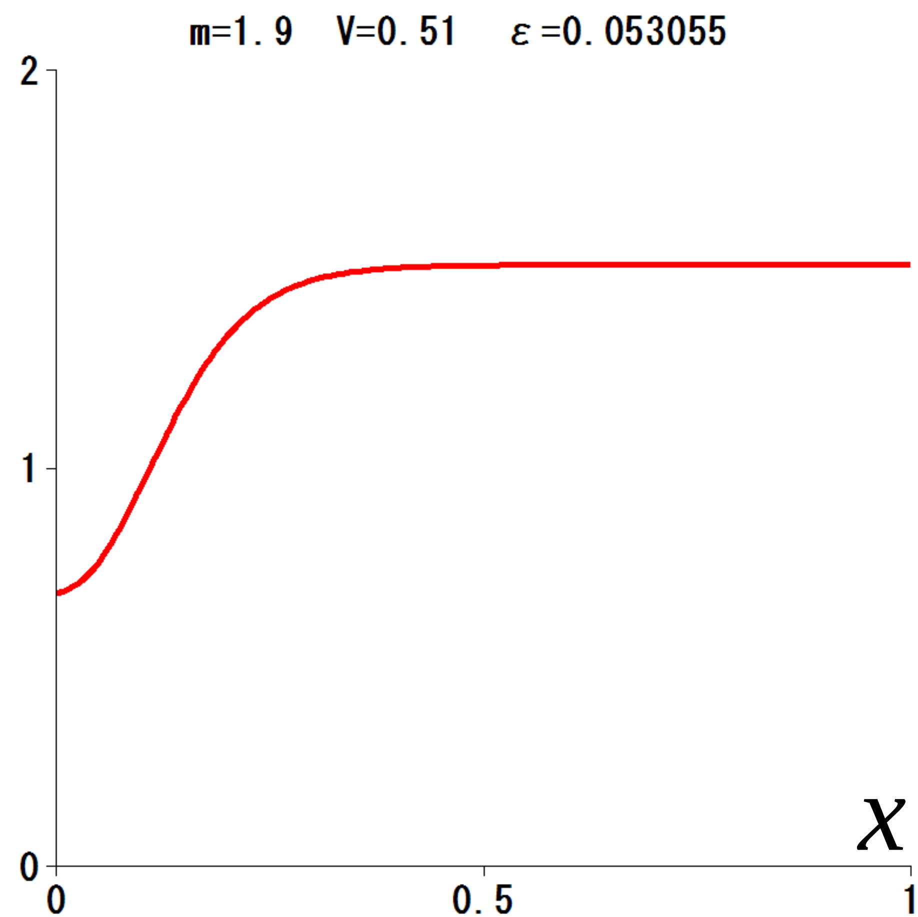
不安定

$\tilde{V} \to 0.45,\ \varepsilon^2 \to 0$ の形状

$m = 1.9$





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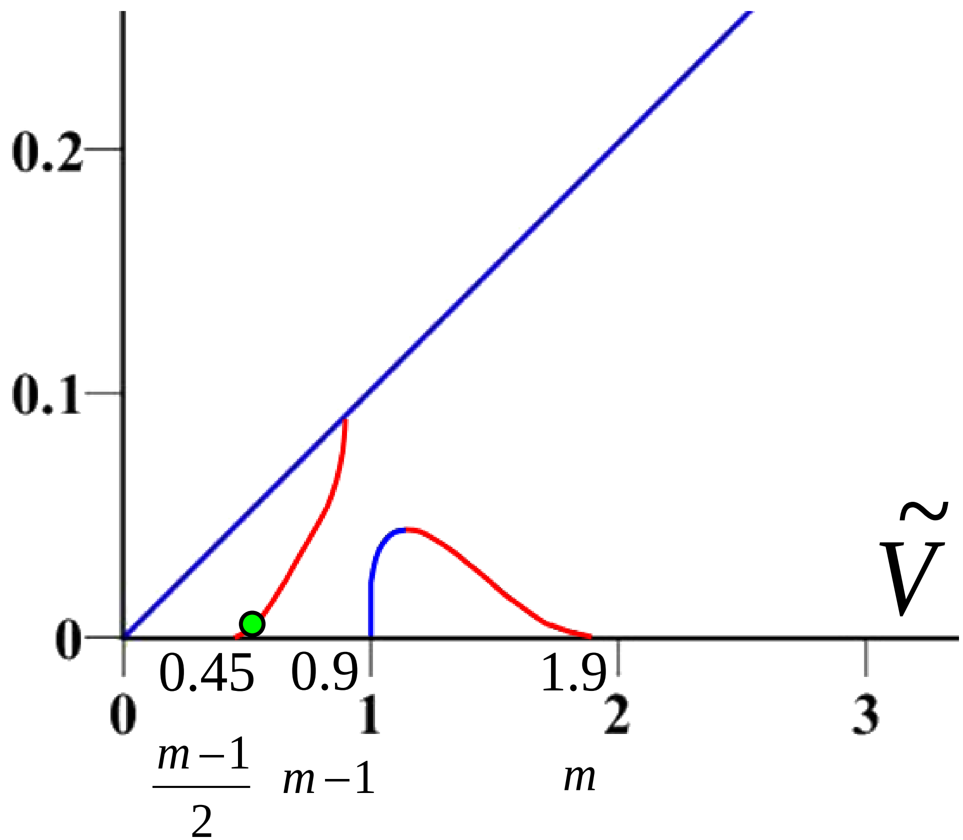
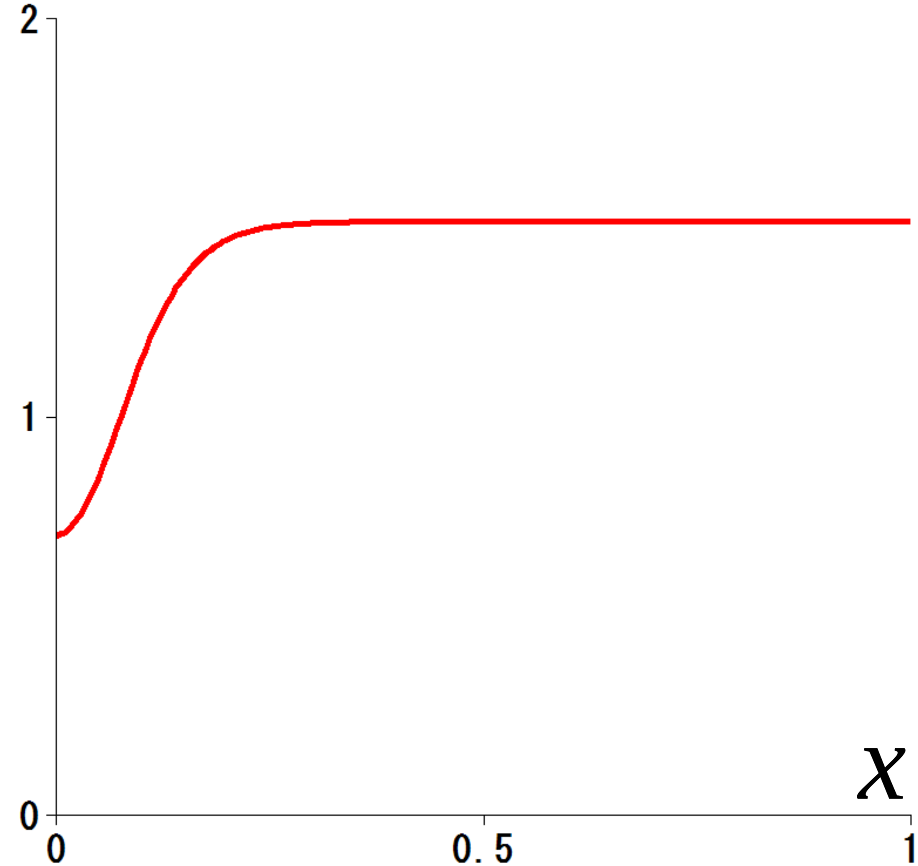
不安定

$\tilde{V} \to 0.45,\ \varepsilon^2 \to 0$ の形状

$$m = 1.9$$





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不安定

$\tilde{V} \to 0.45,\ \varepsilon^2 \to 0$ の形状

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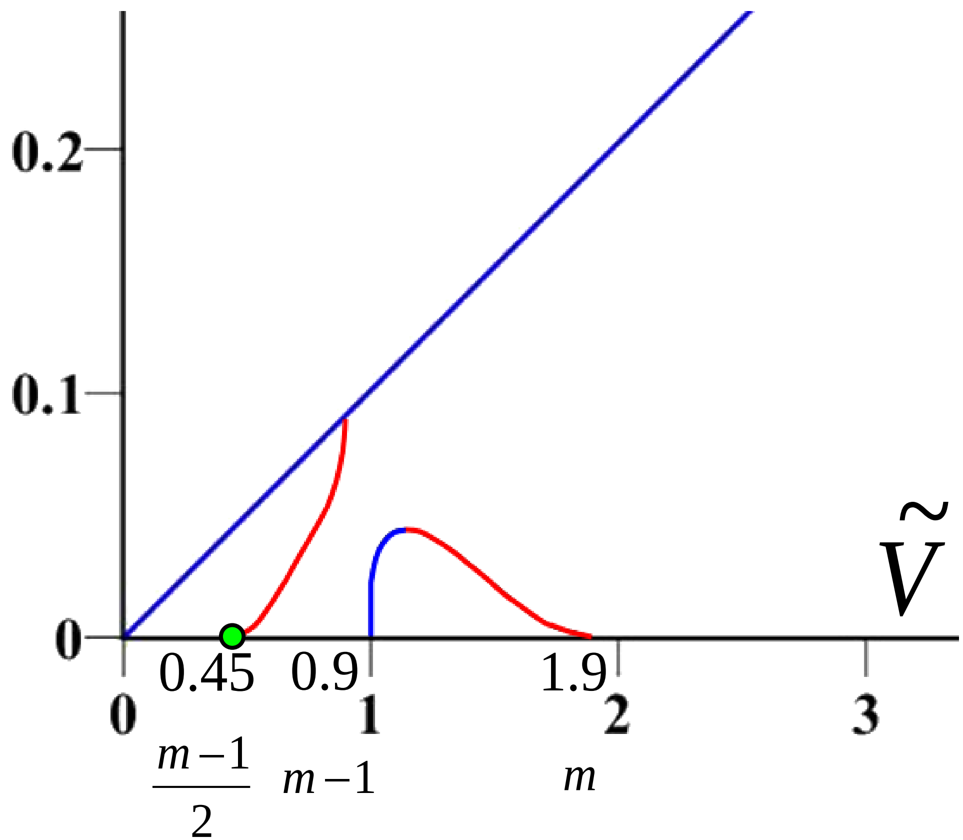

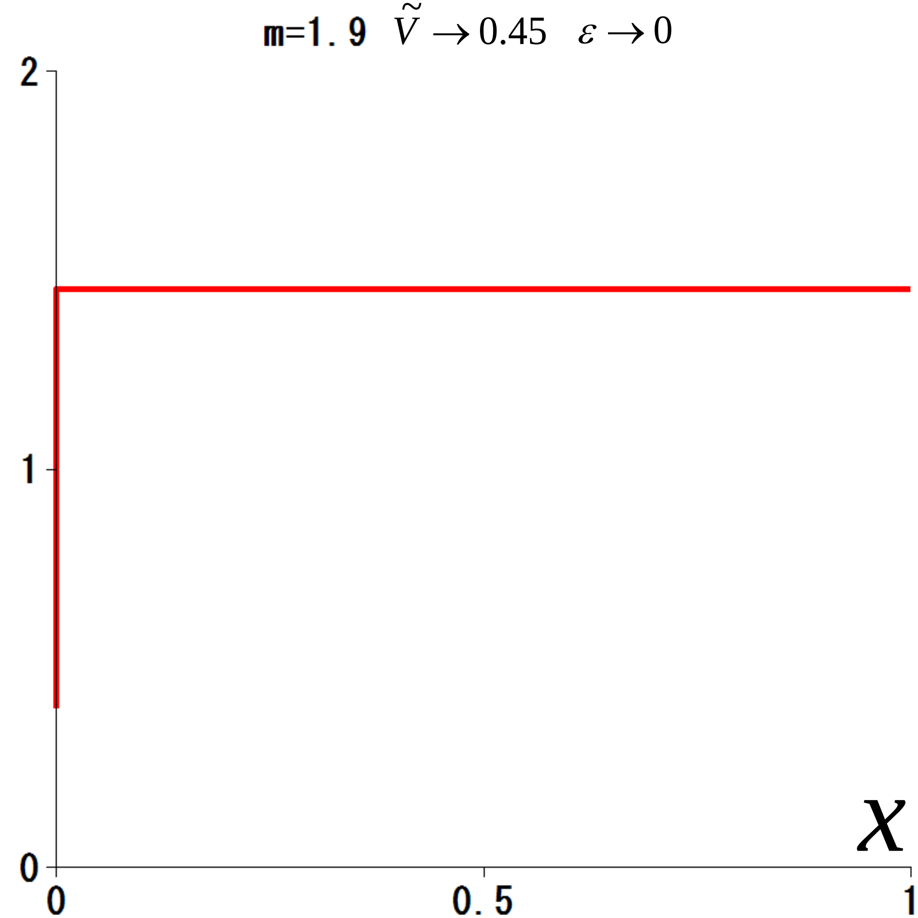

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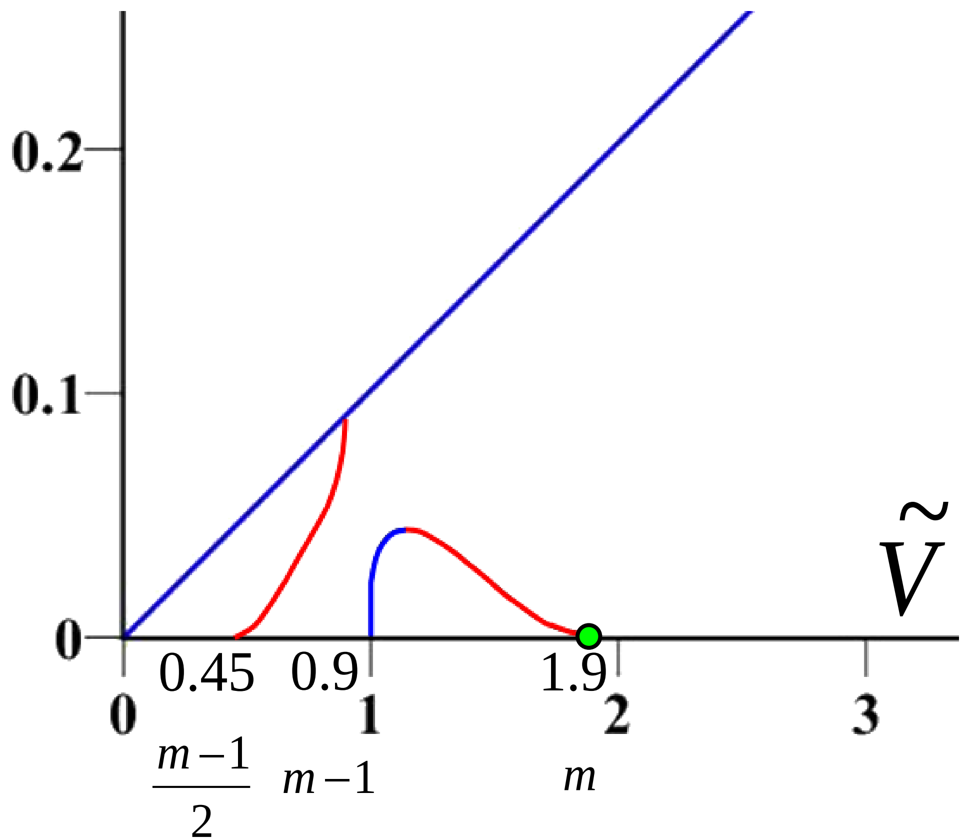
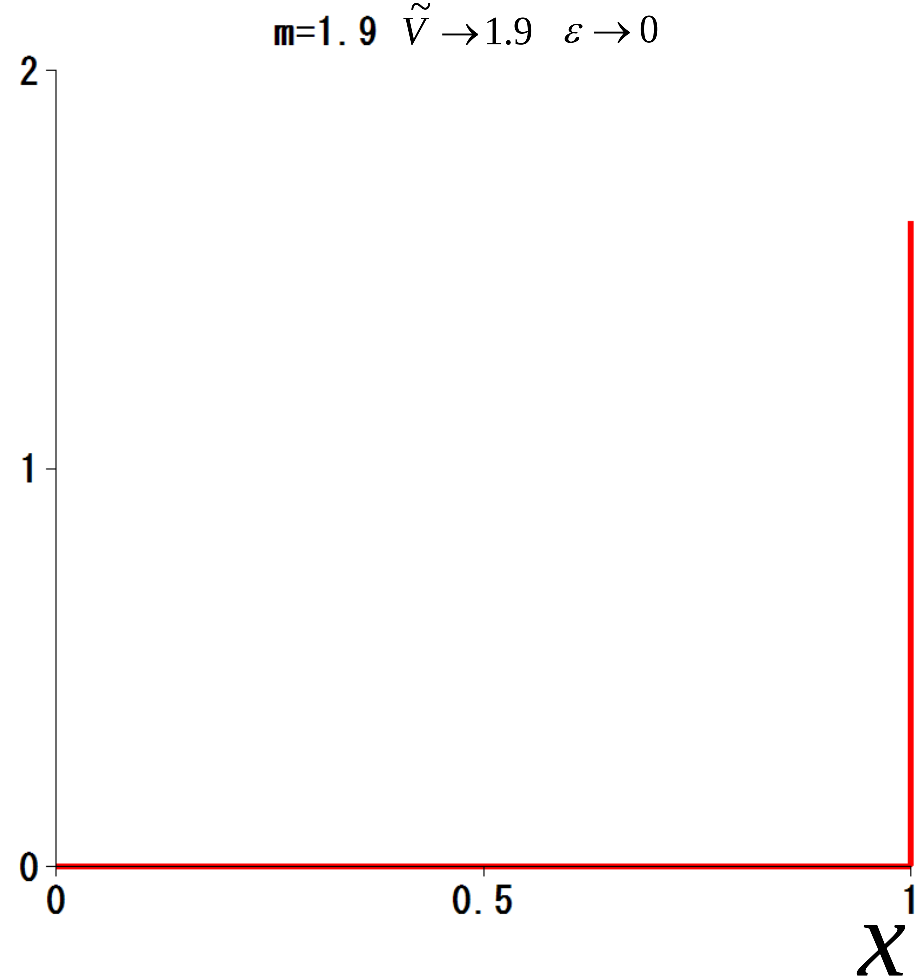
不安定

$\tilde{V} \to 1.9, \varepsilon^2 \to 0$ の形状

$$m = 1.9$$





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不安定

$\tilde{V} \to 1.9, \varepsilon^2 \to 0$ の形状

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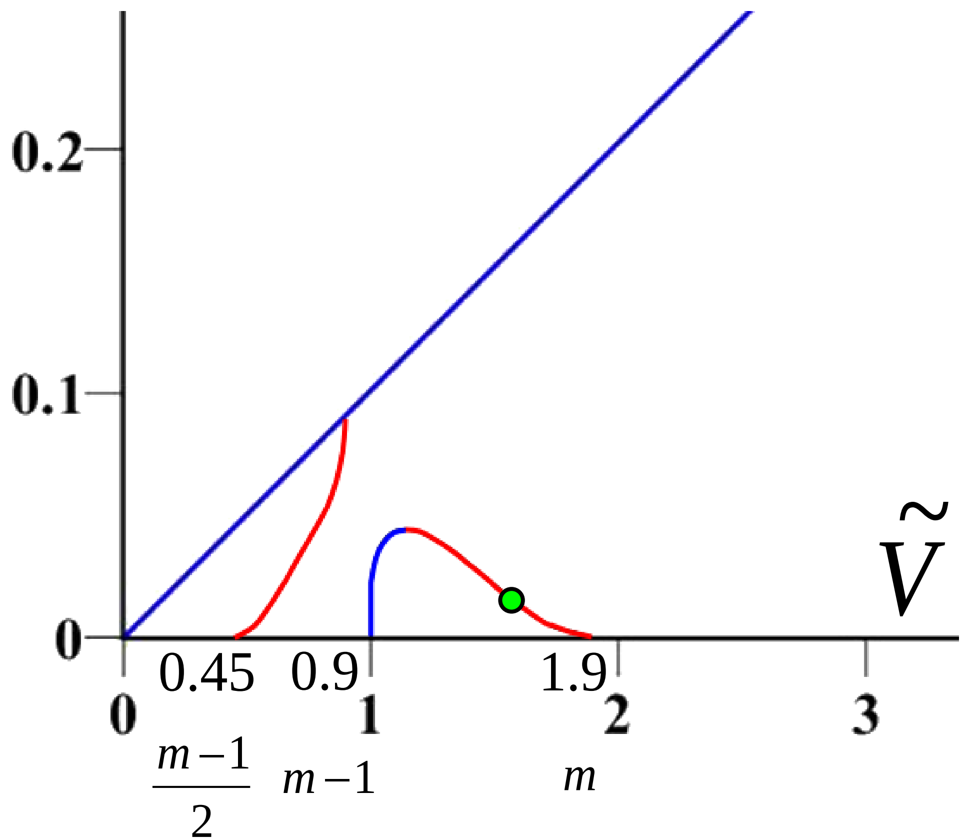

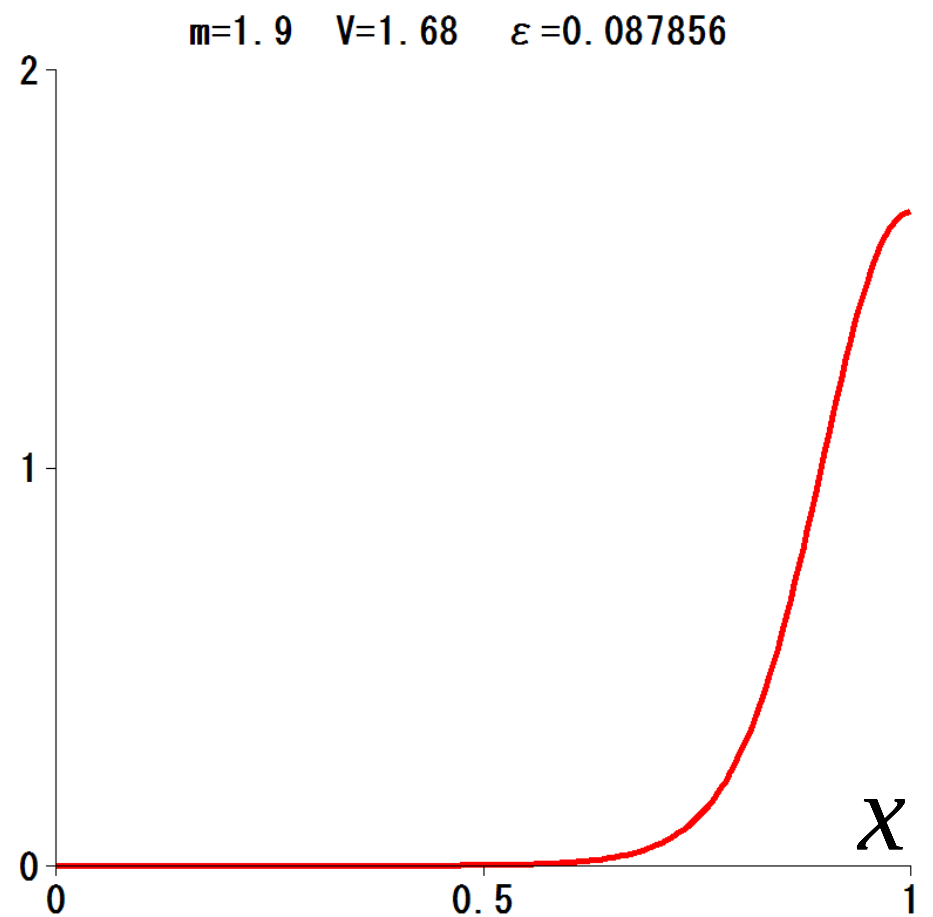

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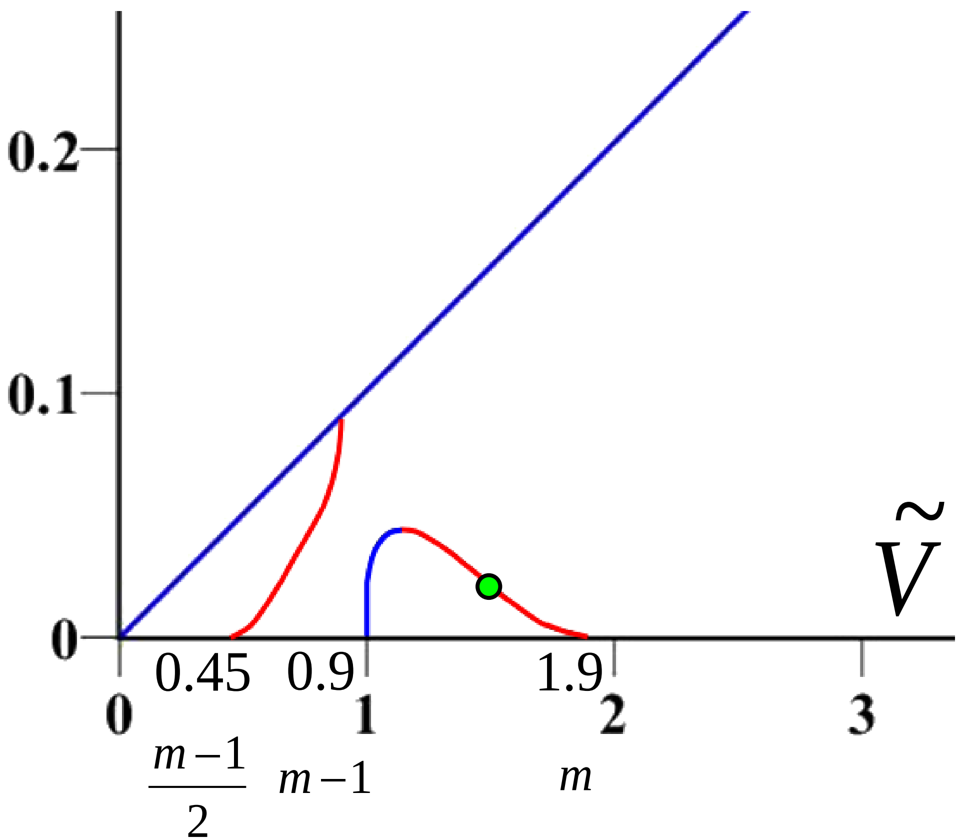
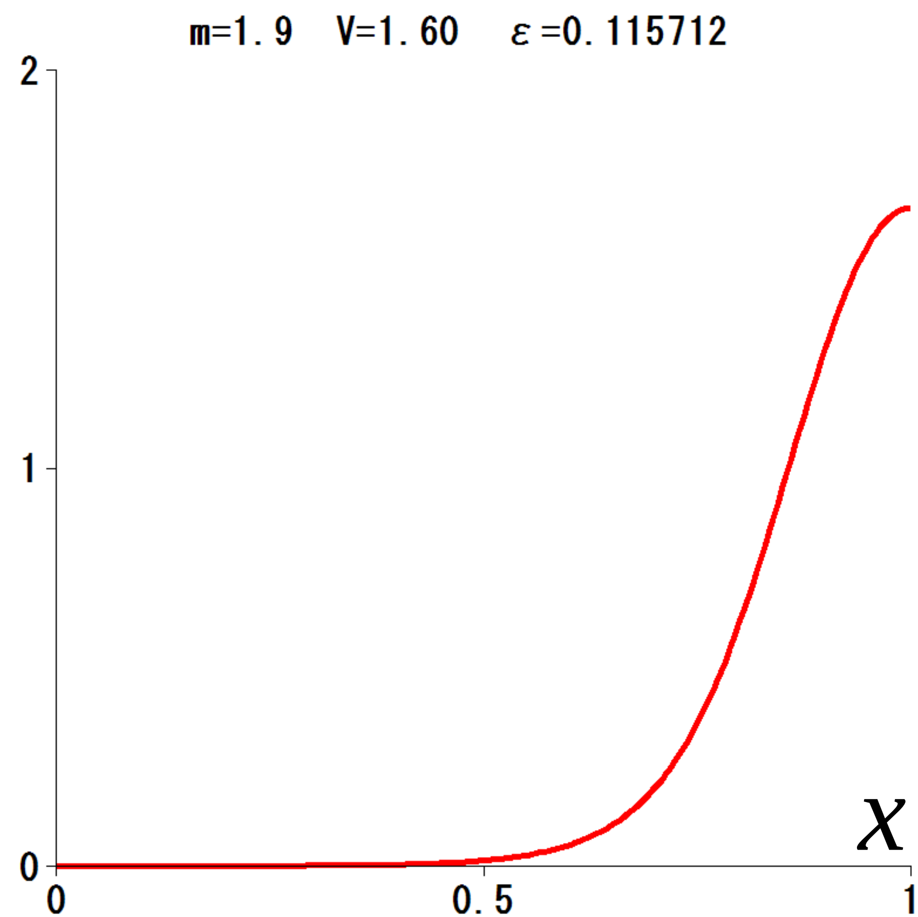
不安定

$\tilde{V} \to 1.9, \varepsilon^2 \to 0$ の形状

$$m = 1.9$$





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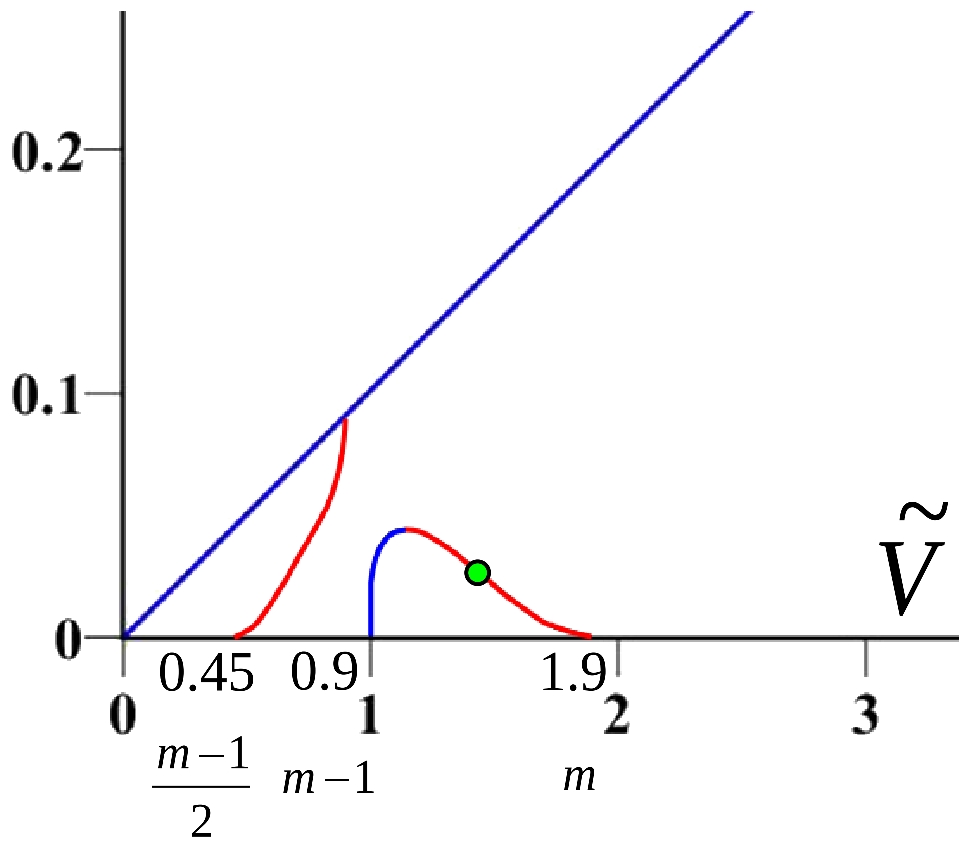
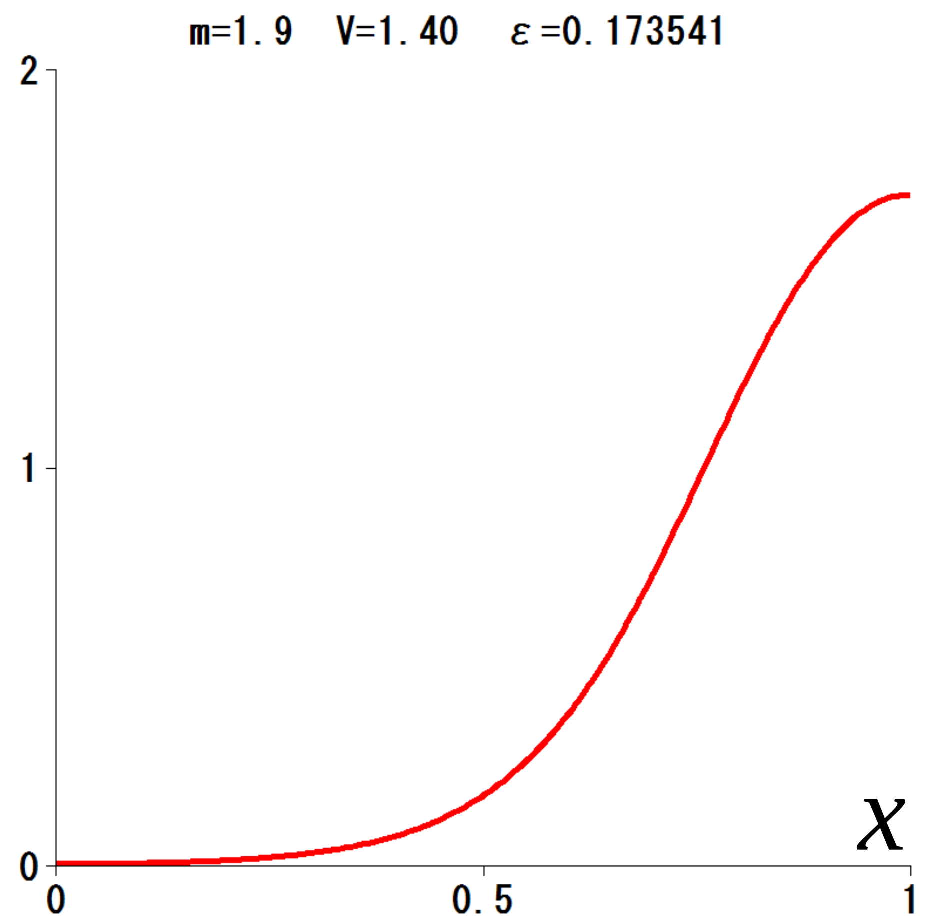
不安定

$\tilde{V} \to 1.9, \varepsilon^2 \to 0$ の形状

$$m = 1.9$$





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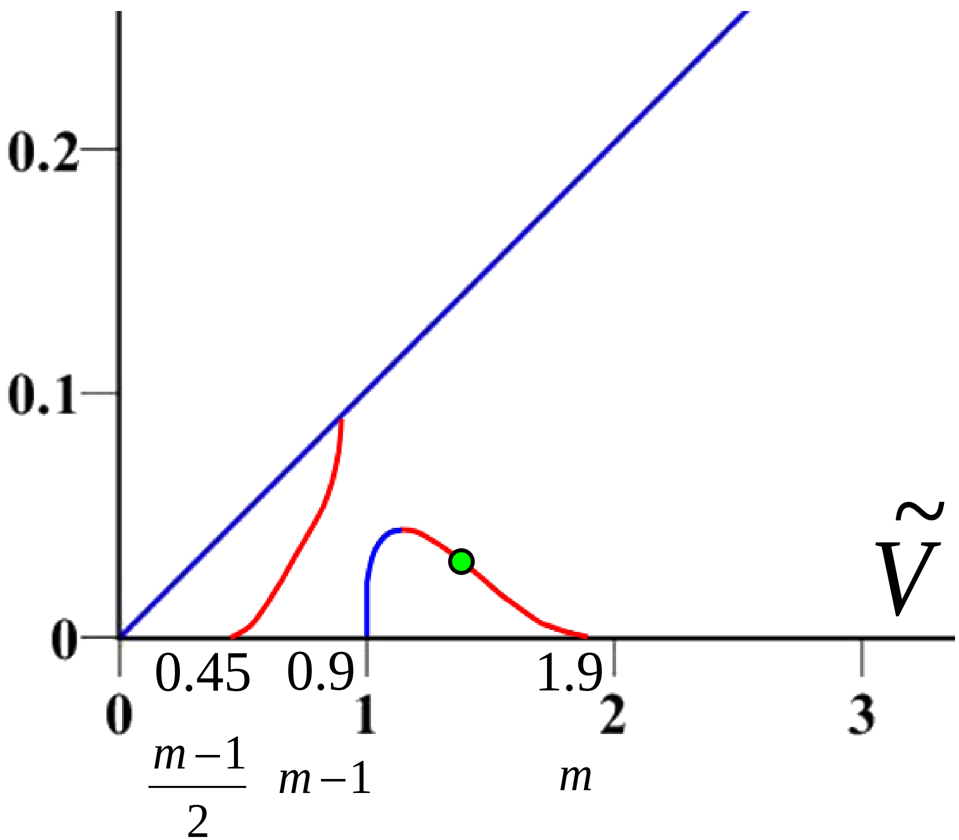
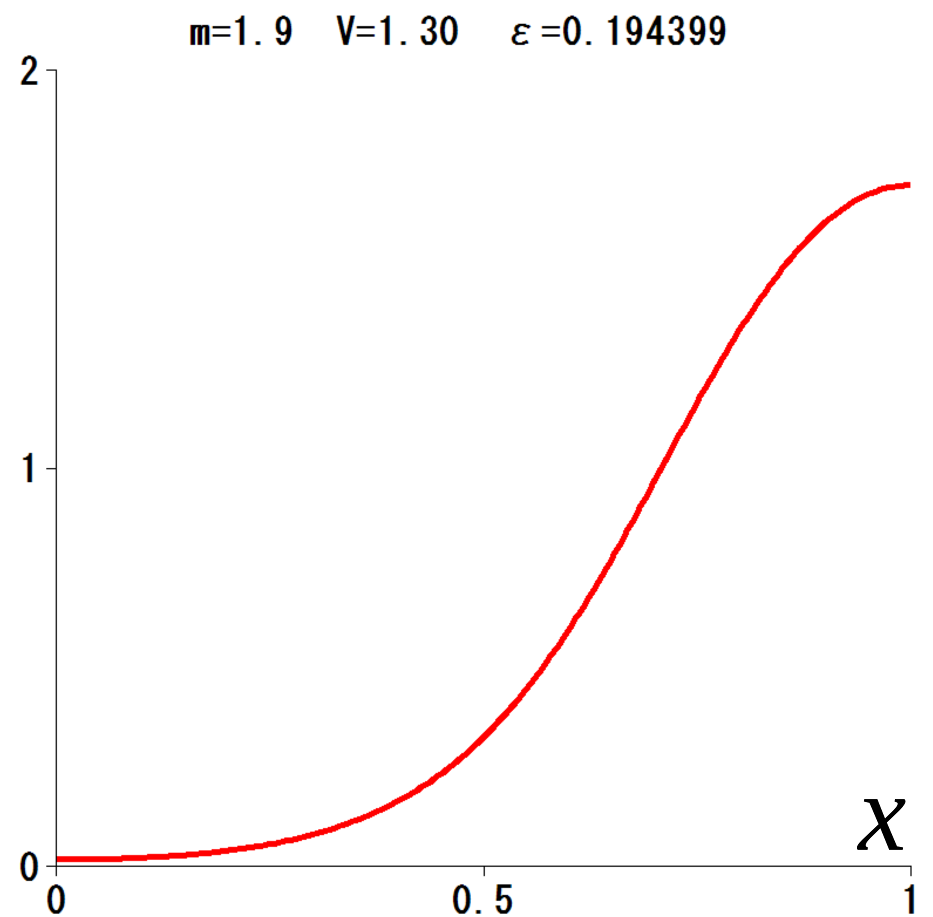
不安定

$\tilde{V} \to 1.9, \varepsilon^2 \to 0$ の形状

$m = 1.9$



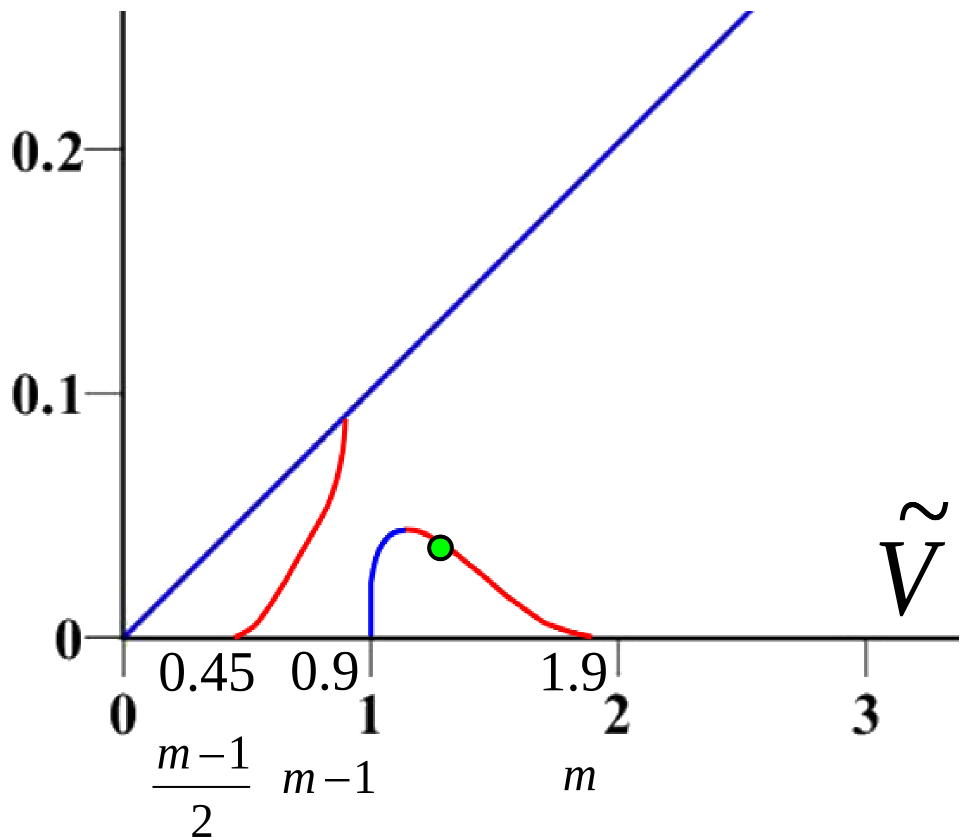


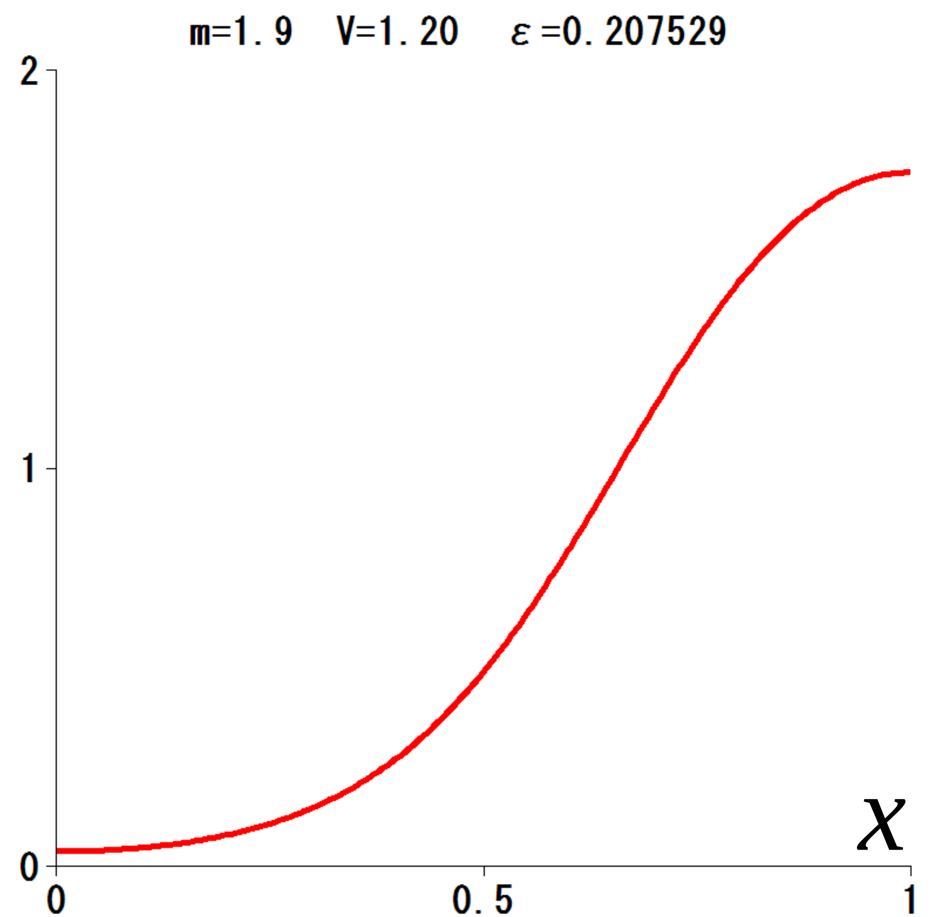
$$(SLP)\begin{cases} \varepsilon^2 W_{xx} + W(W-1)(\tilde{V}+1-W) = 0, \\ W_x(0) = W_x(1) = 0, \\ W'(x) > 0, \\ \int_0^1 W(x)\,\mathrm{d}x + \tilde{V} = m. \end{cases}$$

不安定

$\tilde{V} \to 1.9, \varepsilon^2 \to 0$ の形状

$$m = 1.9$$





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(SLP)\begin{cases}
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\end{cases}
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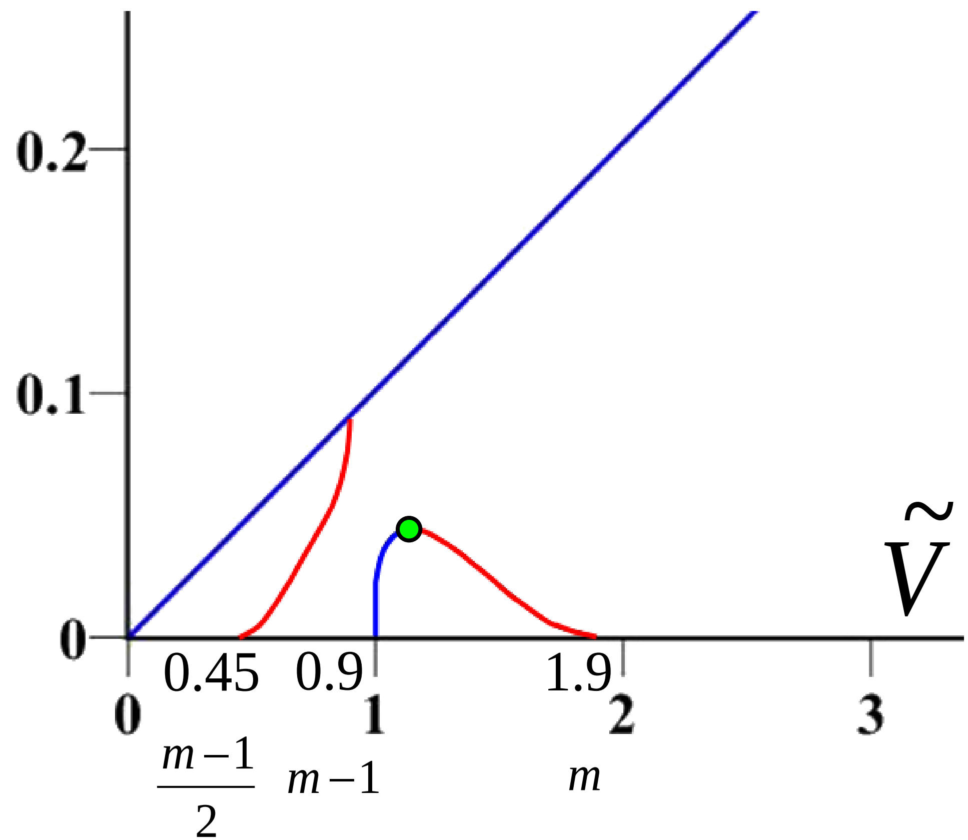
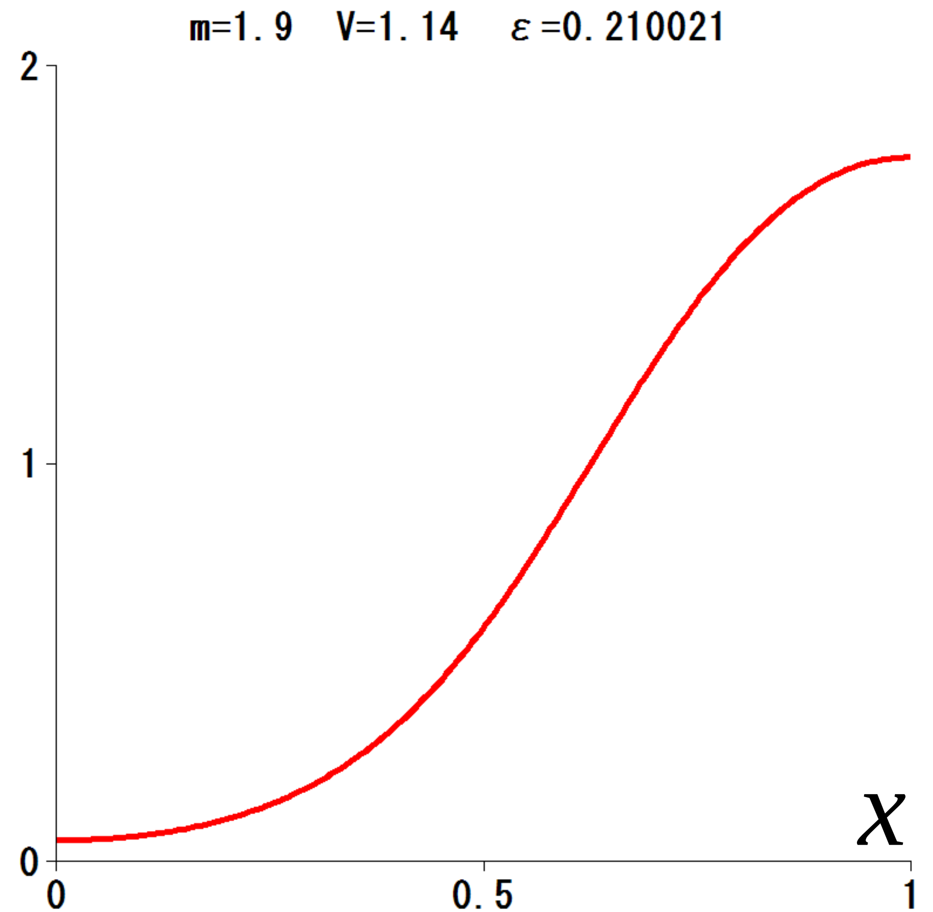
安定

$\tilde{V} \to 1.9, \varepsilon^2 \to 0$ の形状

$$m = 1.9$$





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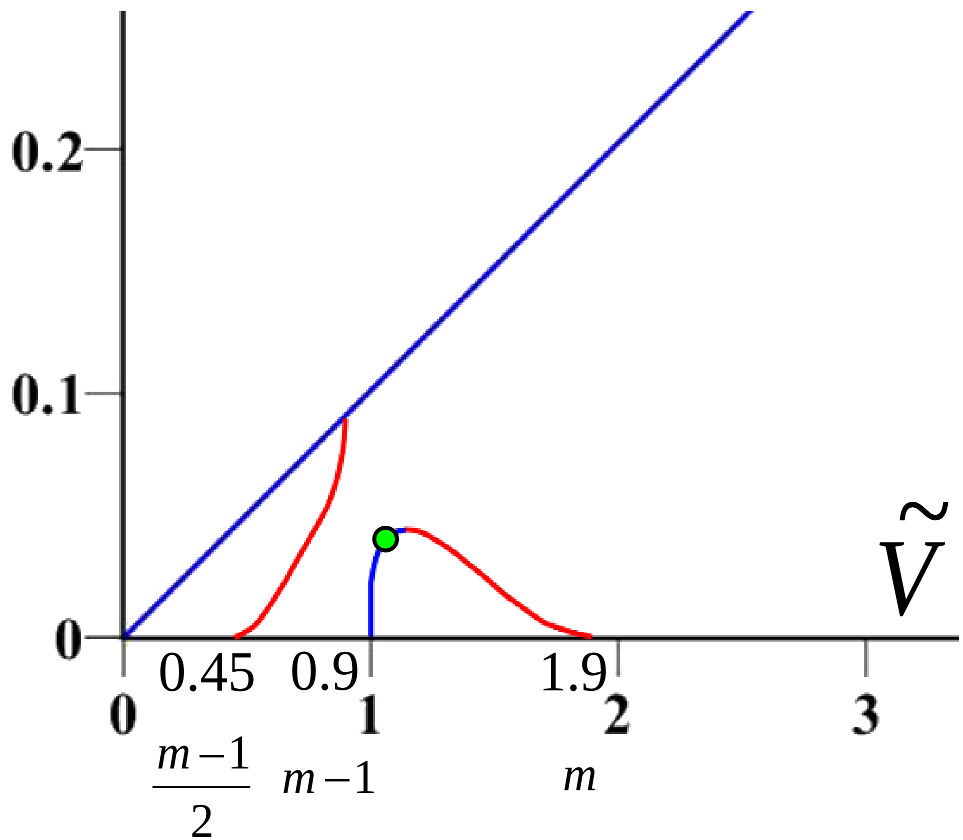
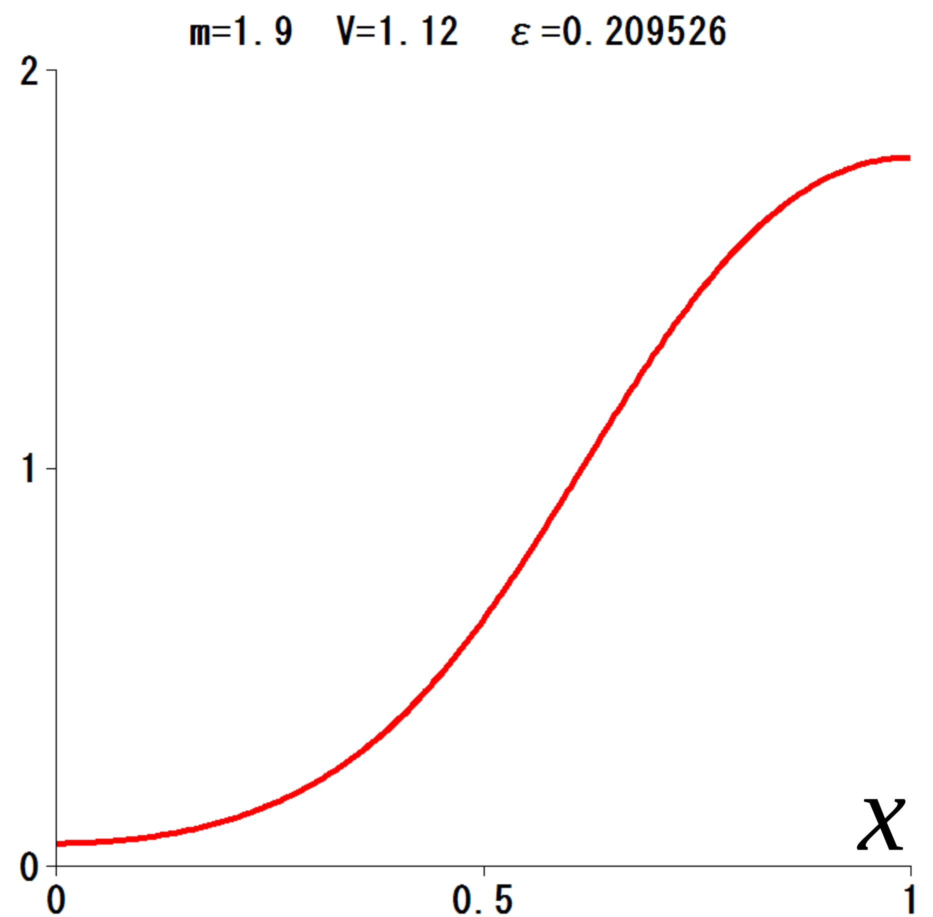
安定

$\tilde{V} \to 1,\ \varepsilon^2 \to 0$ の形状

$$m = 1.9$$





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安定

$\tilde{V} \to 1,\ \varepsilon^2 \to 0$ の形状

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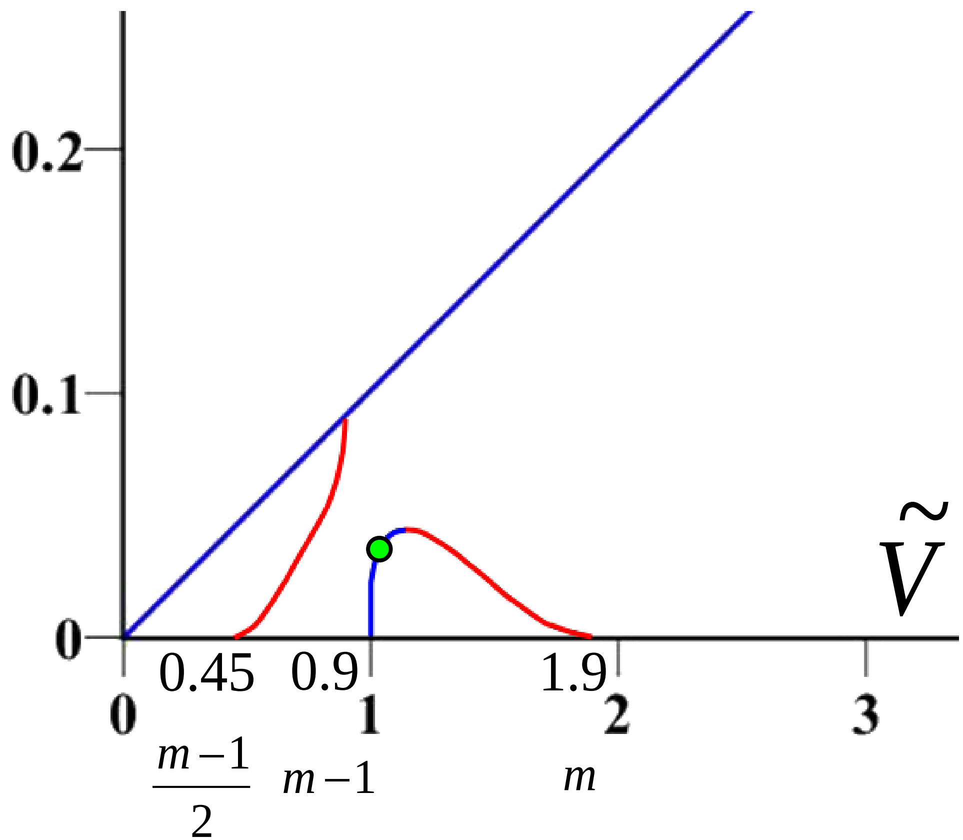

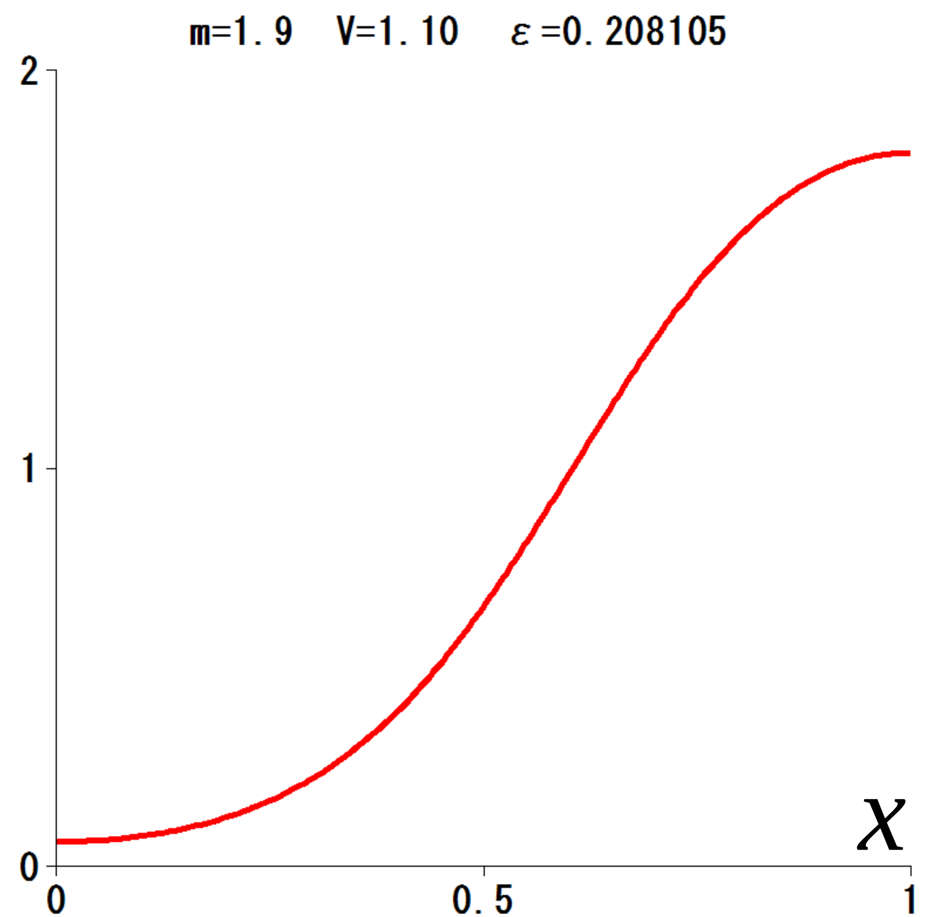

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安定

$\tilde{V} \to 1,\ \varepsilon^2 \to 0$ の形状

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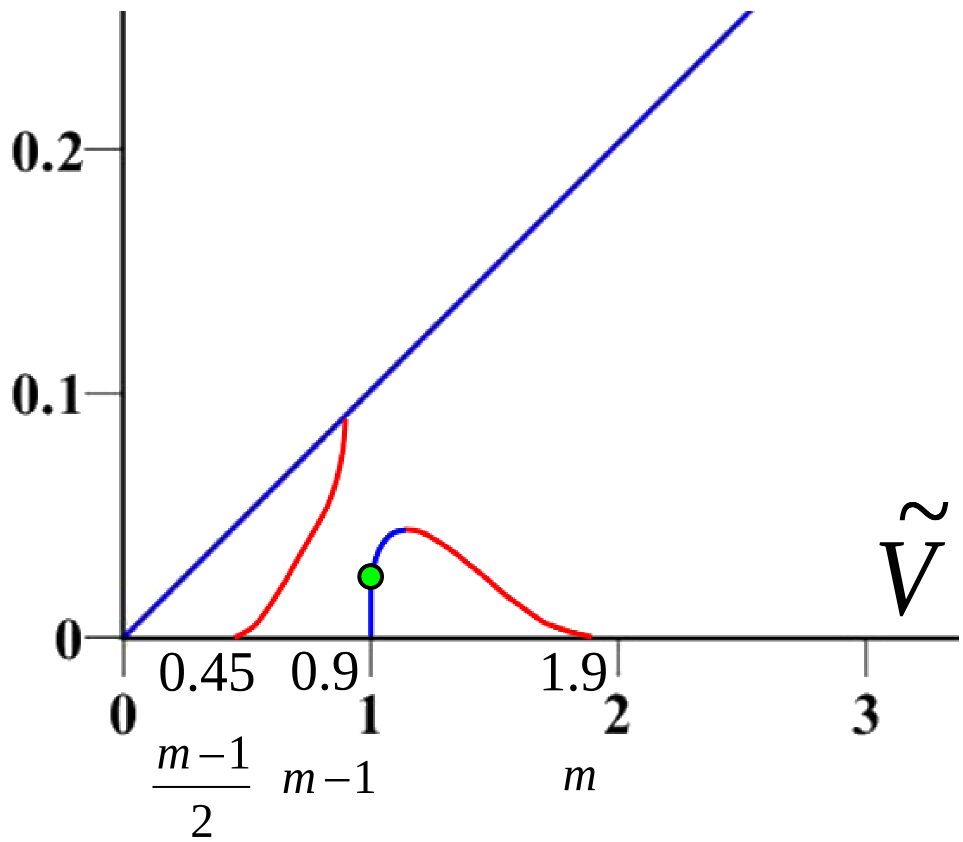

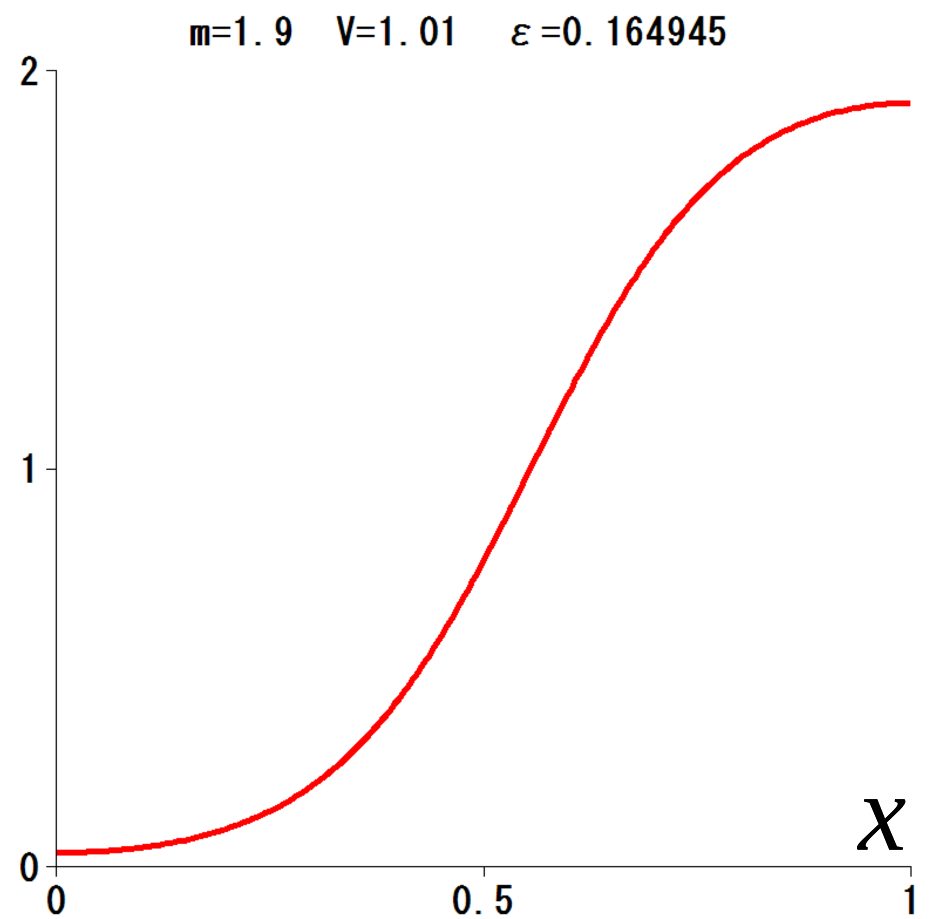

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$\tilde{V} \to 1,\ \varepsilon^2 \to 0$ の形状

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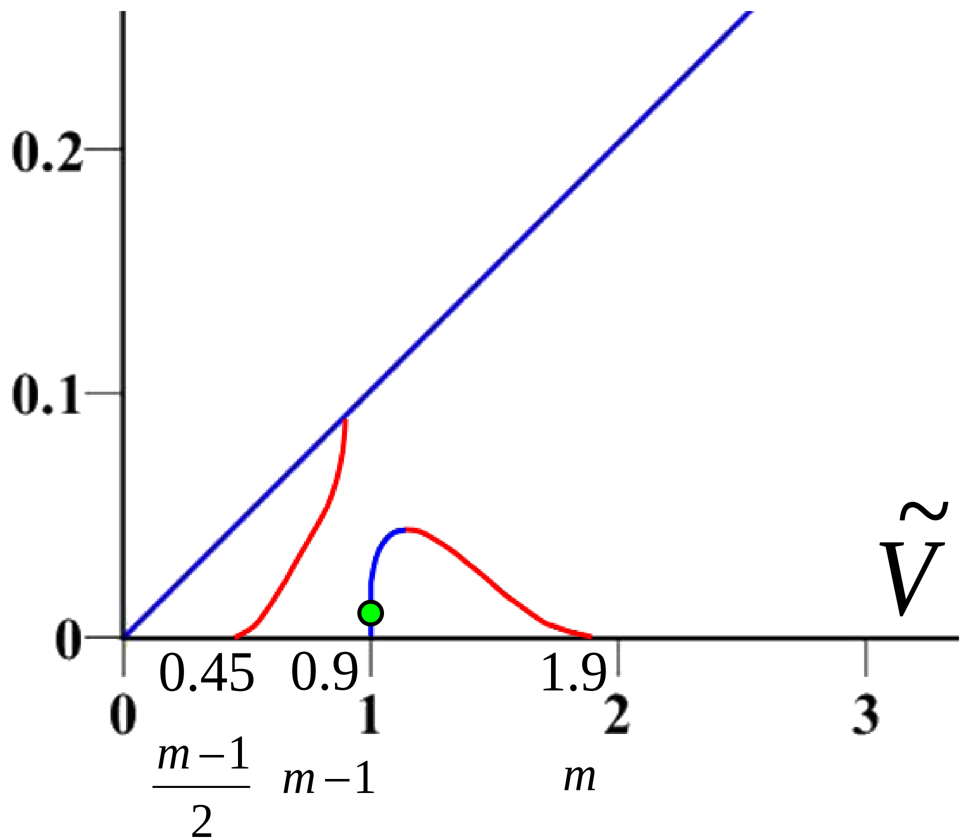

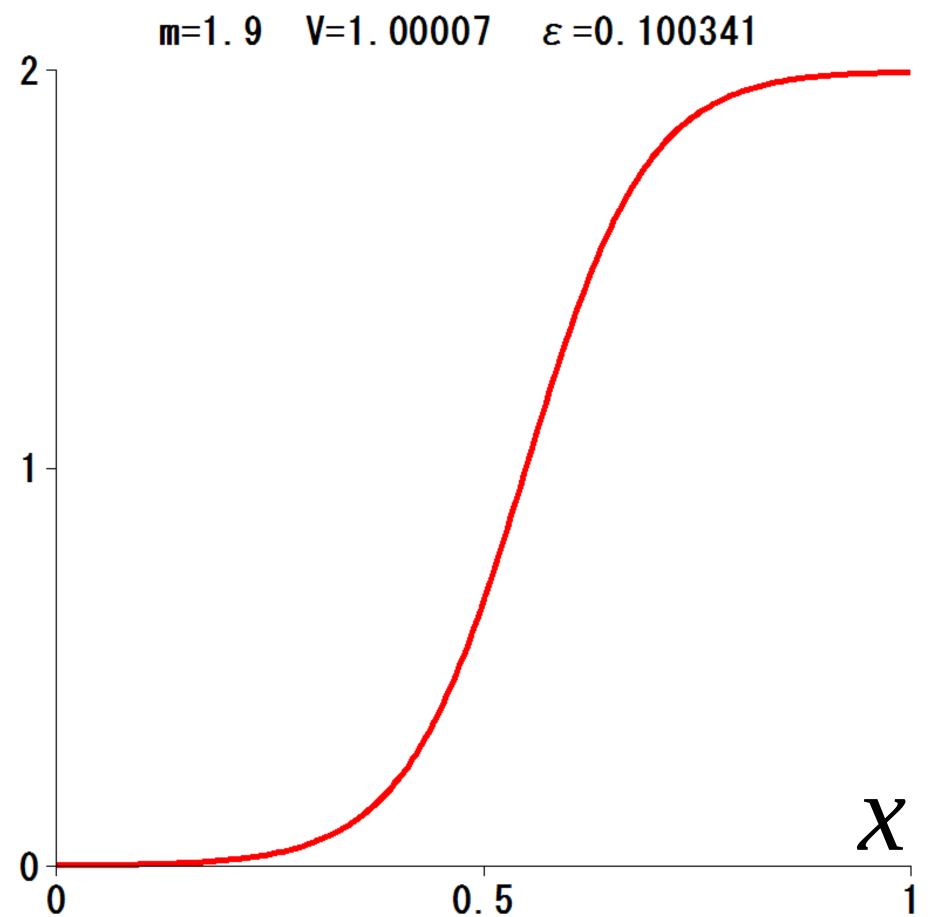

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$\tilde{V} \to 1,\ \varepsilon^2 \to 0$ の形状

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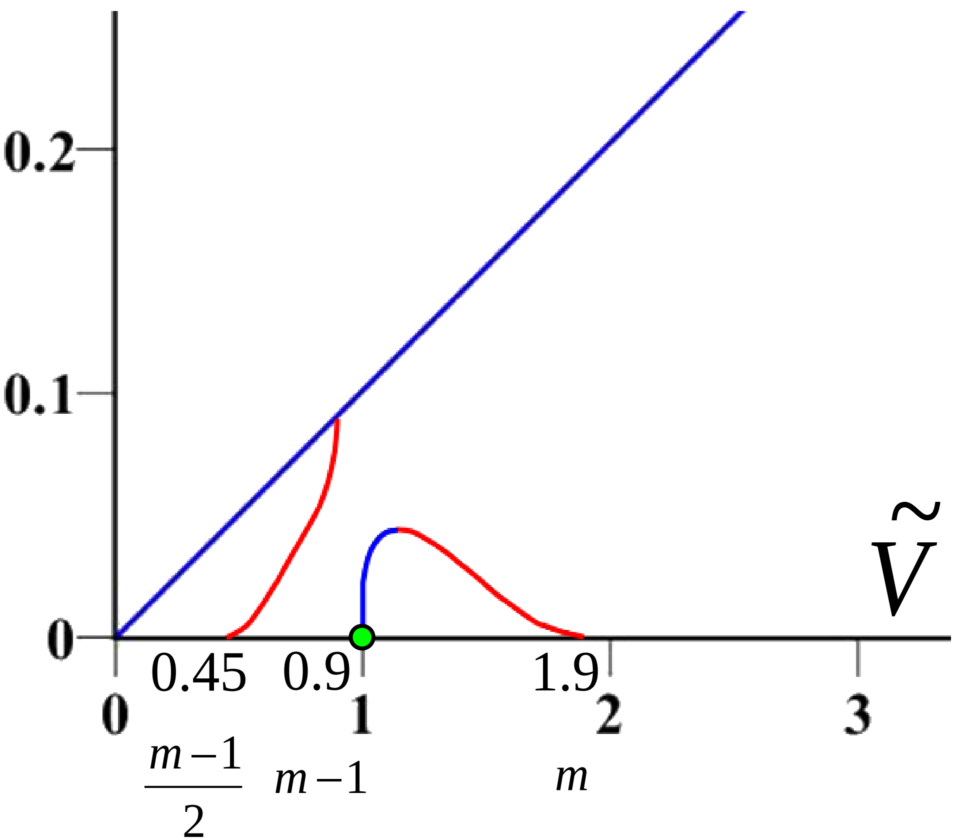

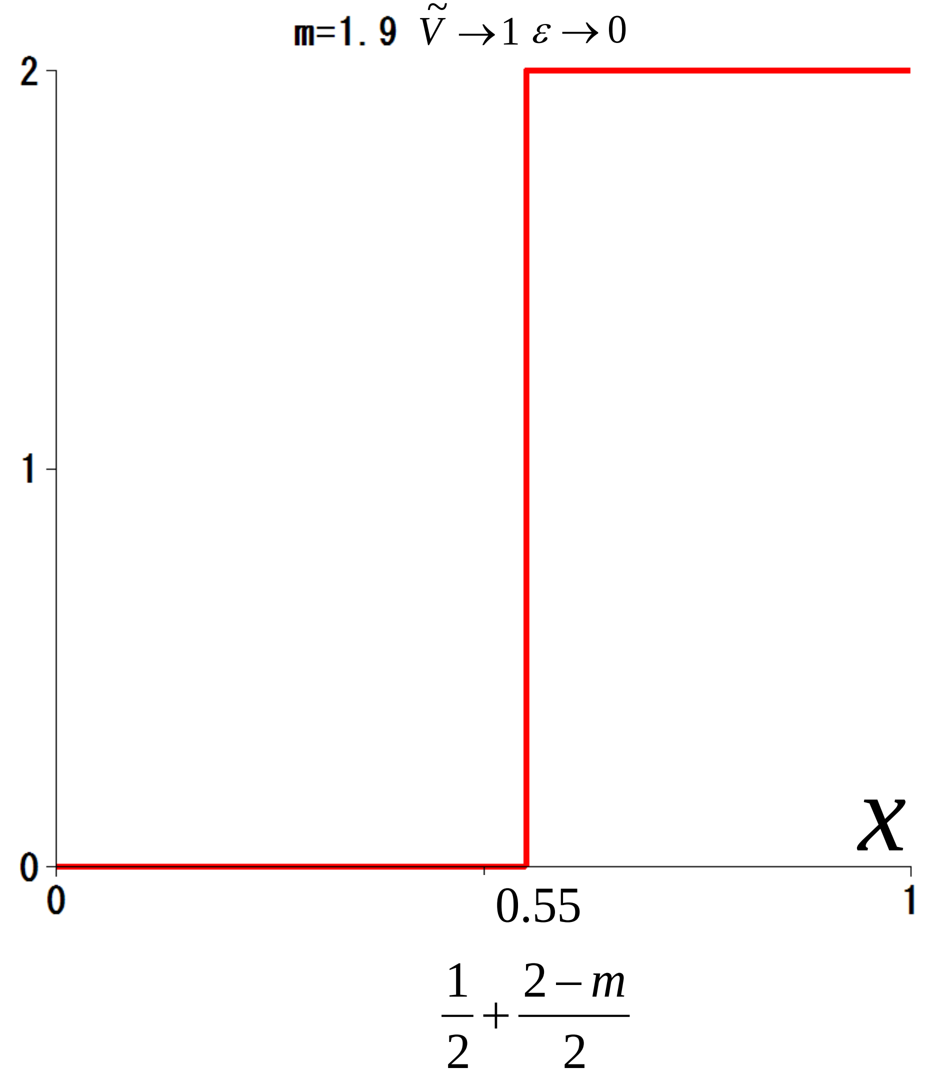

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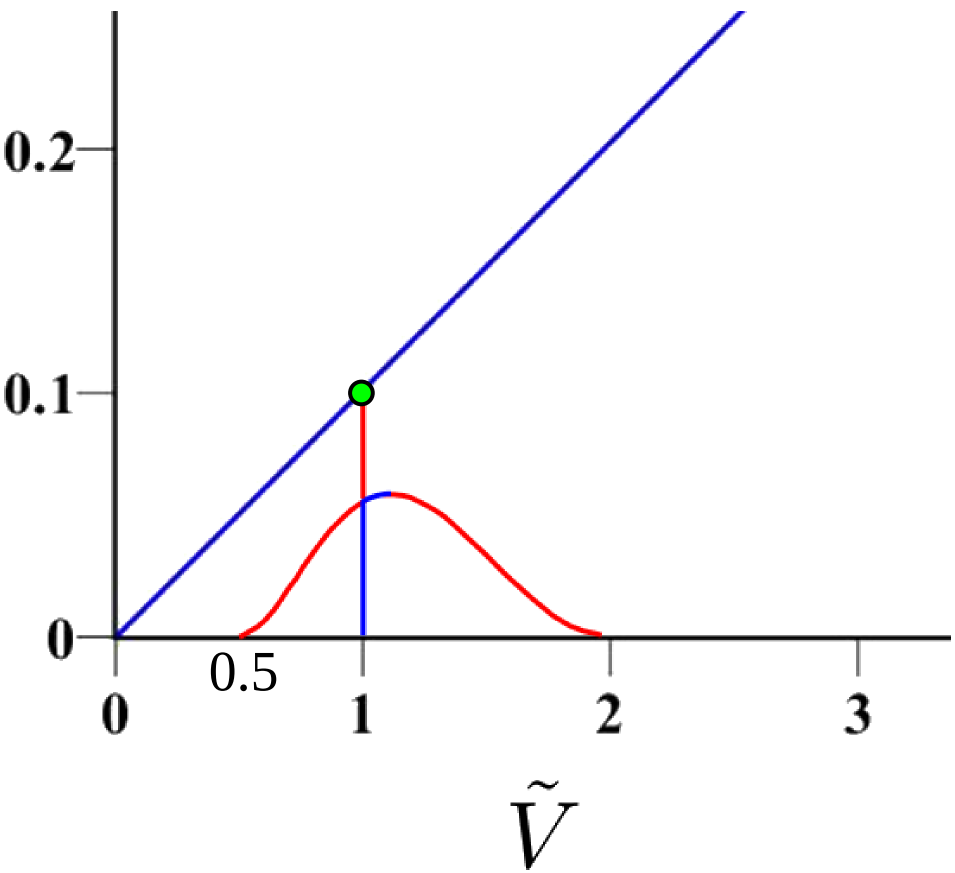
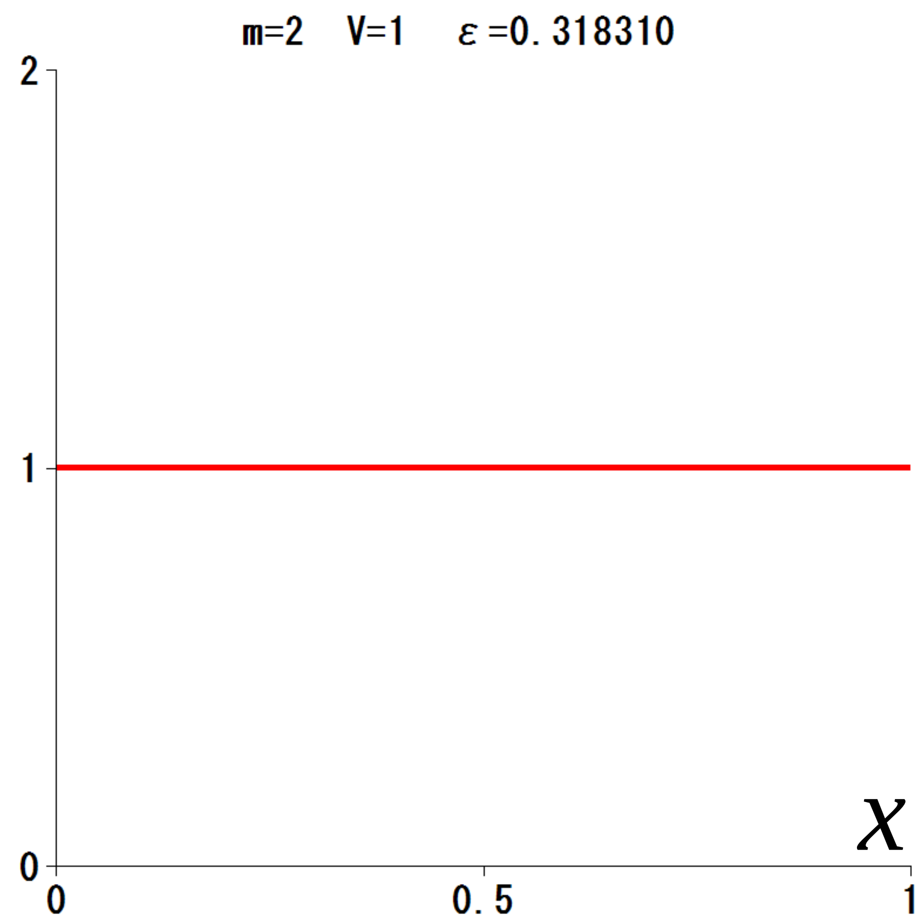
不安定

$\tilde{V} = 1, \quad \varepsilon^2 \to 0$ の形状

$$m = 2$$





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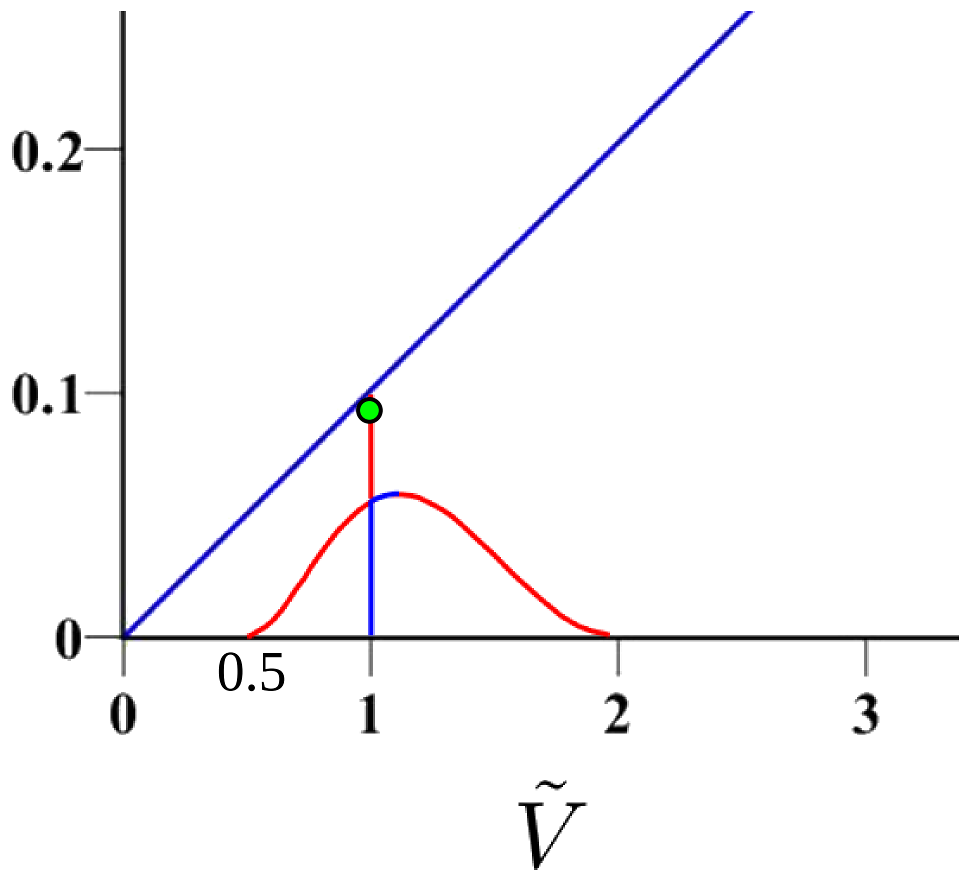

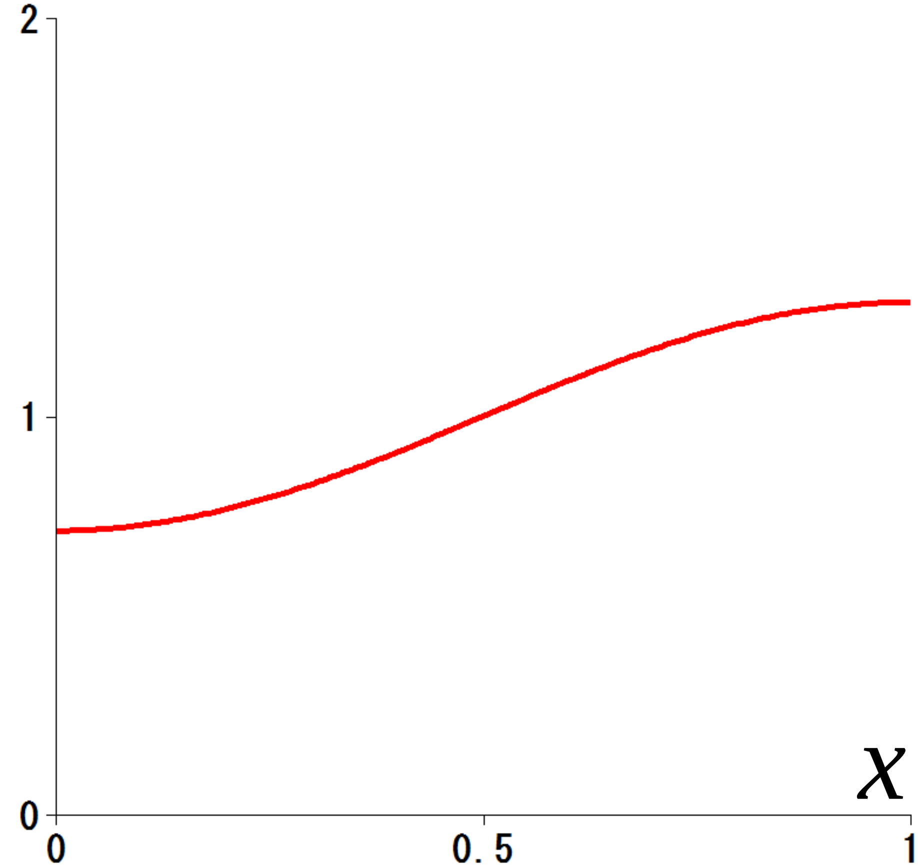

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不安定

$\tilde{V} = 1, \quad \varepsilon^2 \to 0$ の形状

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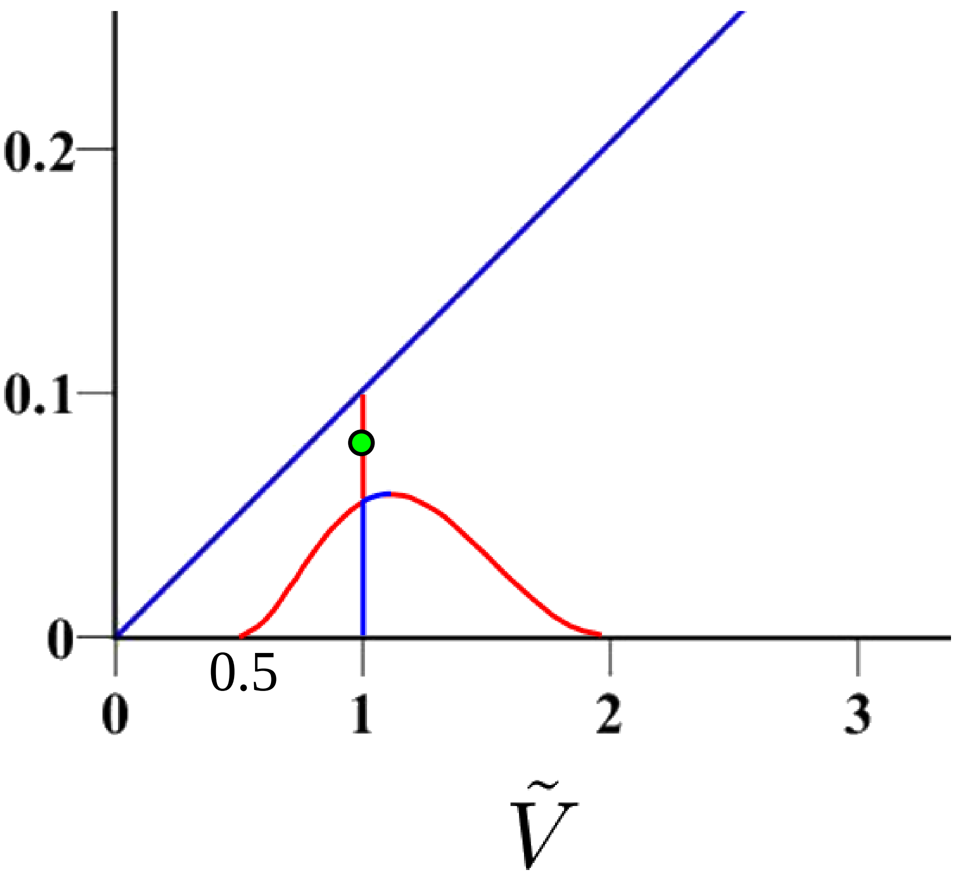

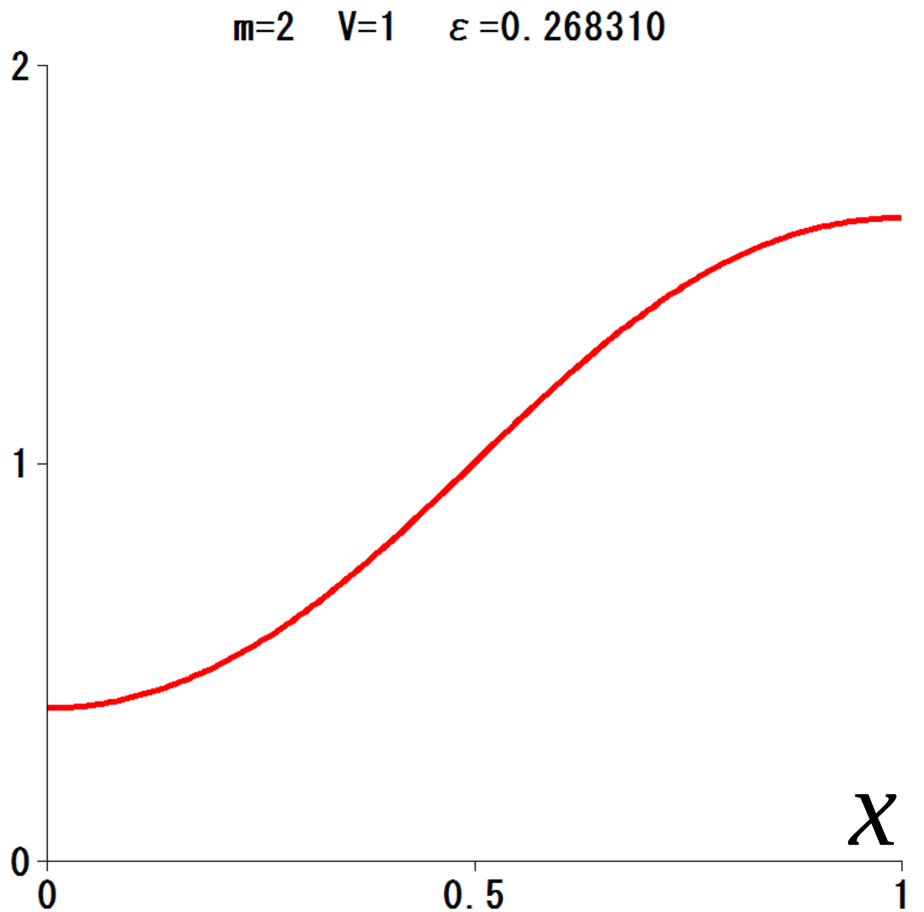

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$\tilde{V} = 1, \quad \varepsilon^2 \to 0$ の形状

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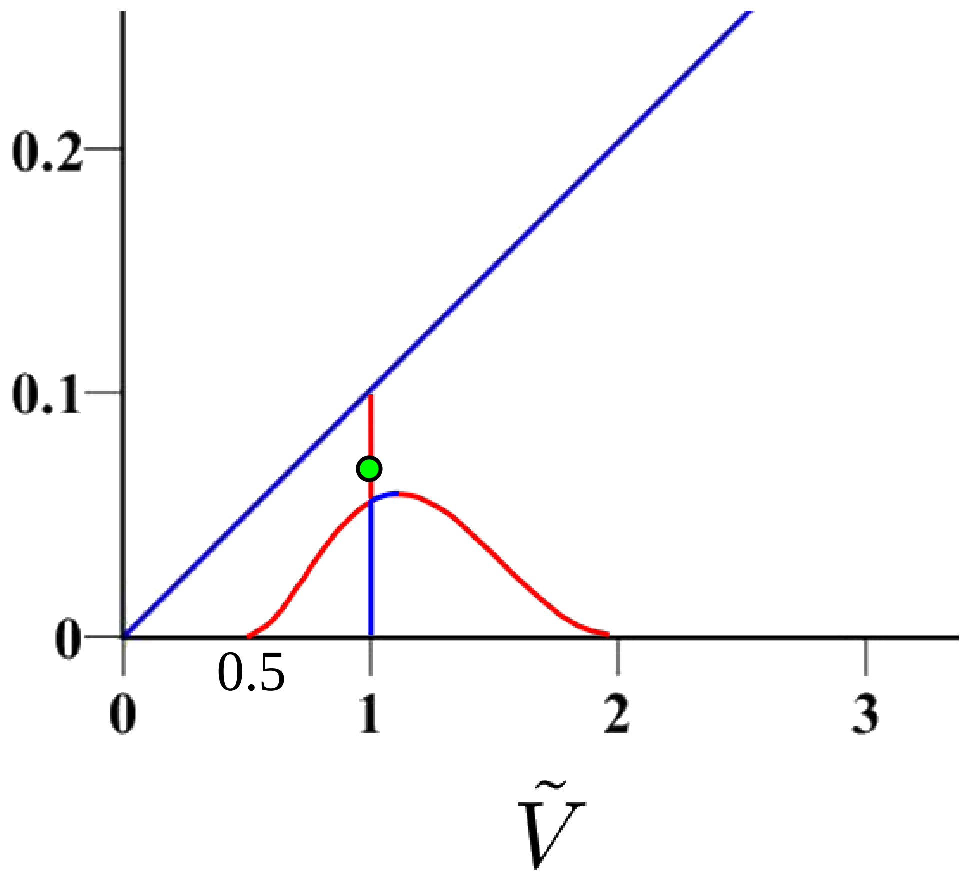

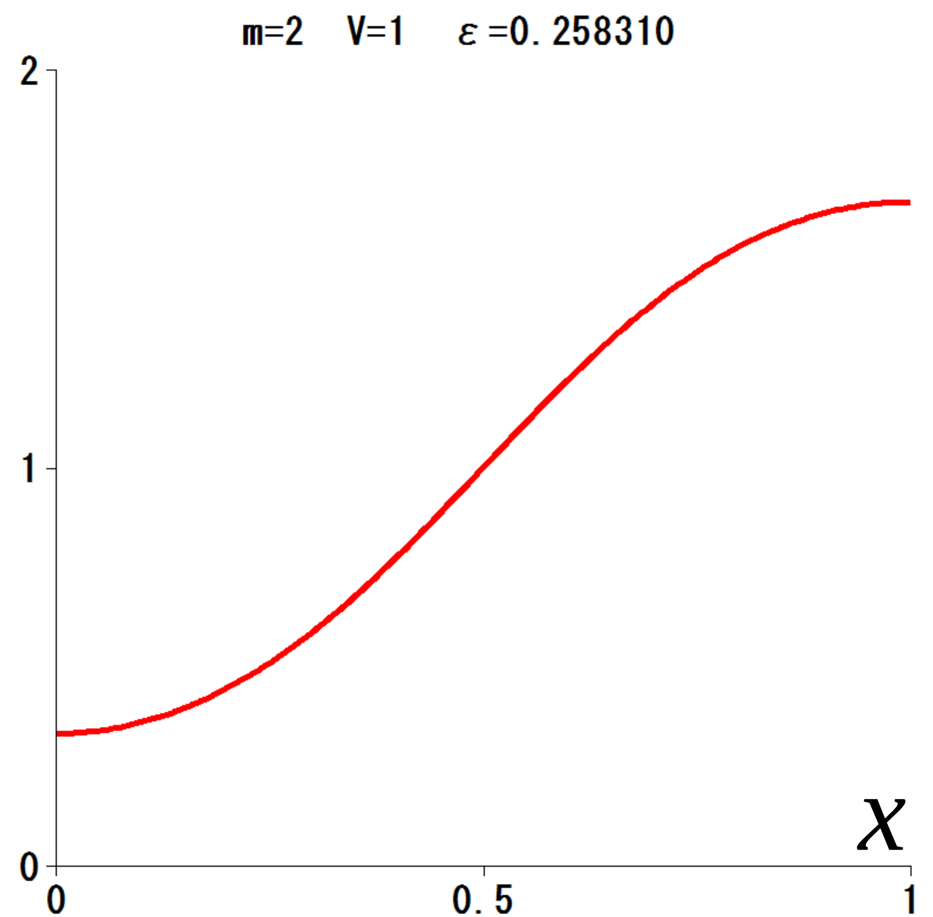

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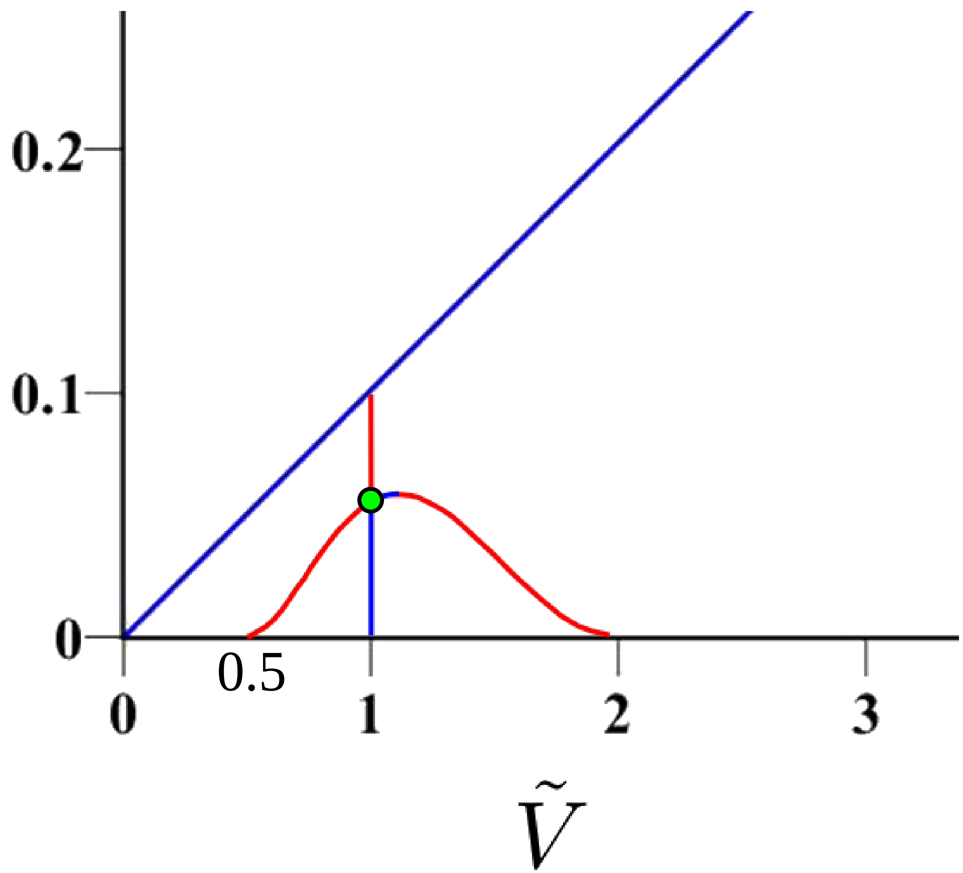
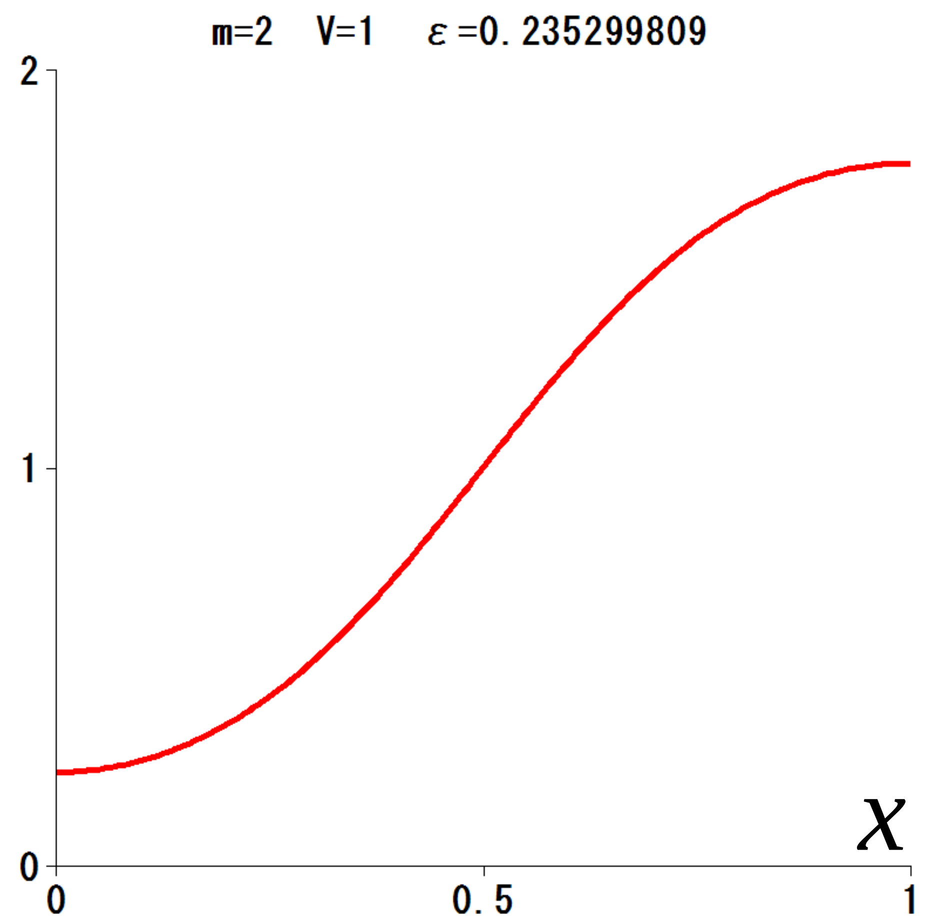
安定

$\tilde{V} = 1, \quad \varepsilon^2 \to 0$ の形状

$$m = 2$$





2次分岐点

$$
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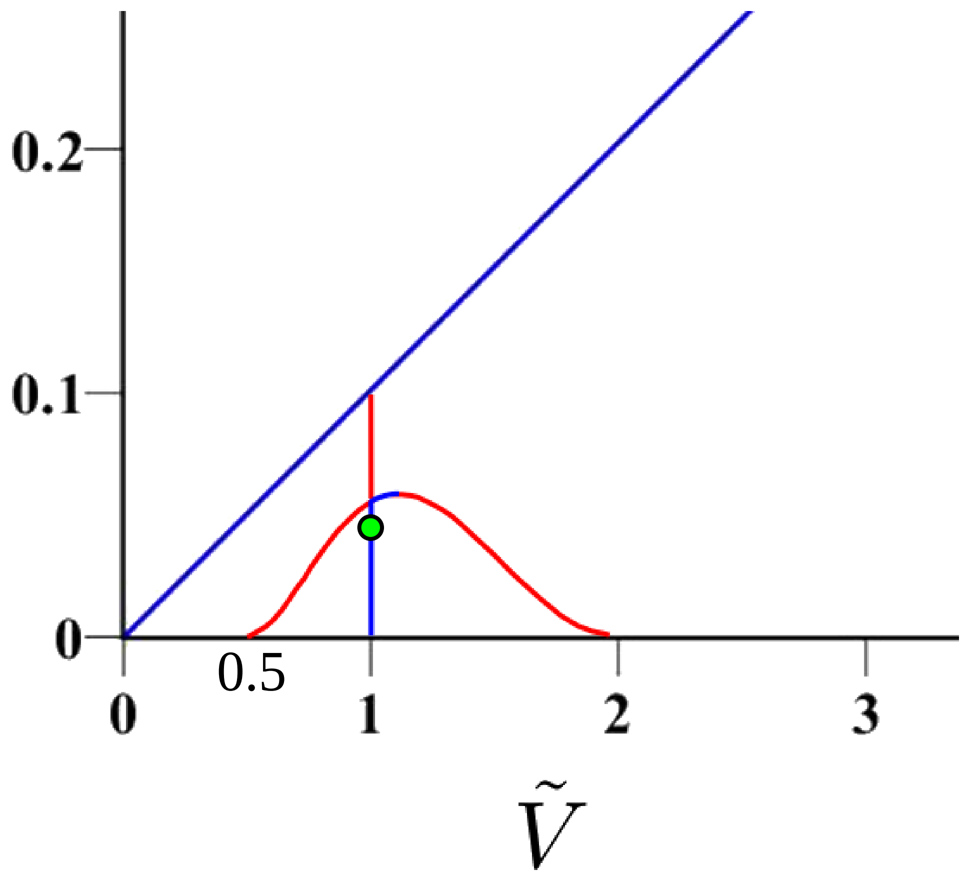
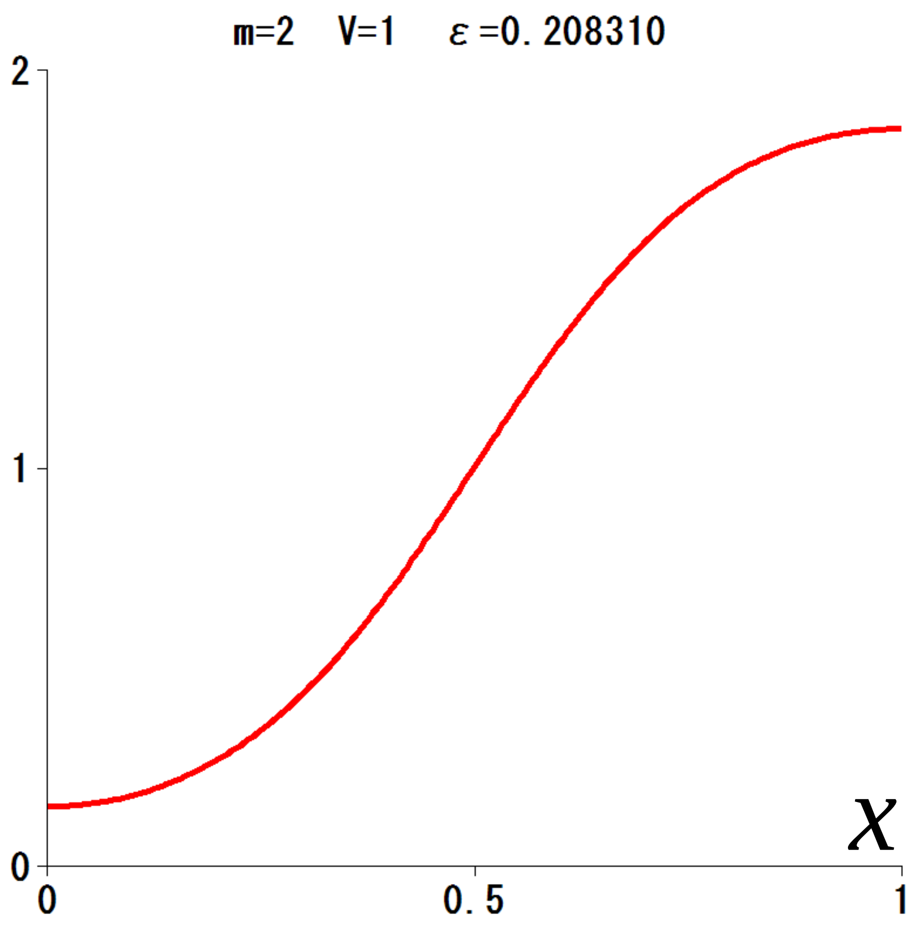
安定

$\tilde{V} = 1, \quad \varepsilon^2 \to 0$ の形状

$m = 2$





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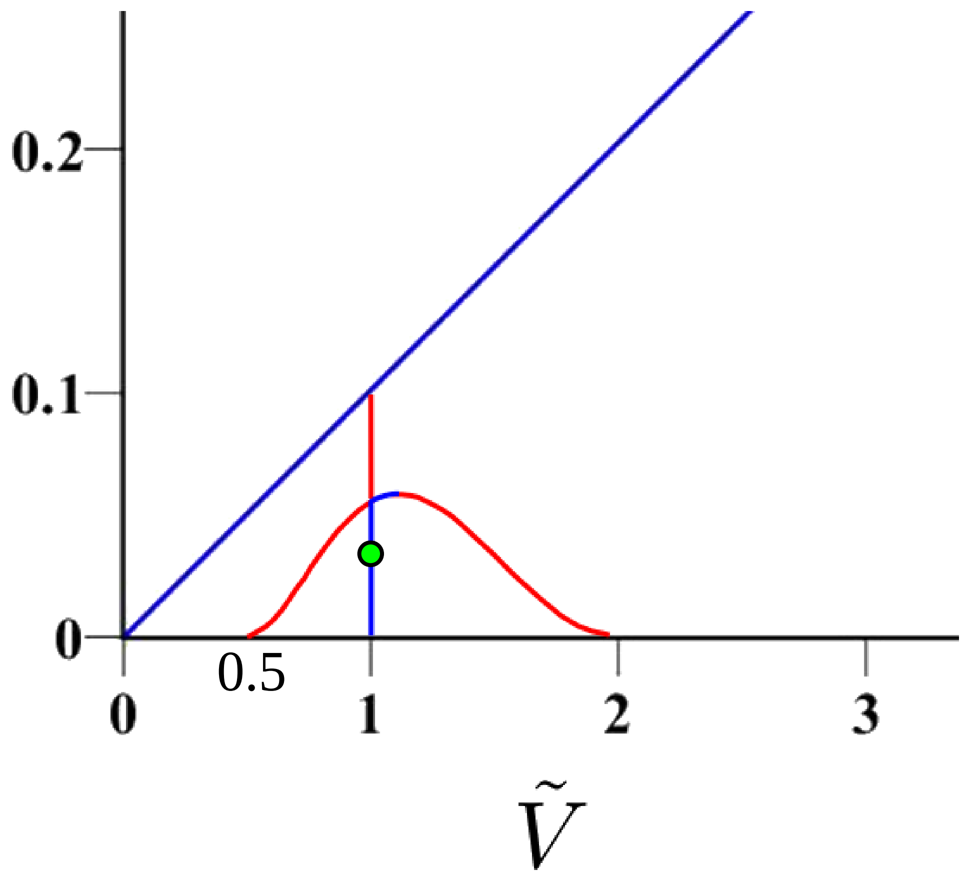
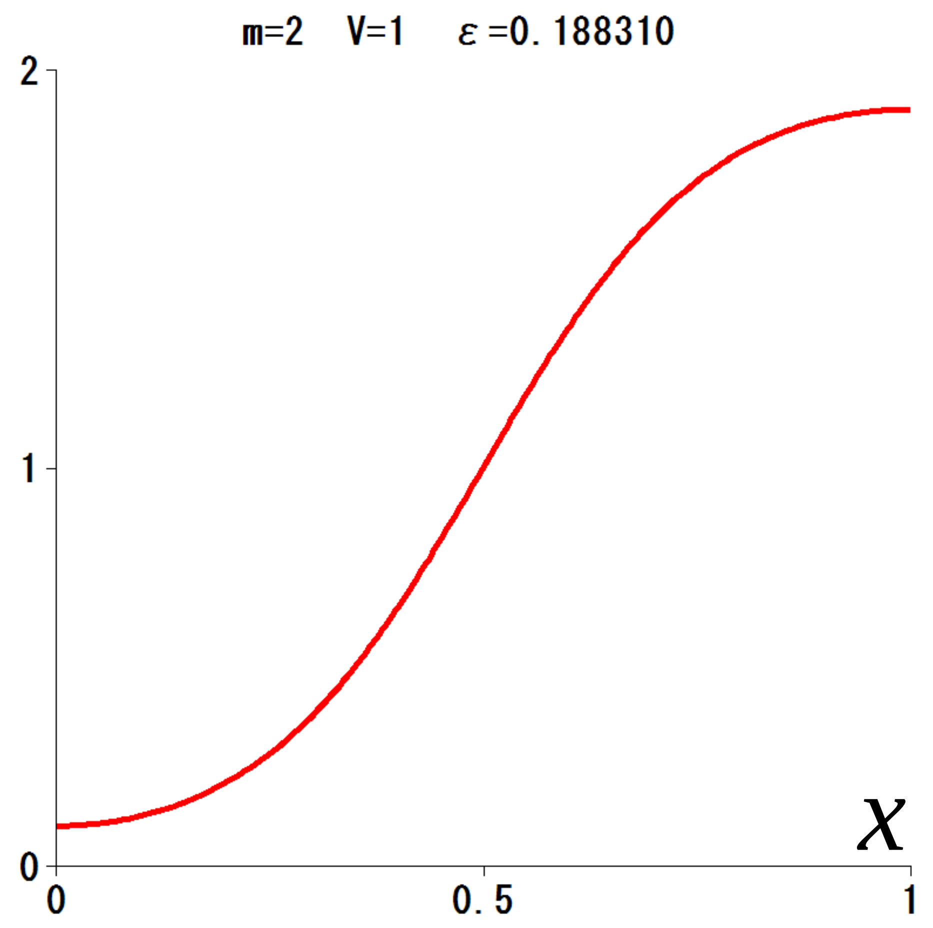
安定

$\tilde{V} = 1, \quad \varepsilon^2 \to 0$ の形状

$m = 2$





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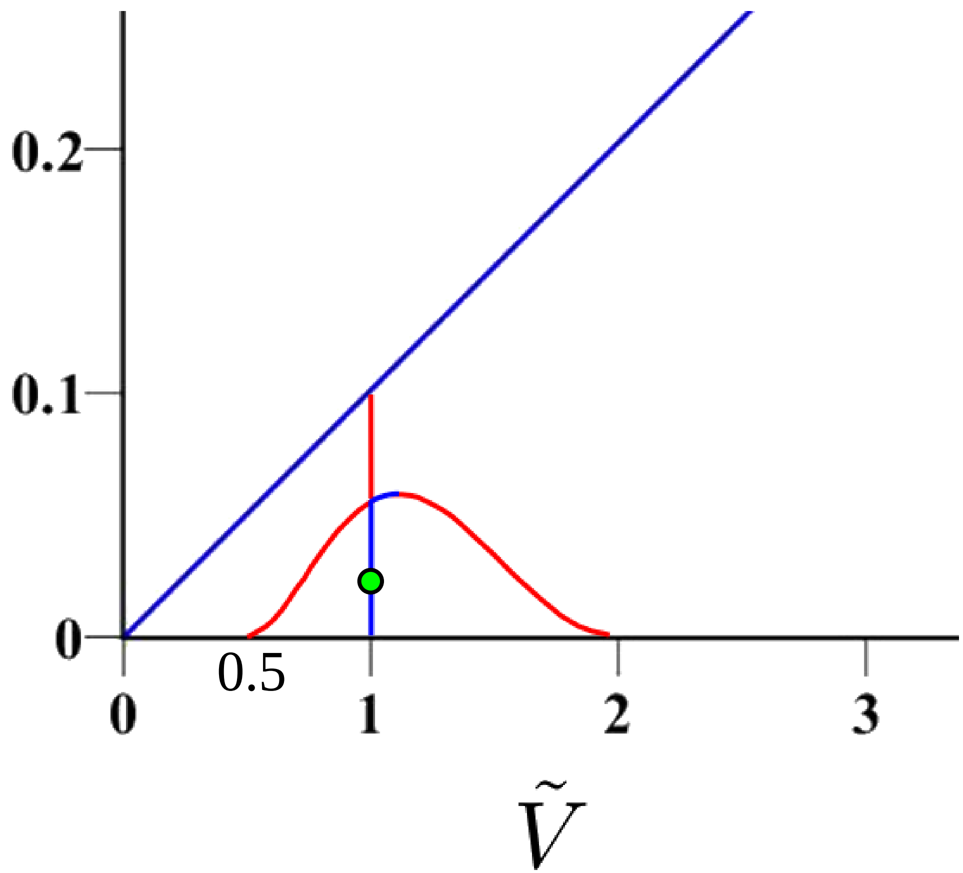
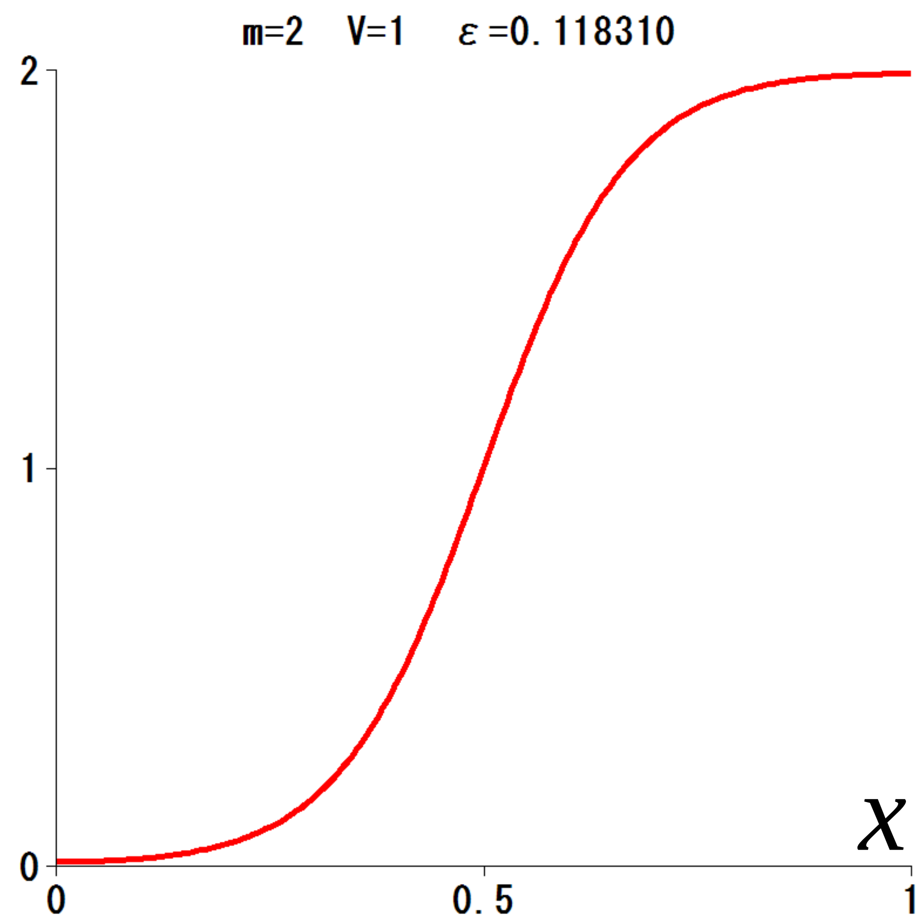
安定

$\tilde{V} = 1,\quad \varepsilon^2 \to 0$ の形状

$$m = 2$$





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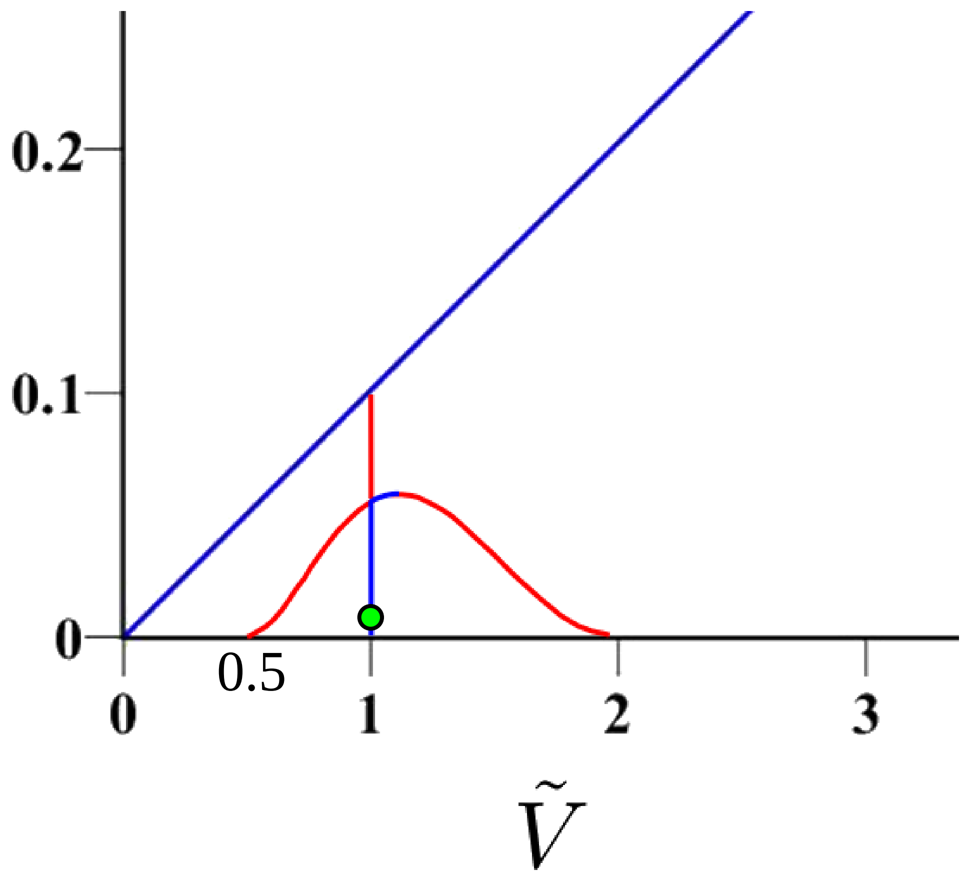
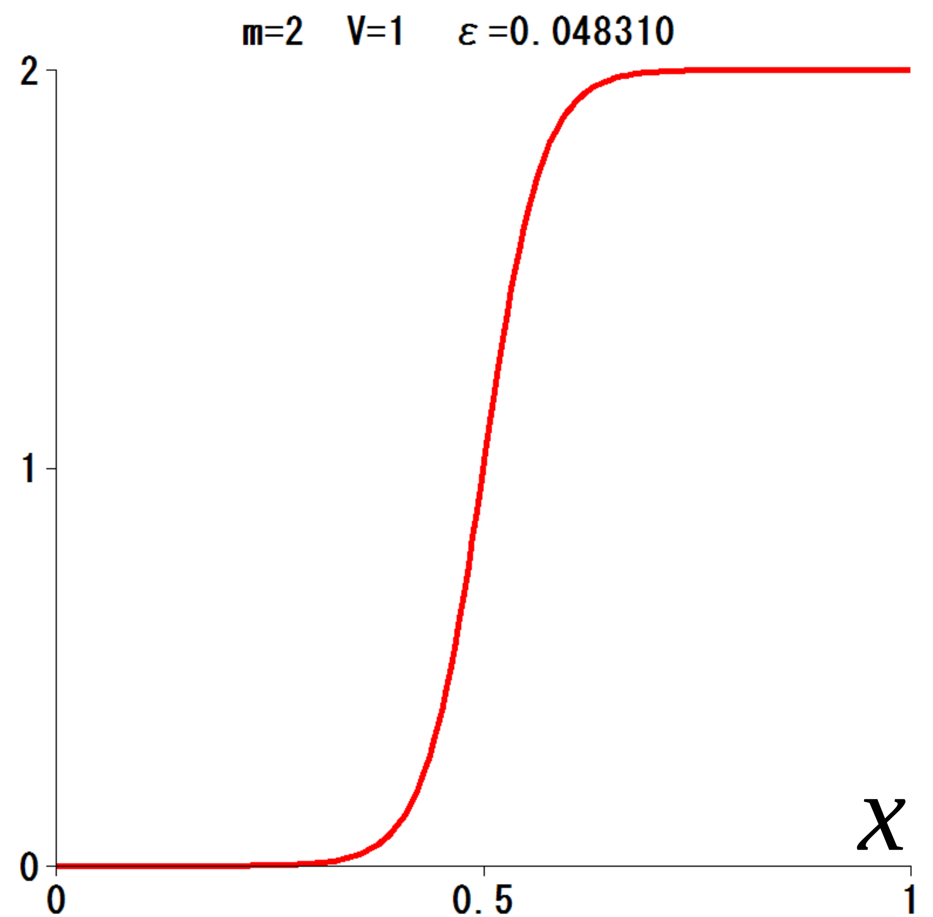
安定

$\tilde{V} = 1, \quad \varepsilon^2 \to 0$ の形状

$m = 2$





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安定

$\tilde{V} = 1, \quad \varepsilon^2 \to 0$ の形状

$$
m = 2
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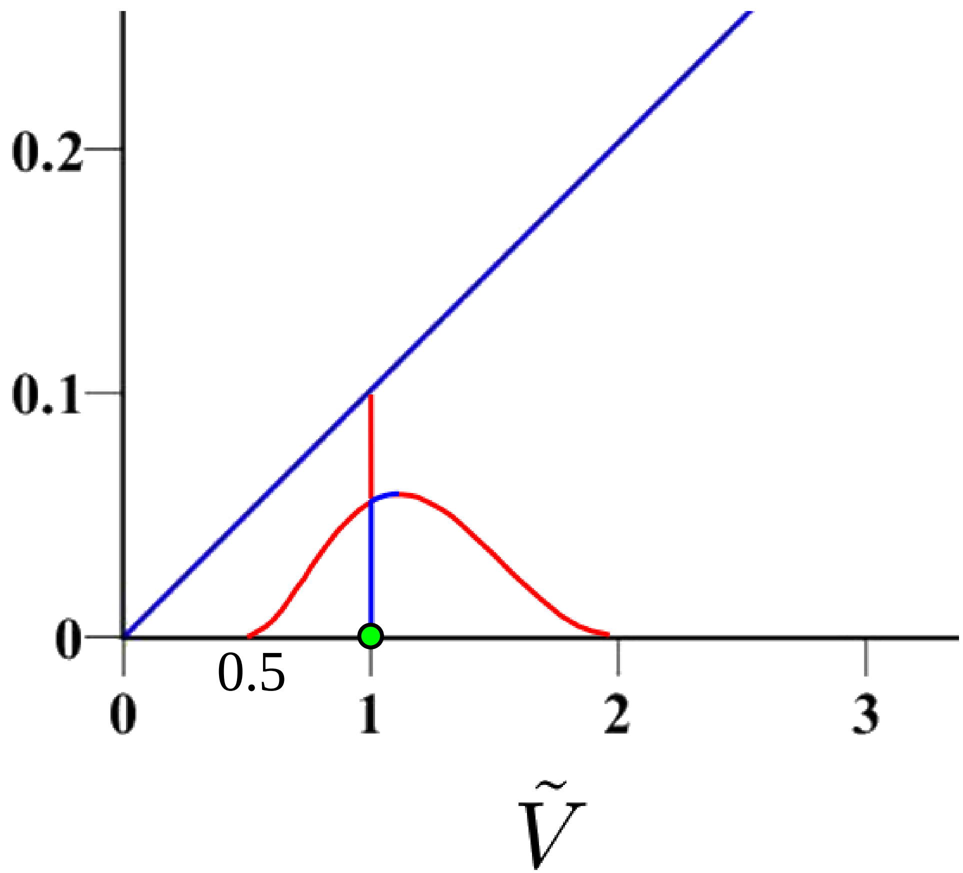

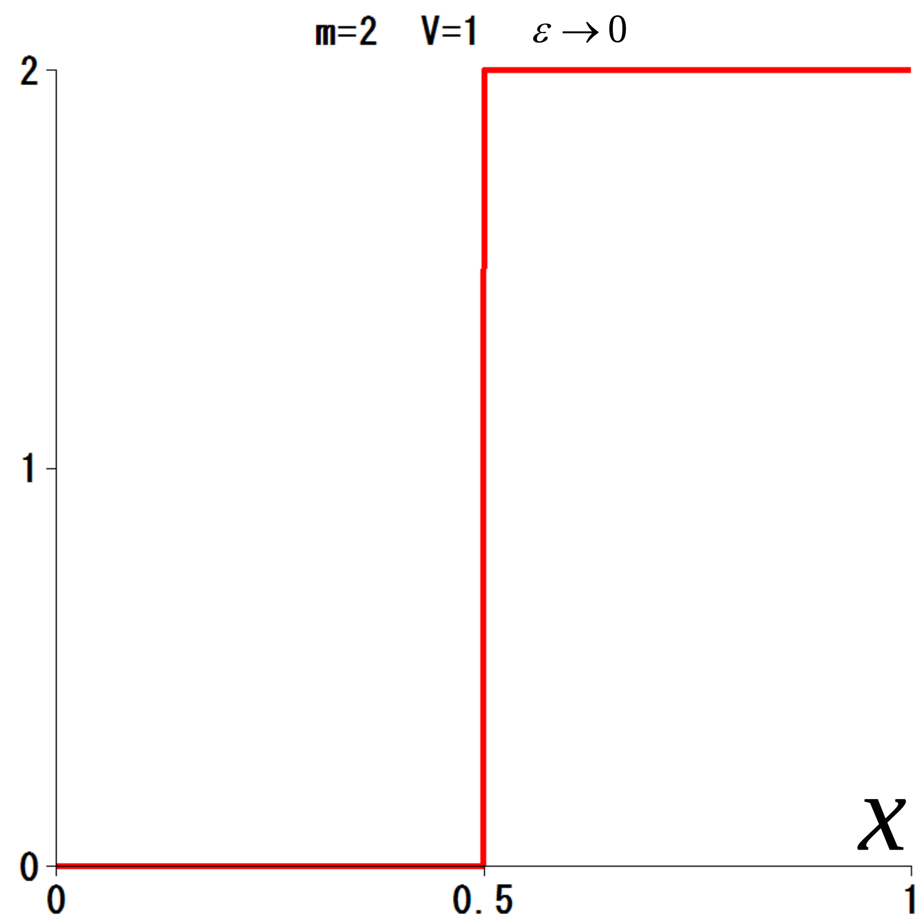

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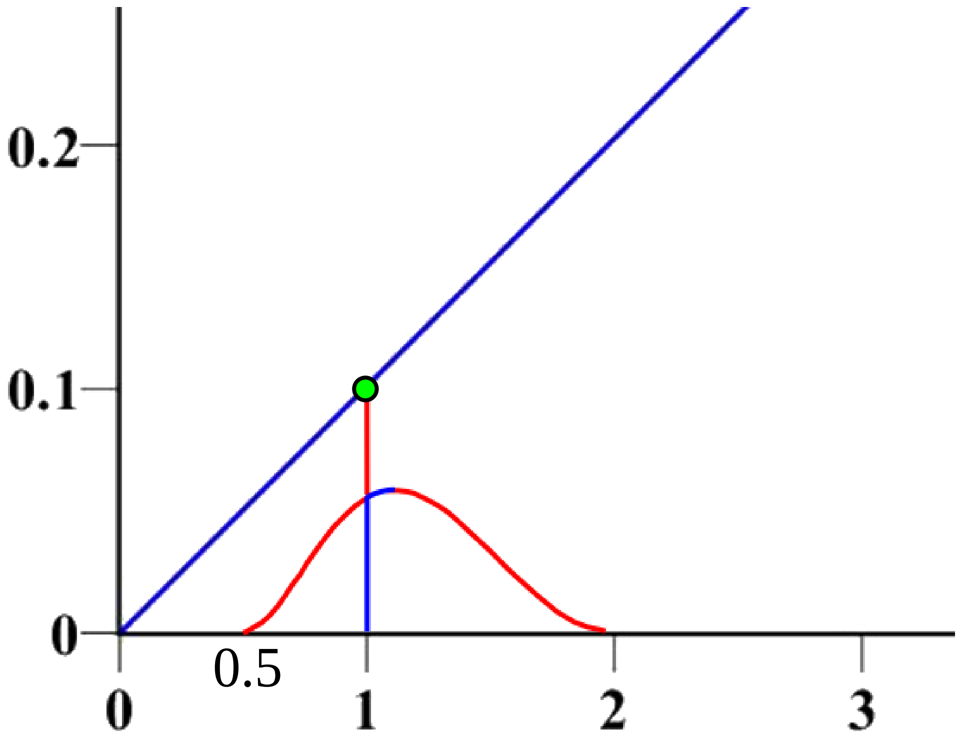
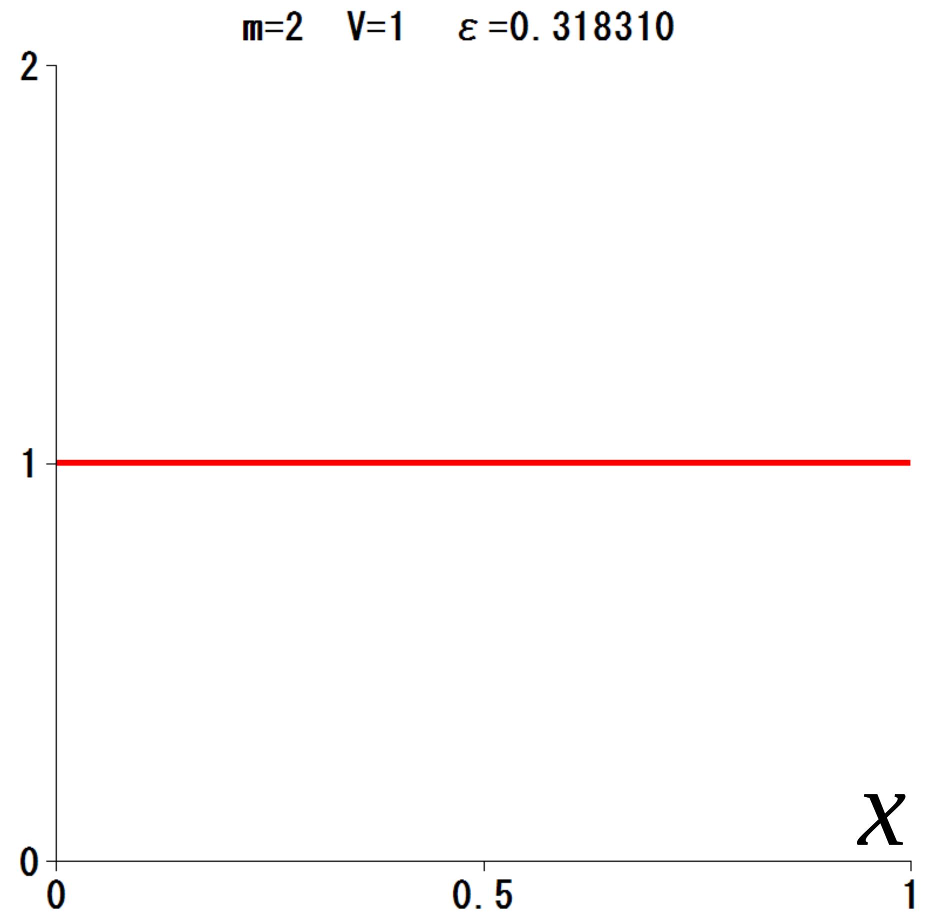
不安定

$\tilde{V} \to 2,\ \varepsilon^2 \to 0$ の形状

$$m = 2$$



$\tilde{V}$



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不安定

$\tilde{V} \to 2,\ \varepsilon^2 \to 0$ の形状

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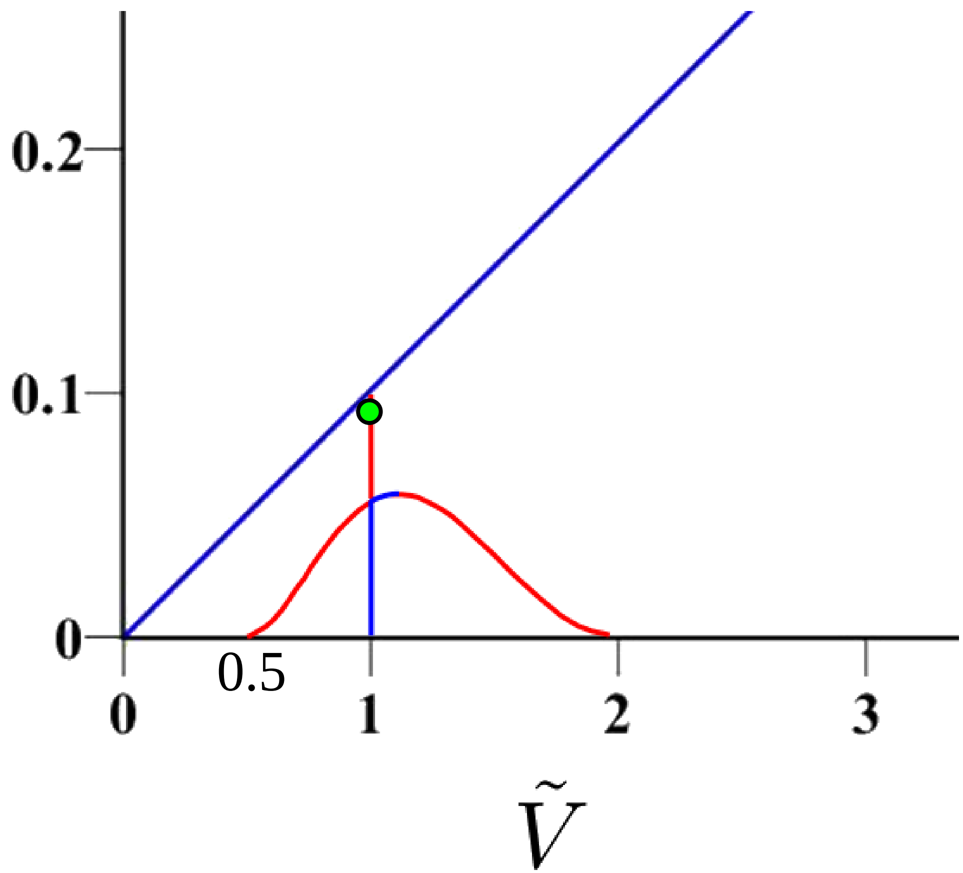

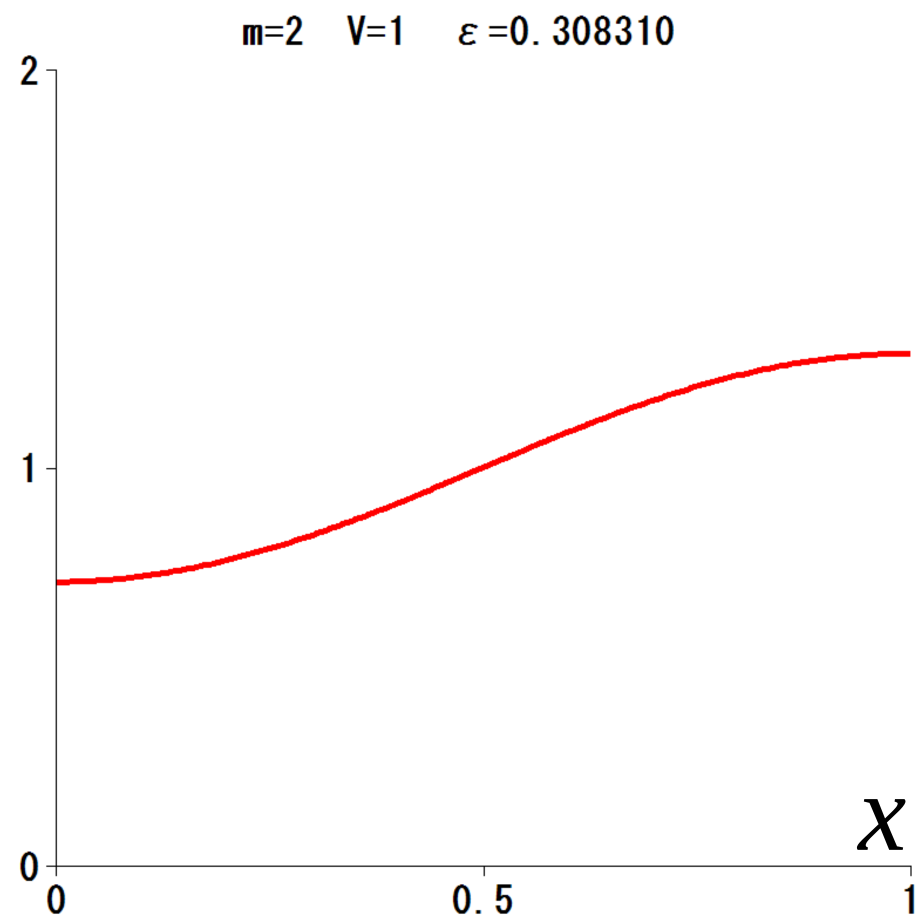

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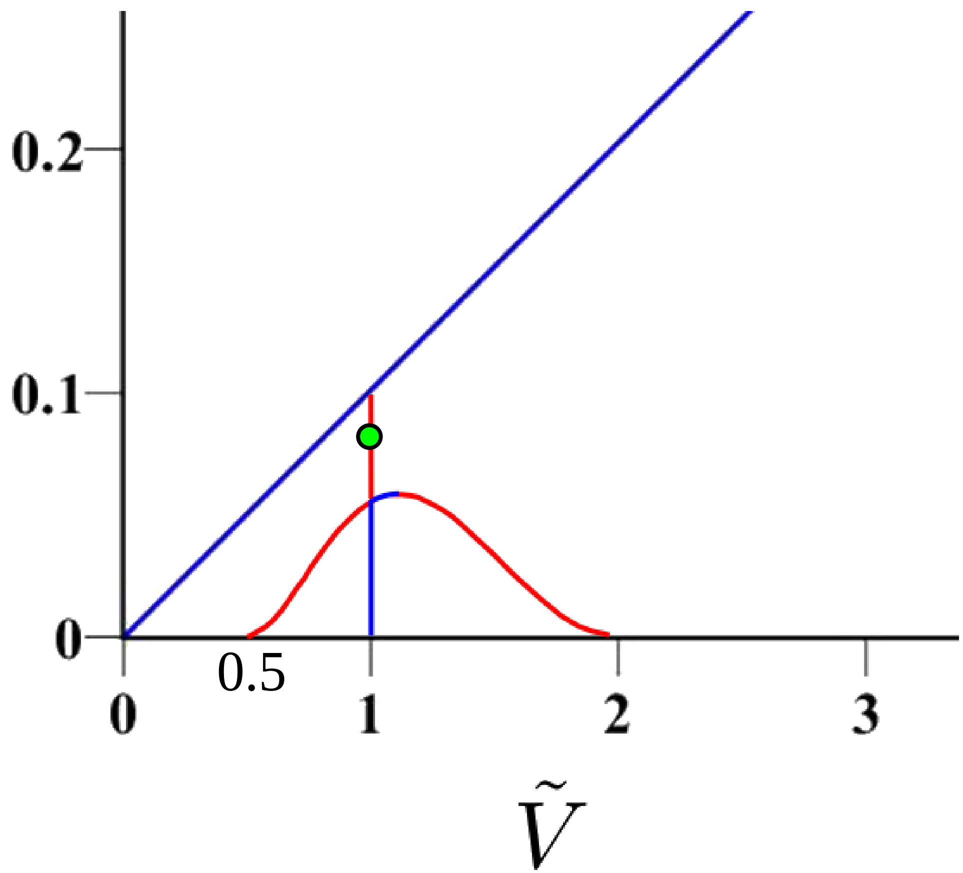

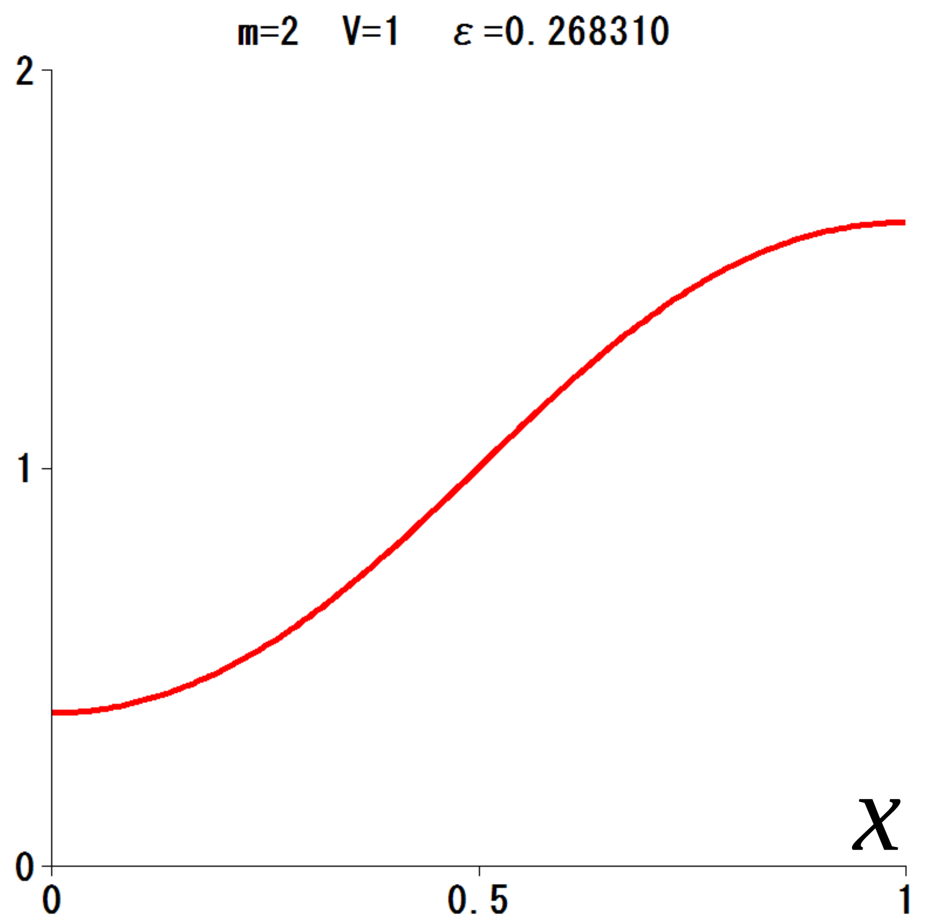

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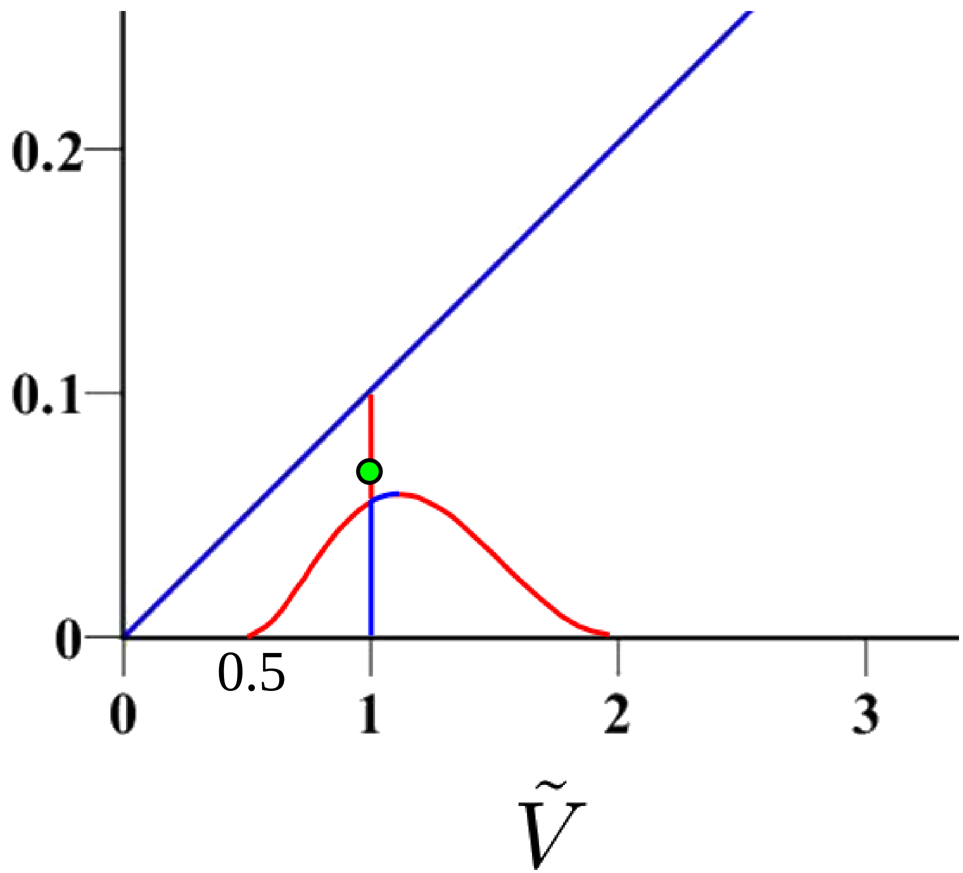

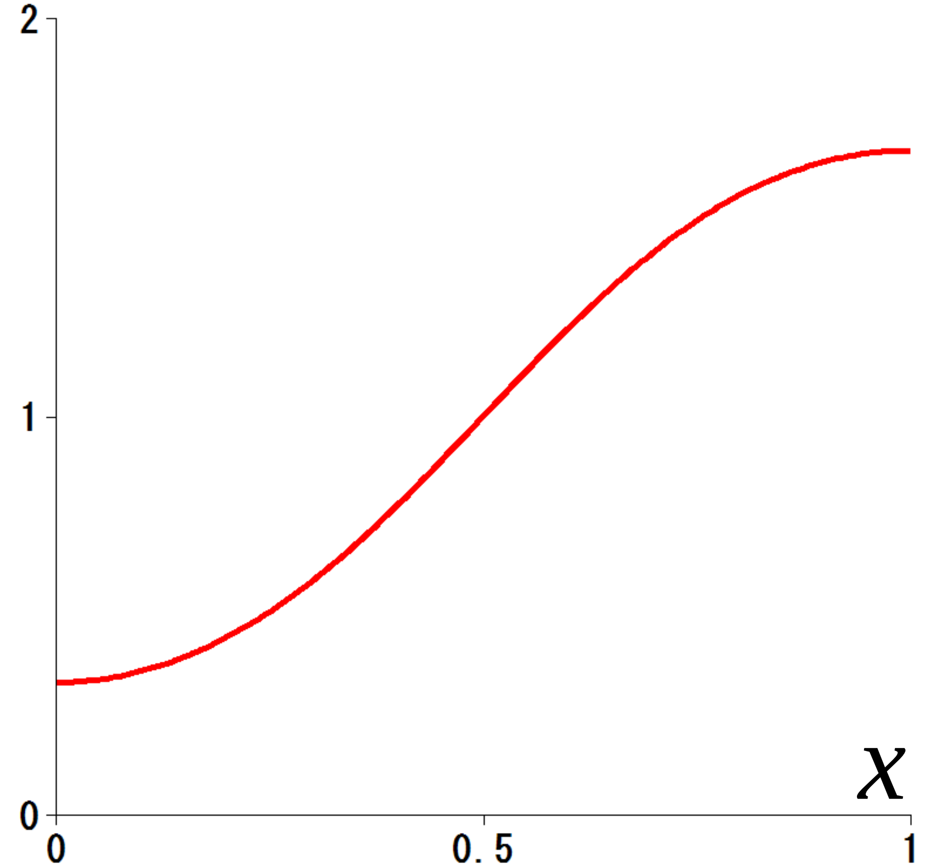

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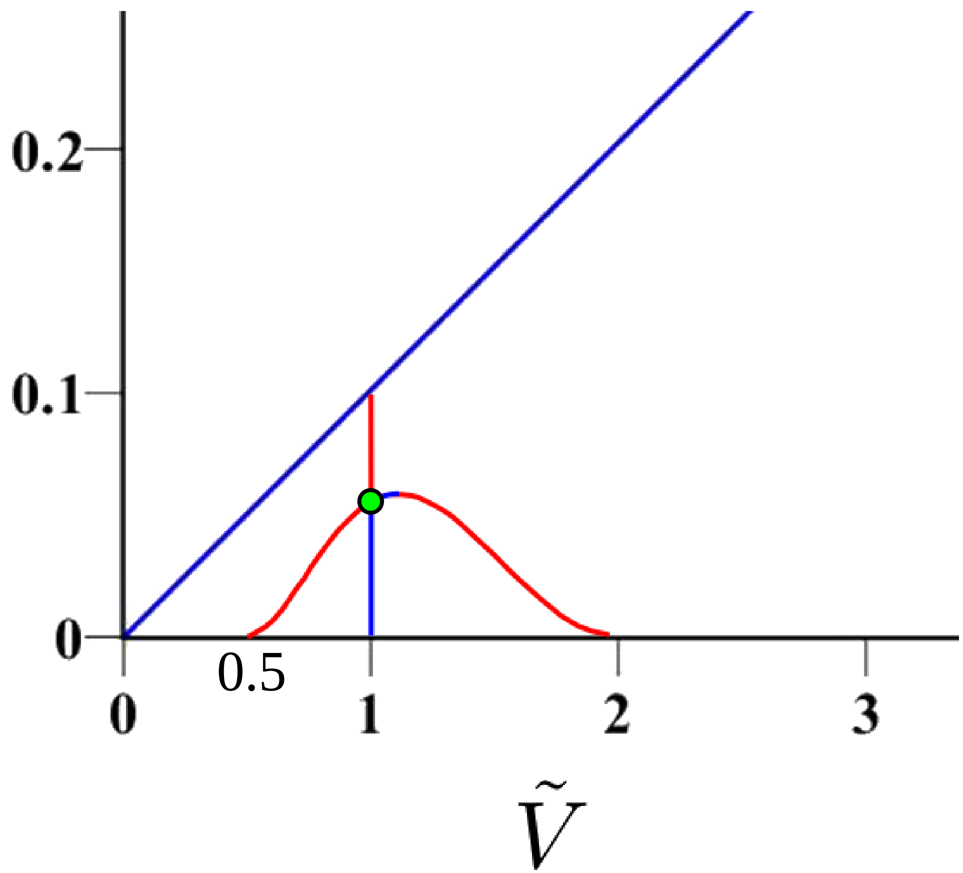

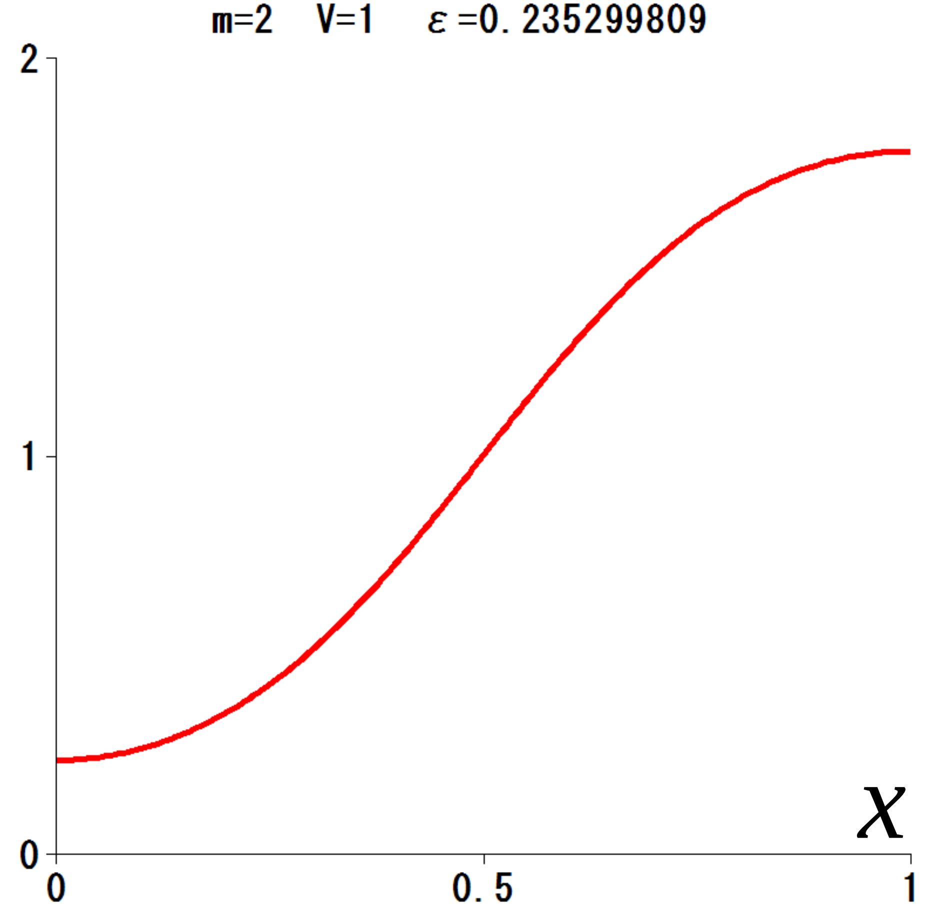

２次分岐点

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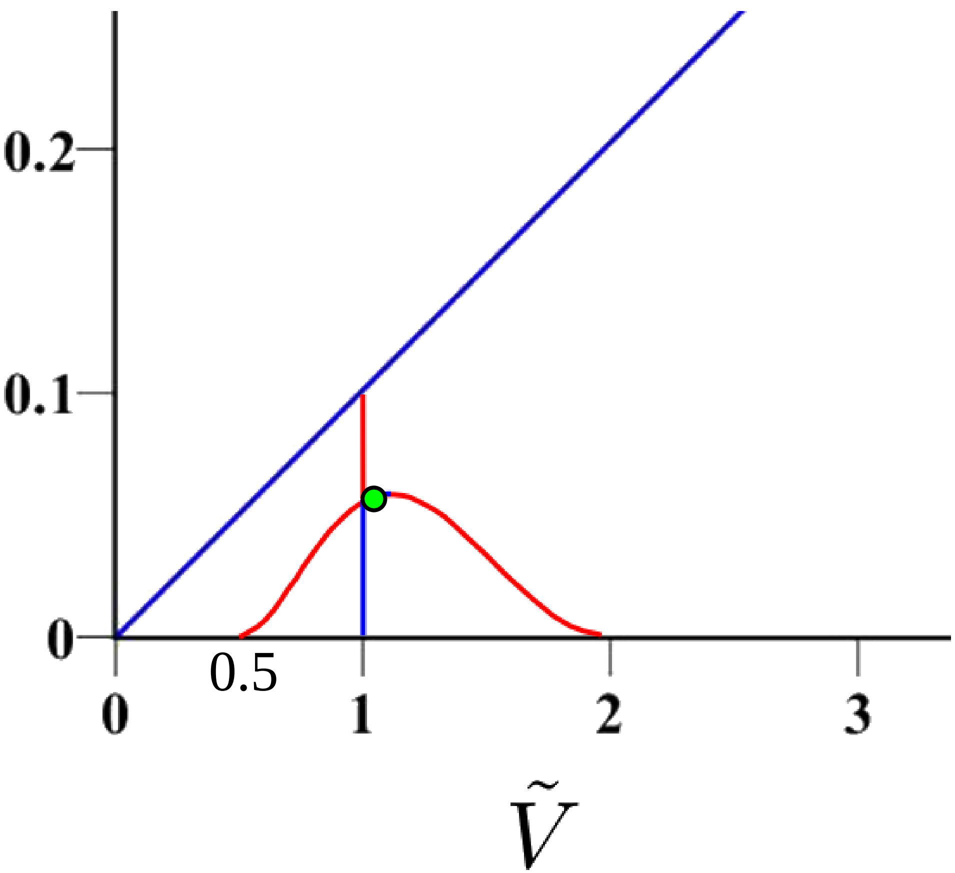

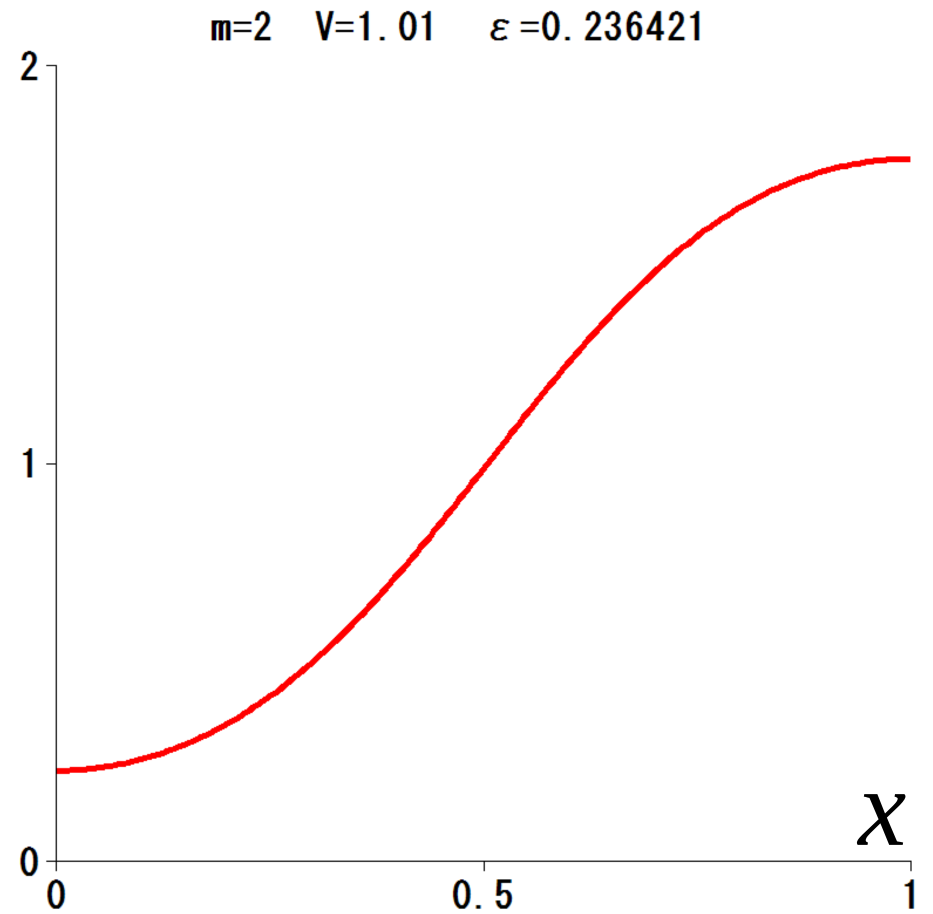

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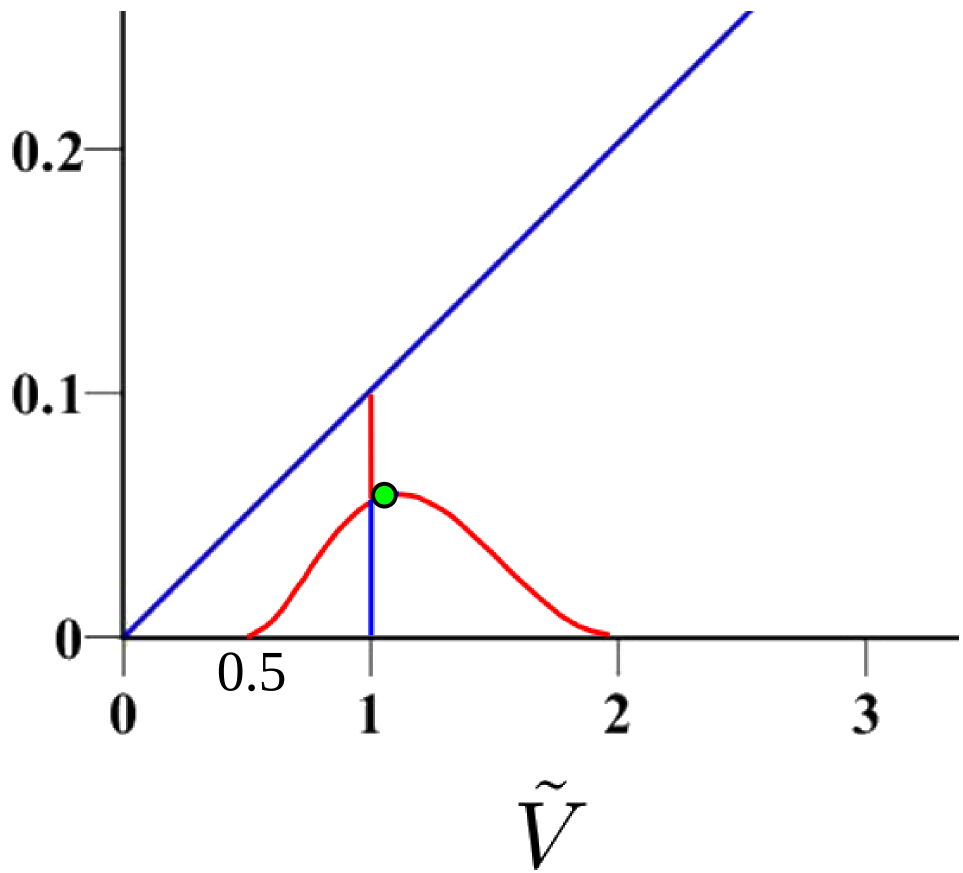
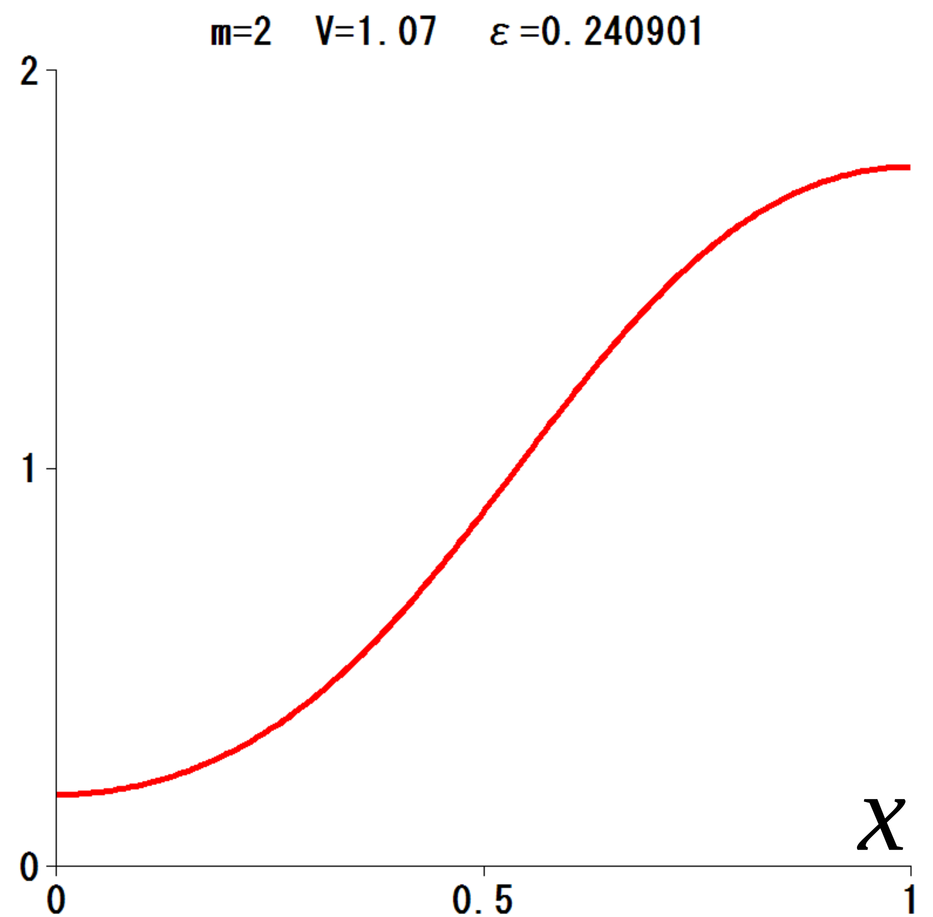
安定

$\tilde{V} \to 2,\ \varepsilon^2 \to 0$ の形状

$m = 2$



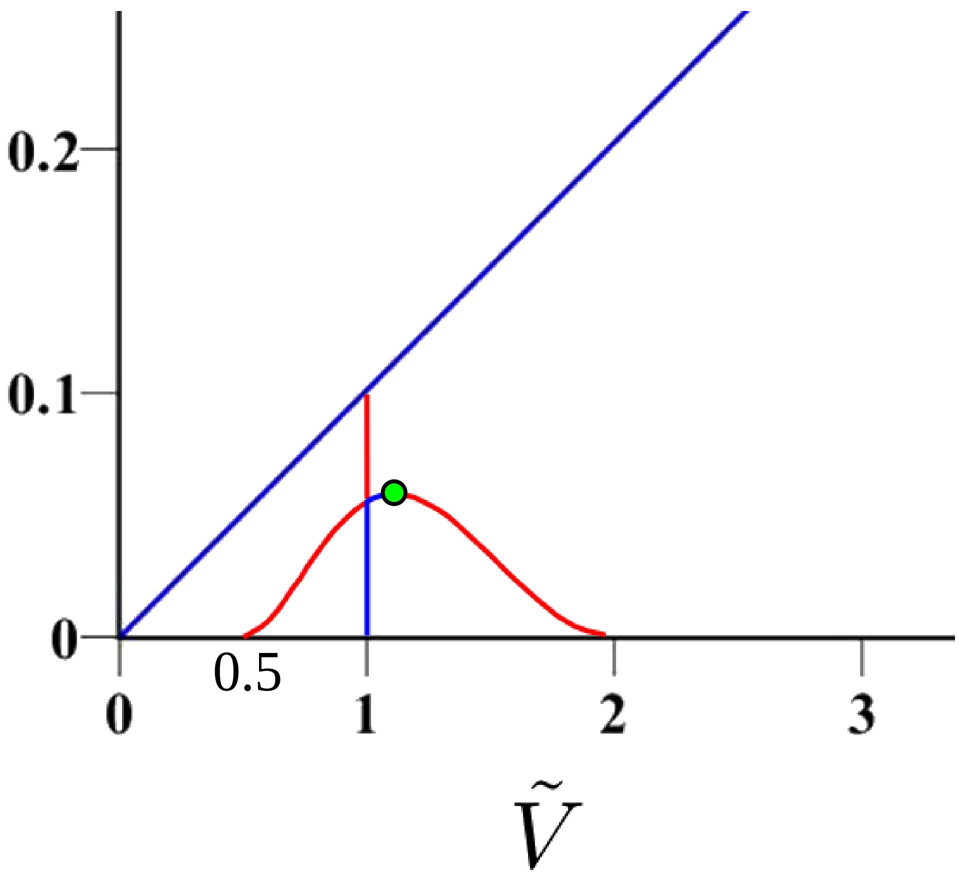


$$(SLP)\begin{cases} \varepsilon^2 W_{xx} + W(W-1)(\tilde{V}+1-W) = 0, \\ W_x(0) = W_x(1) = 0, \\ W'(x) > 0, \\ \int_0^1 W(x)\,\mathrm{dx} + \tilde{V} = m. \end{cases}$$

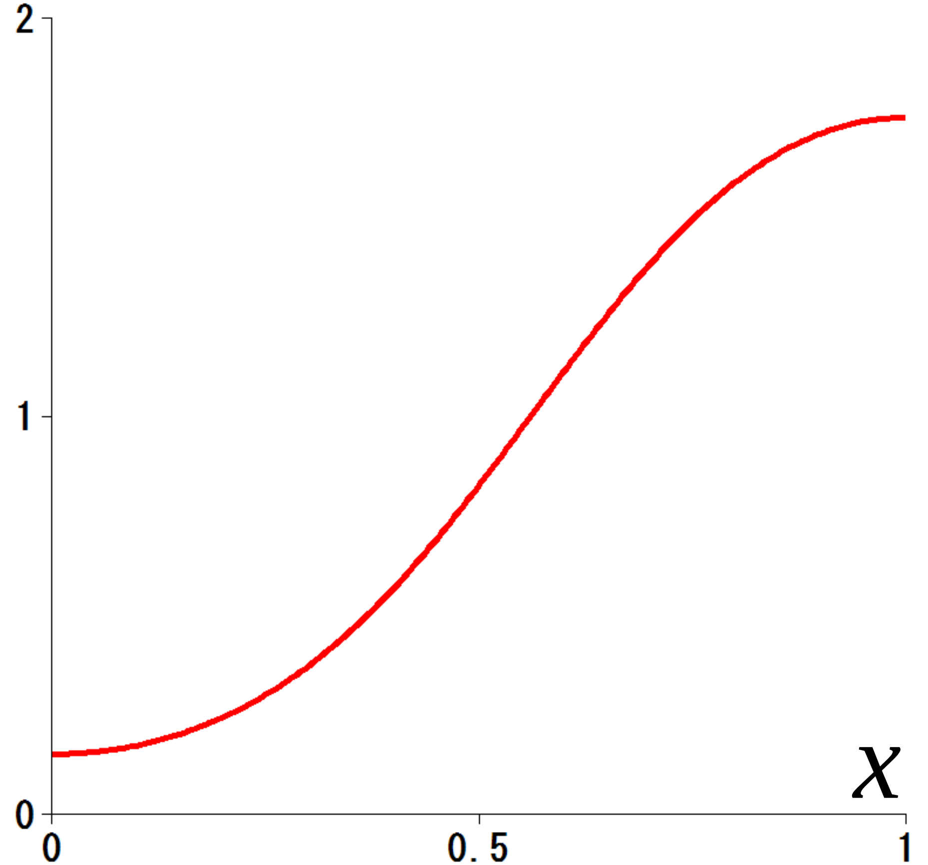
安定

$\tilde{V} \to 2,\ \varepsilon^2 \to 0$ の形状

$$m = 2$$



m=2 V=1.11 ε=0.241856



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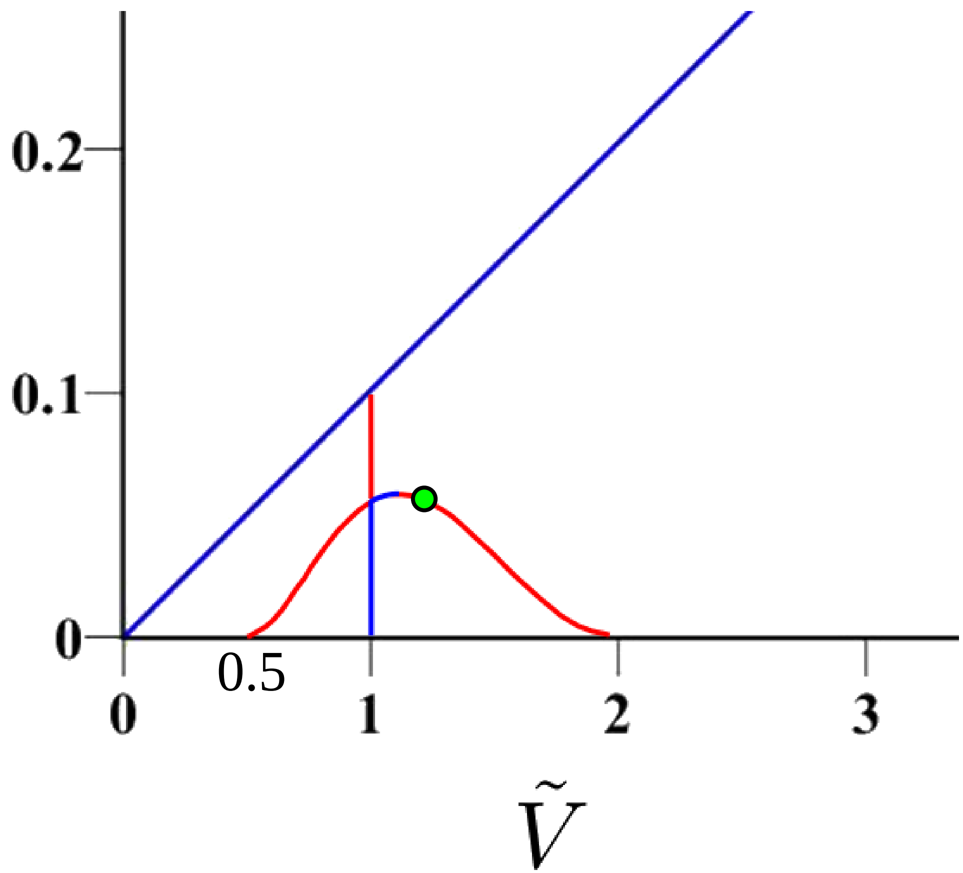
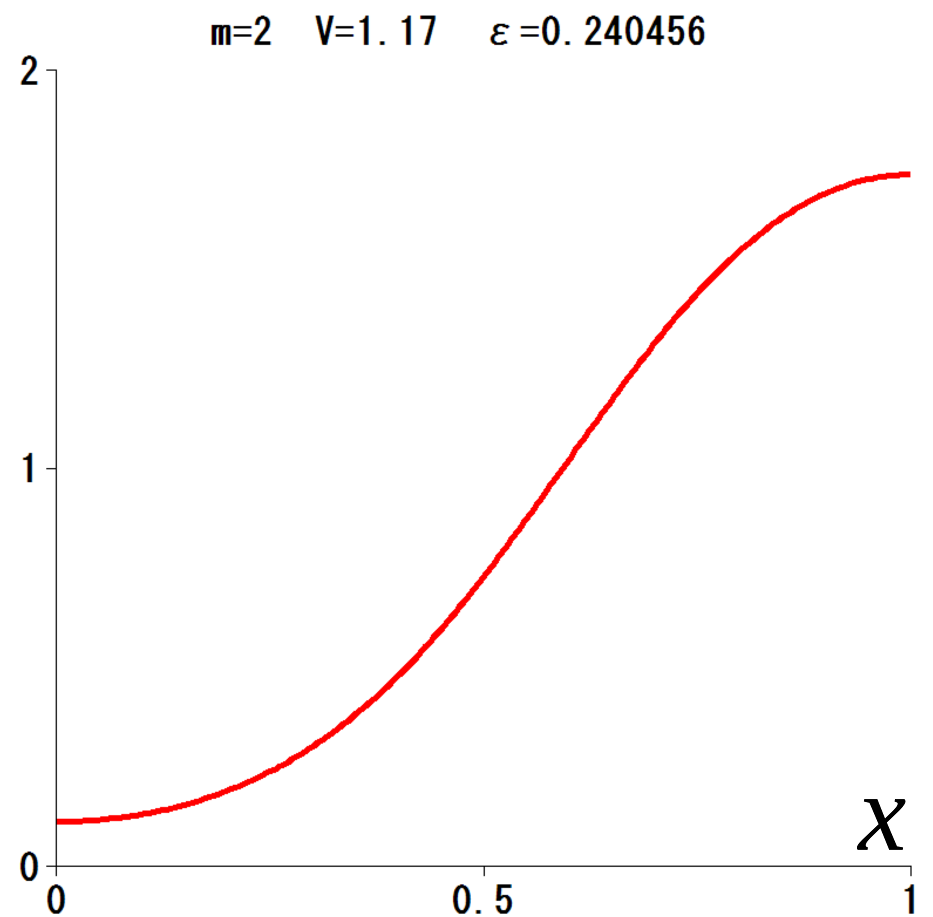
不安定

$\tilde{V} \to 2,\ \varepsilon^2 \to 0$ の形状

$$m = 2$$





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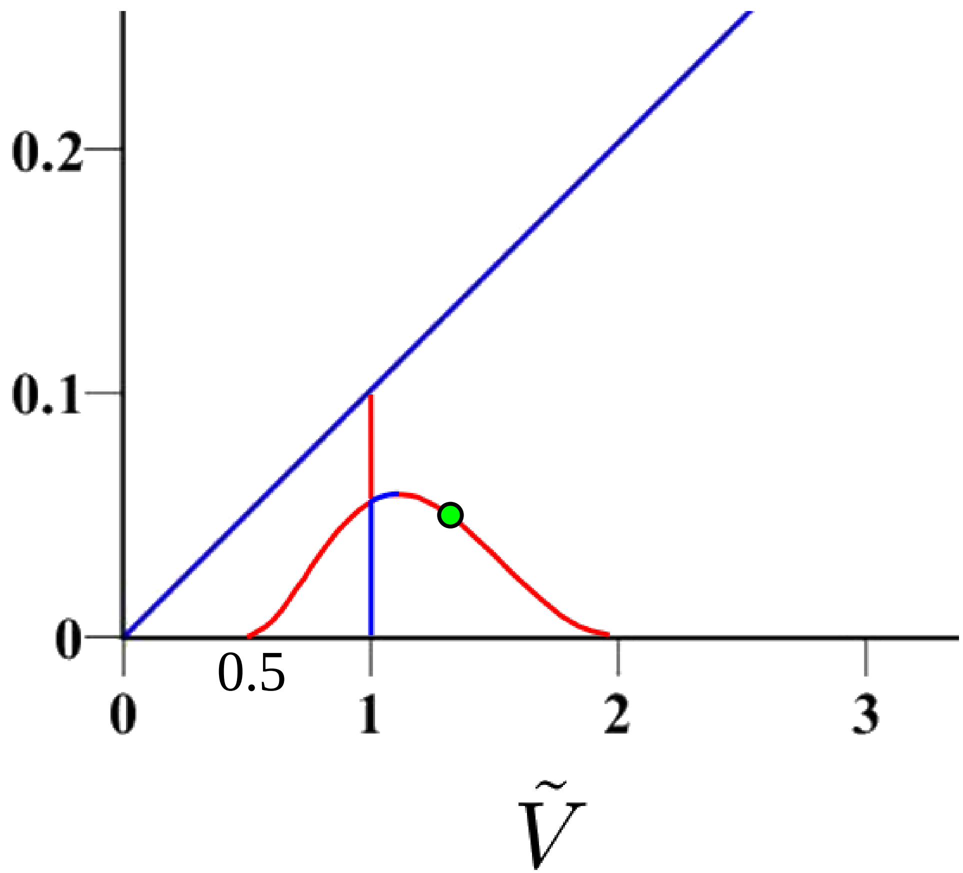
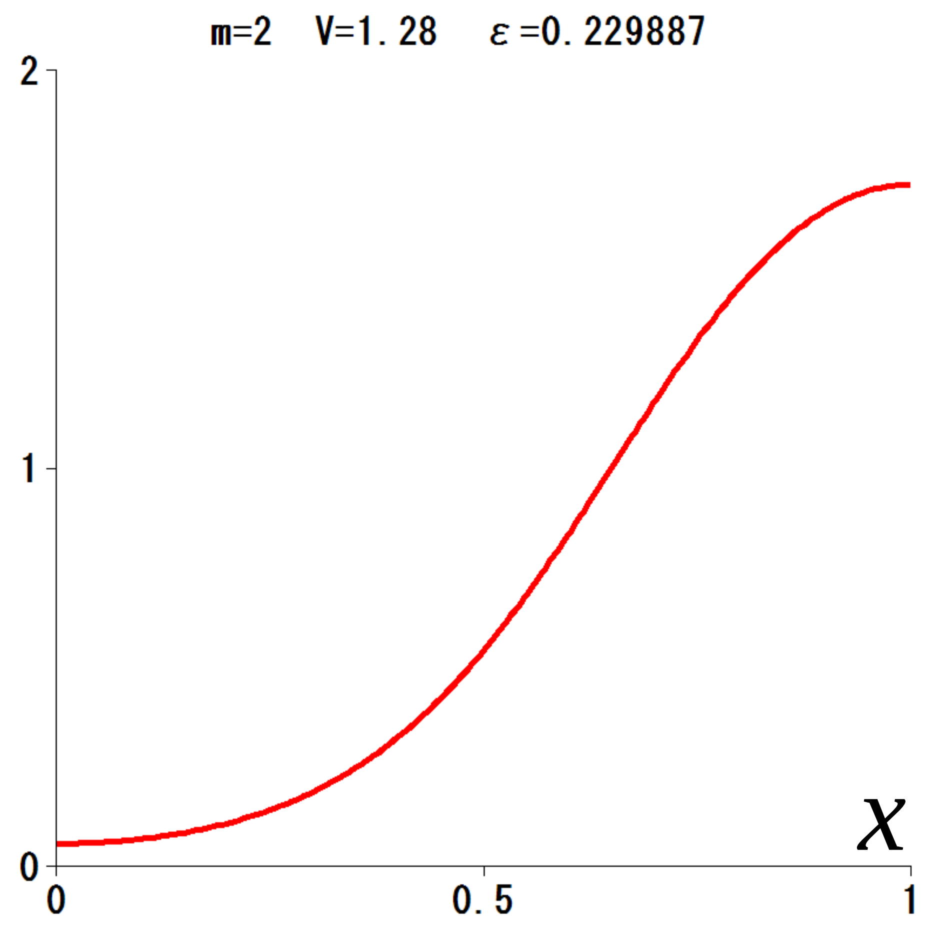
不安定

$\tilde{V} \to 2,\ \varepsilon^2 \to 0$ の形状

$m = 2$





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不安定

$\tilde{V} \to 2,\ \varepsilon^2 \to 0$ の形状

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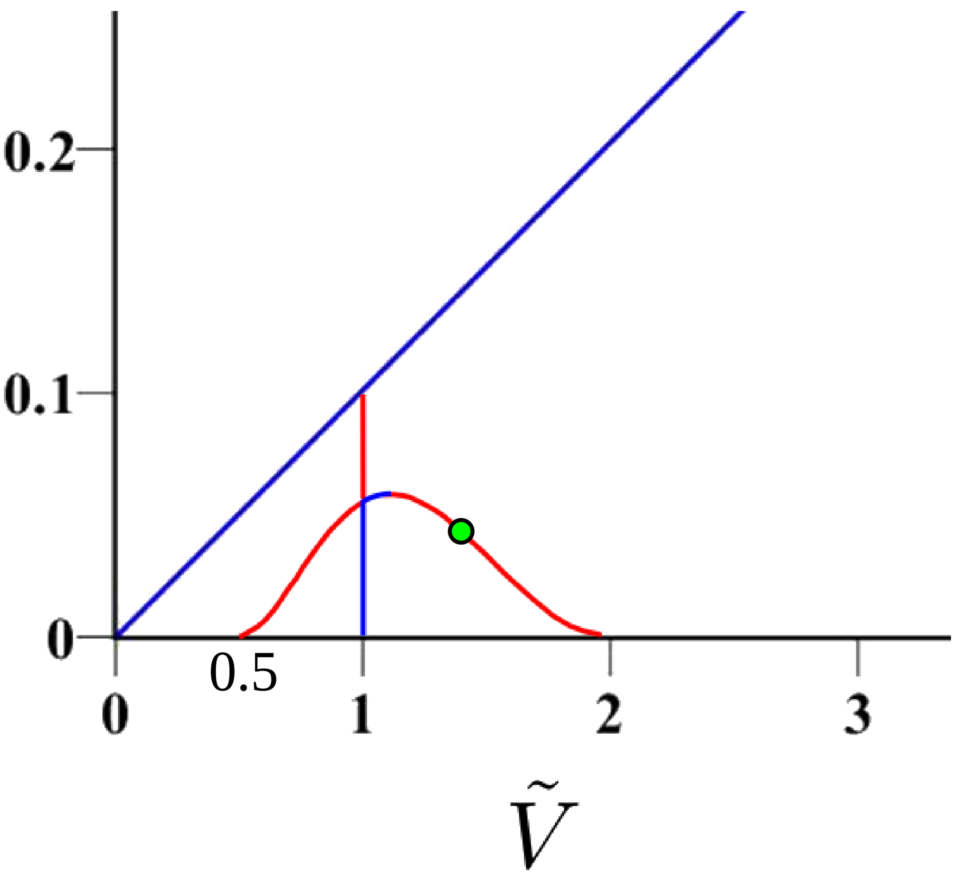

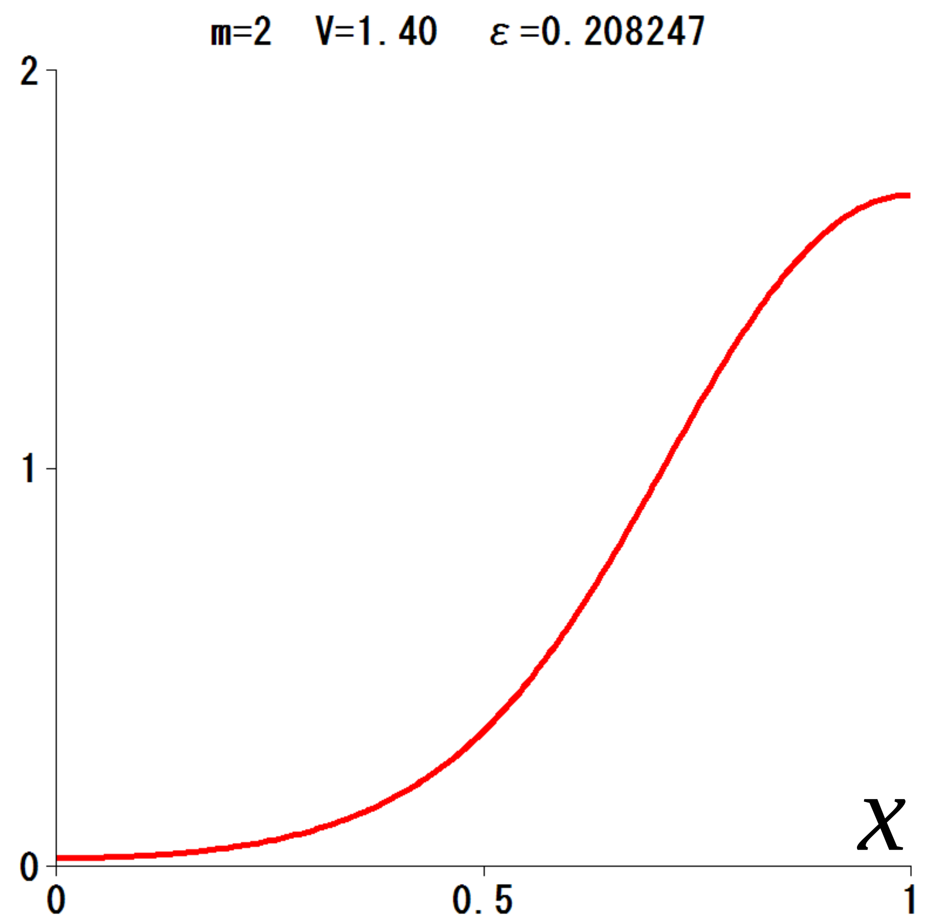

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不安定

$\tilde{V} \to 2,\ \varepsilon^2 \to 0$ の形状

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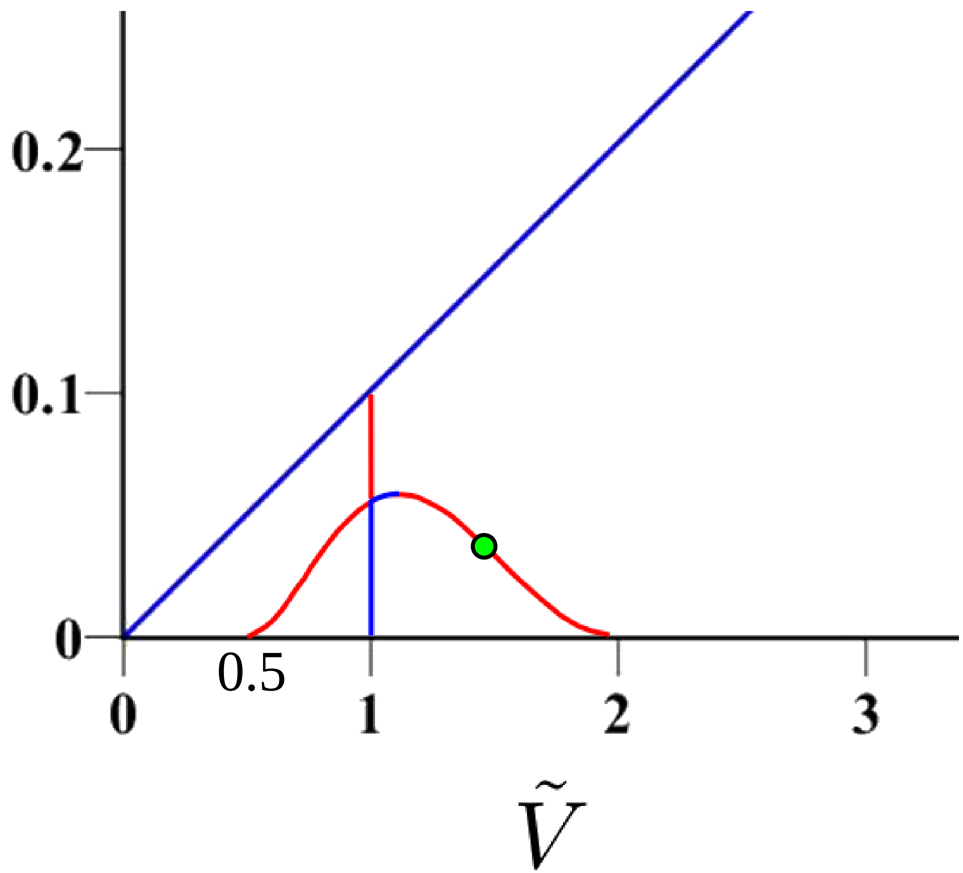

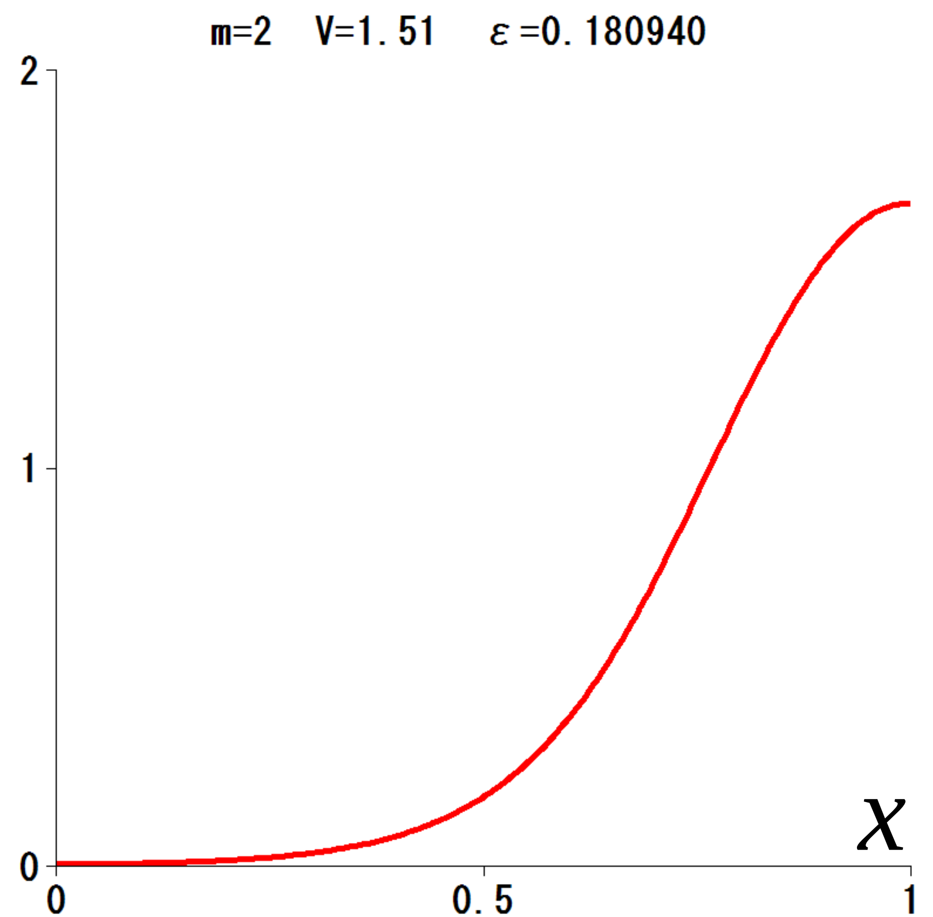

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不安定

$\tilde{V} \to 2,\ \varepsilon^2 \to 0$ の形状

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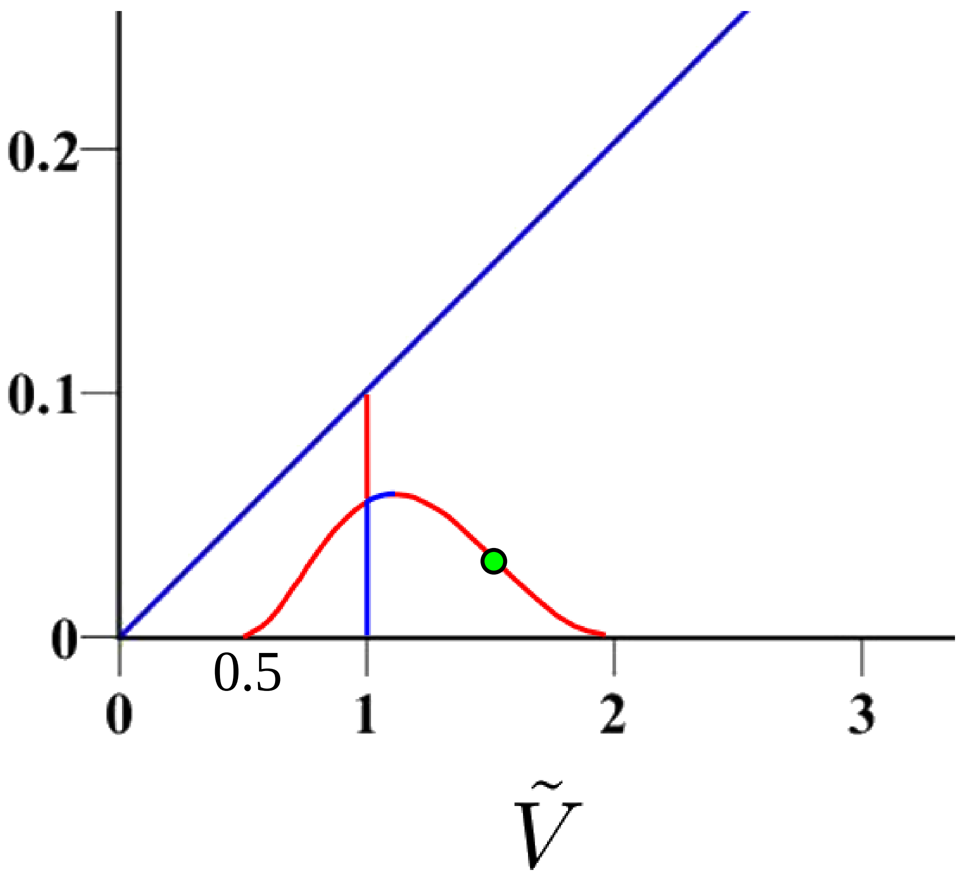
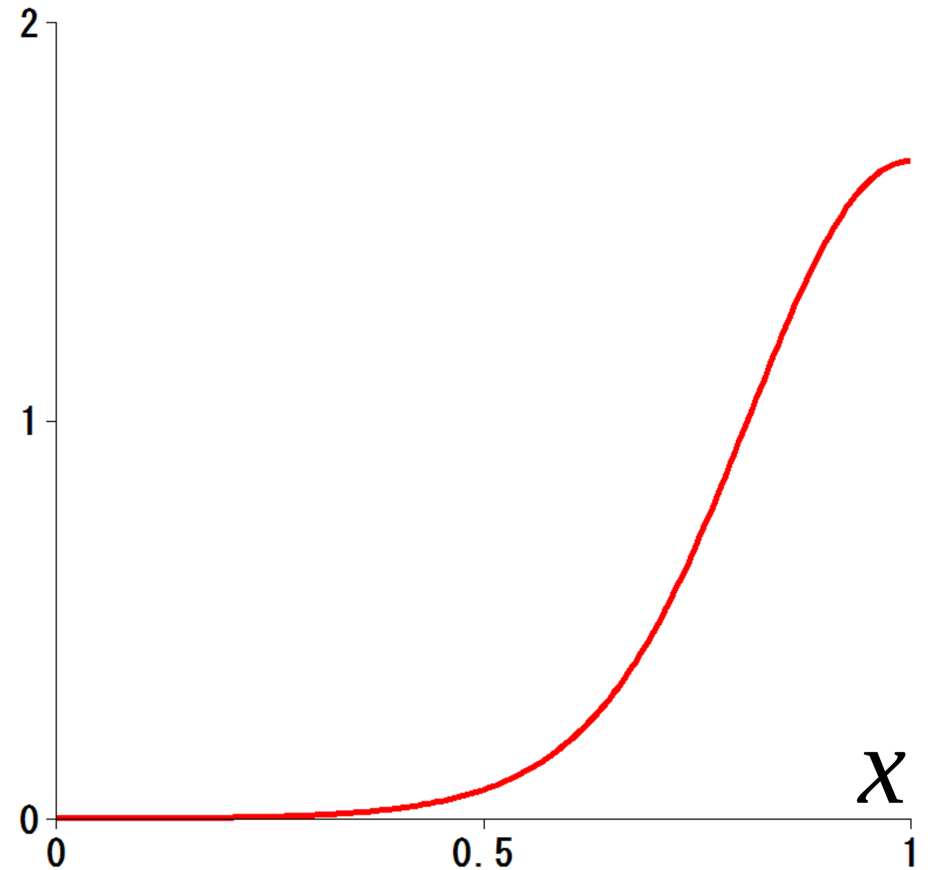


m=2 V=1.60 ε=0.154282



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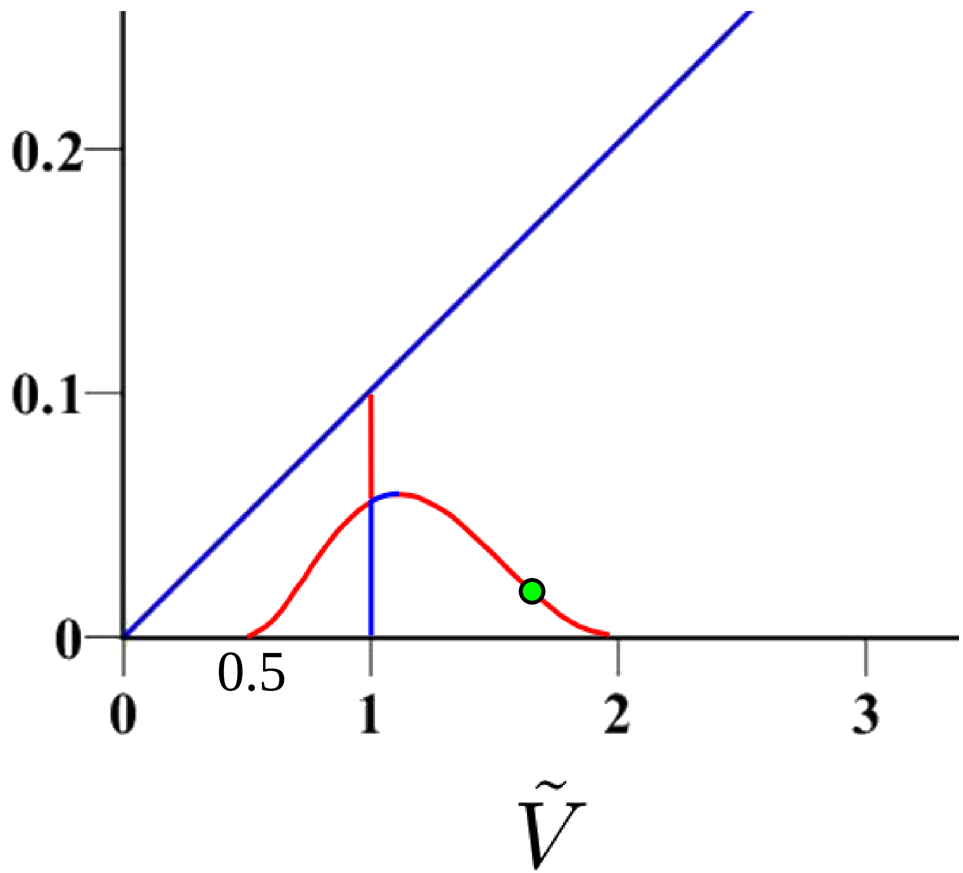
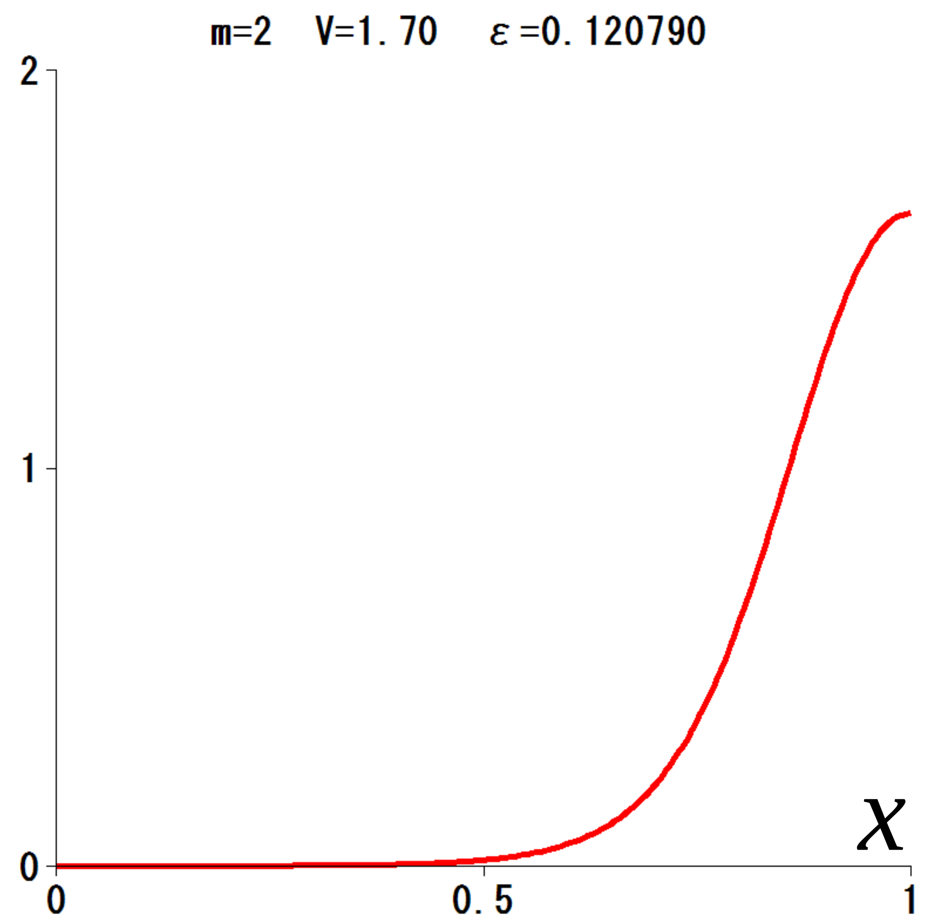
不安定

$\tilde{V} \to 2,\ \varepsilon^2 \to 0$ の形状

$$m = 2$$





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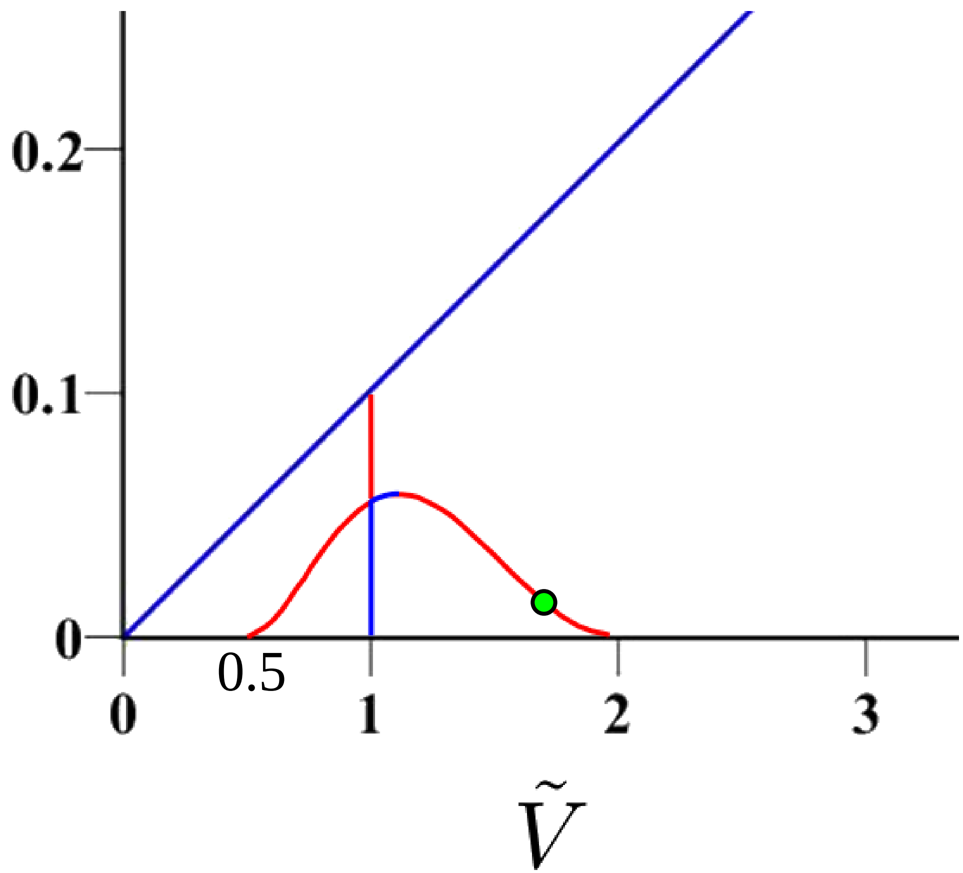
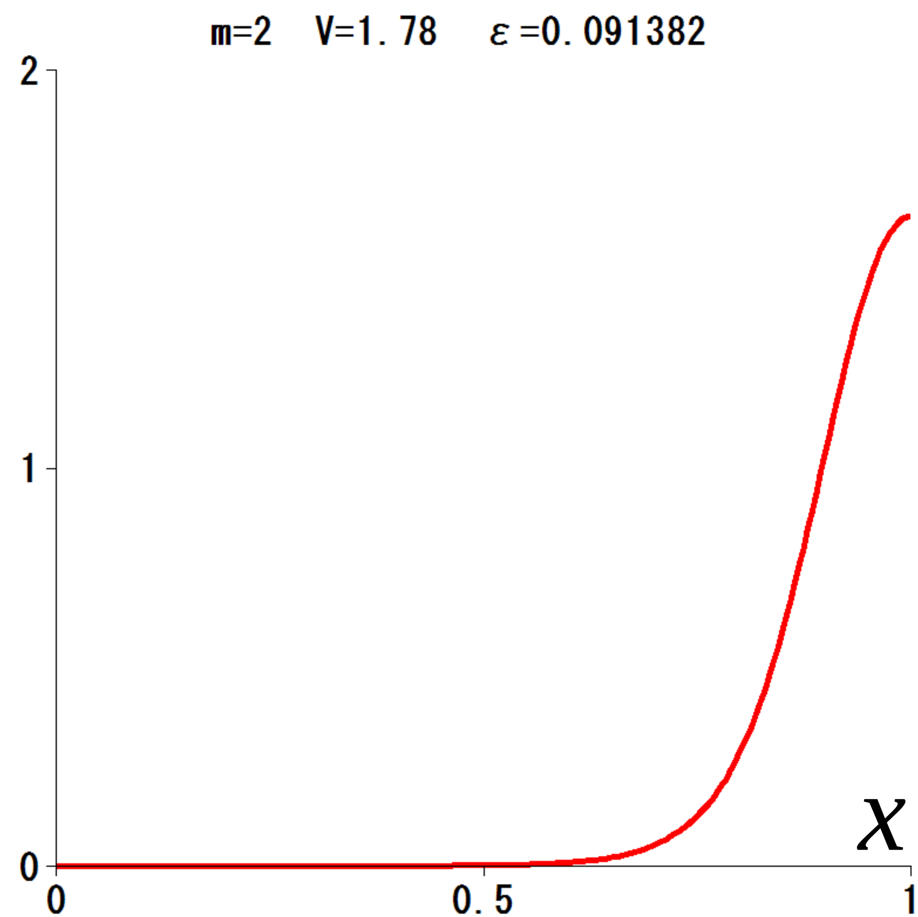
不安定

$\tilde{V} \to 2,\ \varepsilon^2 \to 0$ の形状

$$m = 2$$





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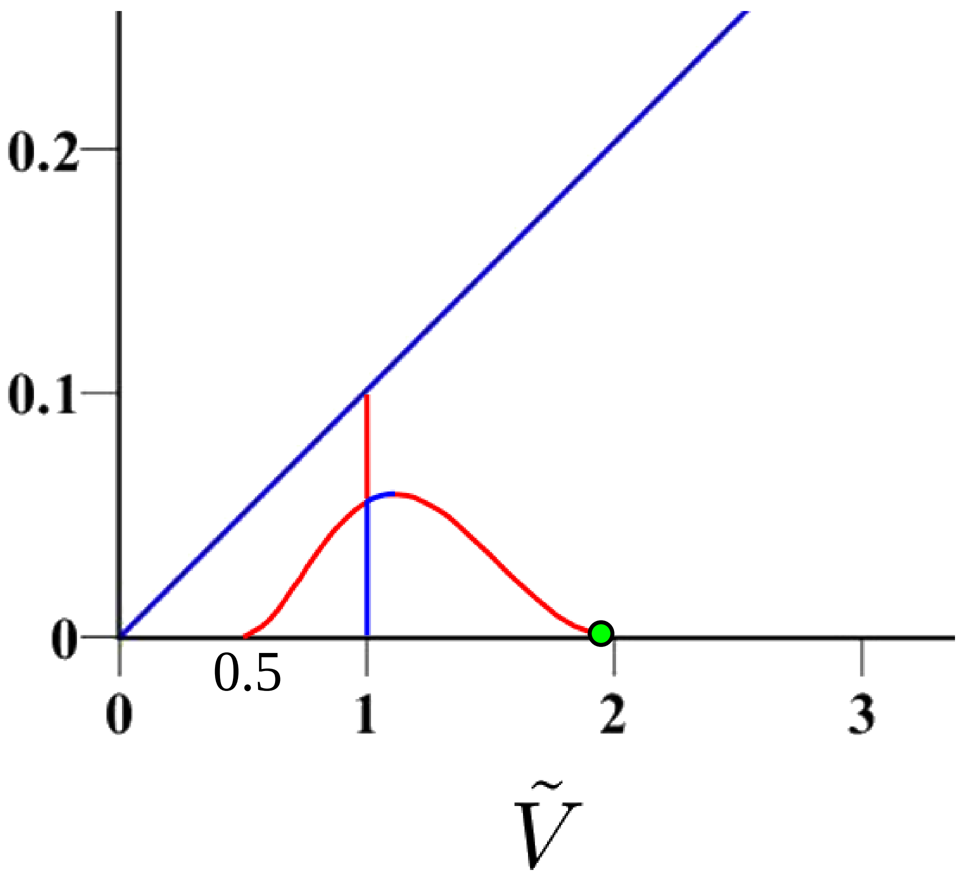
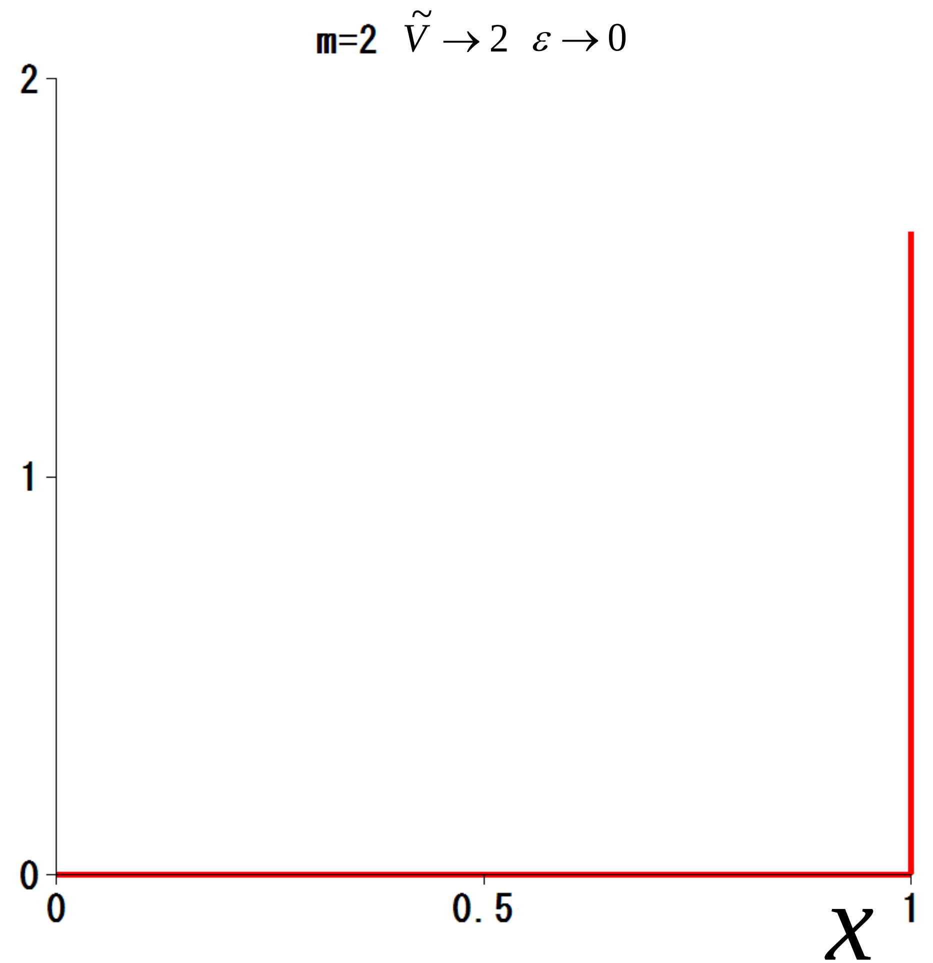
不安定

$\tilde{V} \to 2,\ \varepsilon^2 \to 0$ の形状

$$m = 2$$





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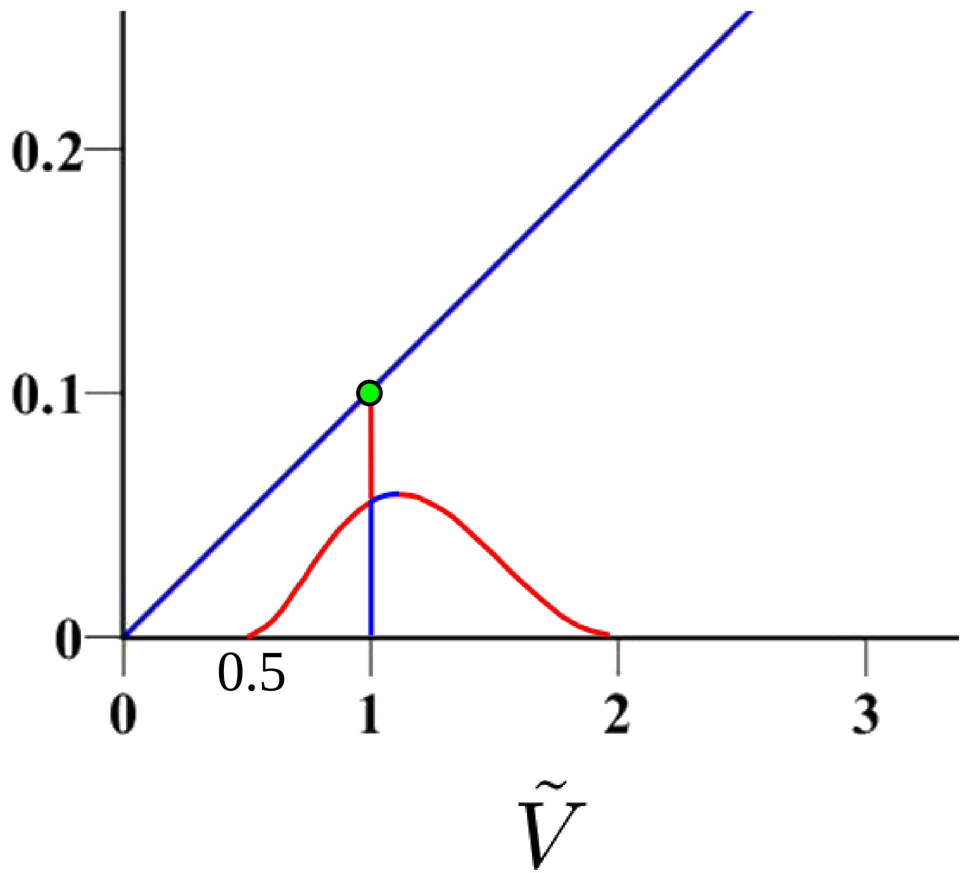
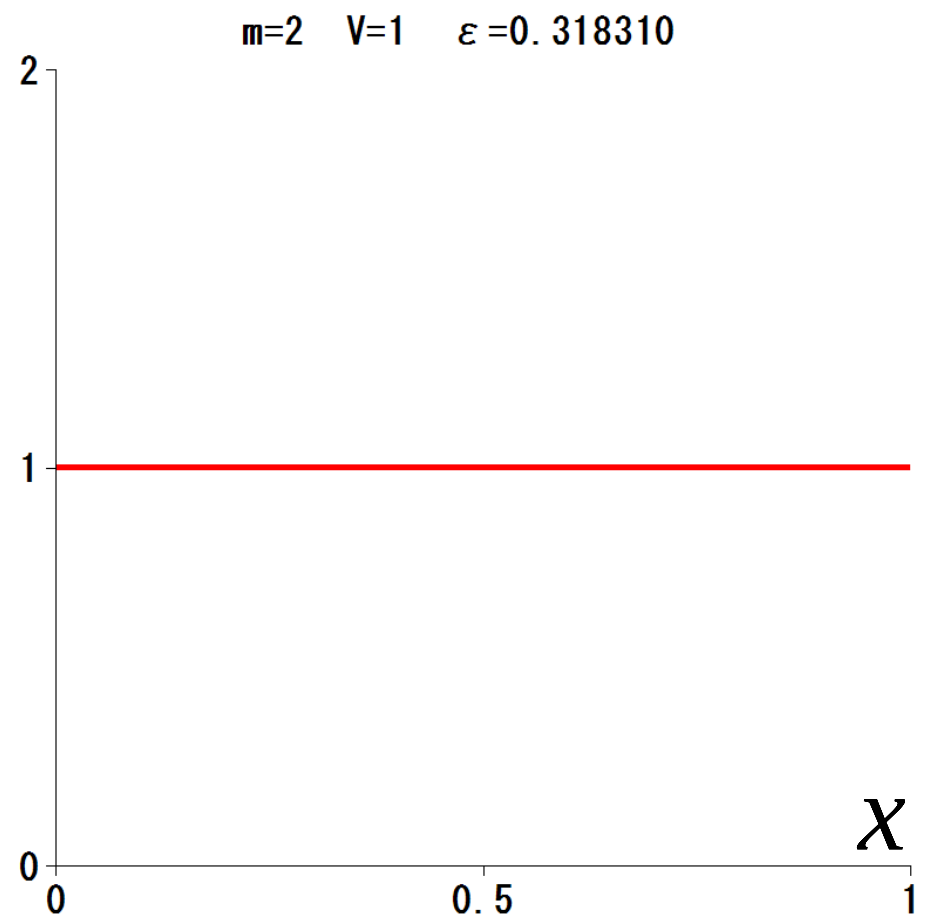
不安定

$\tilde{V} \to 0.5,\ \varepsilon^2 \to 0$ の形状

$m = 2$





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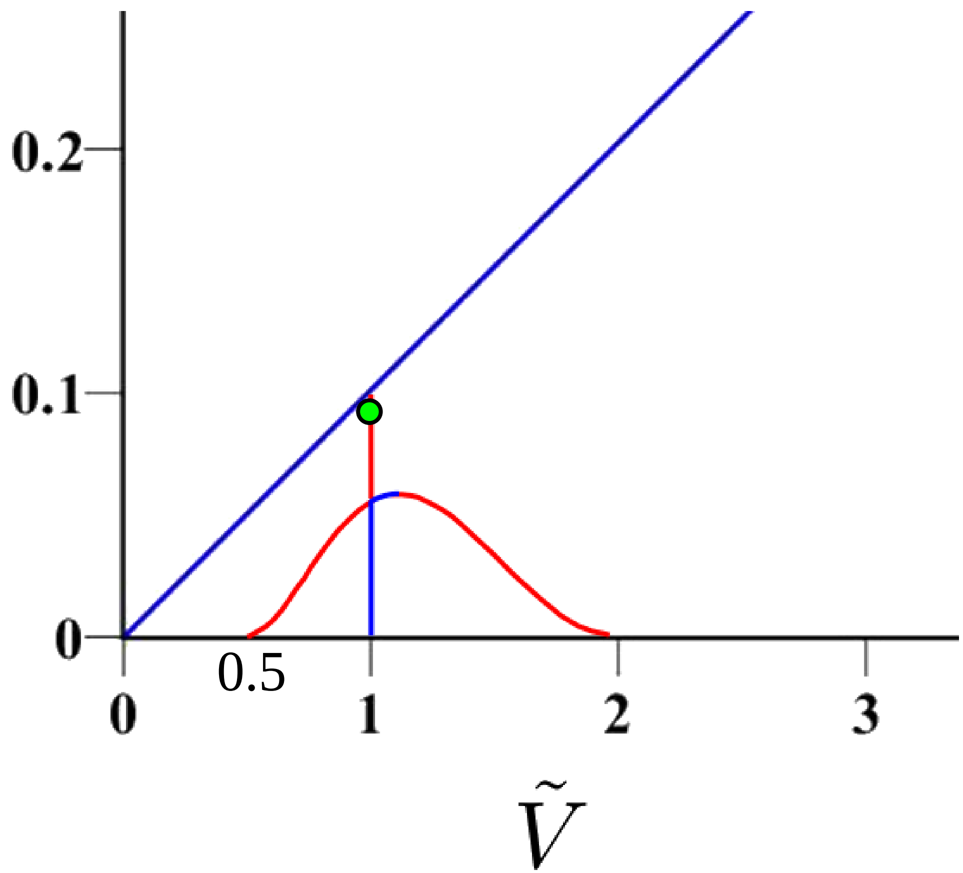
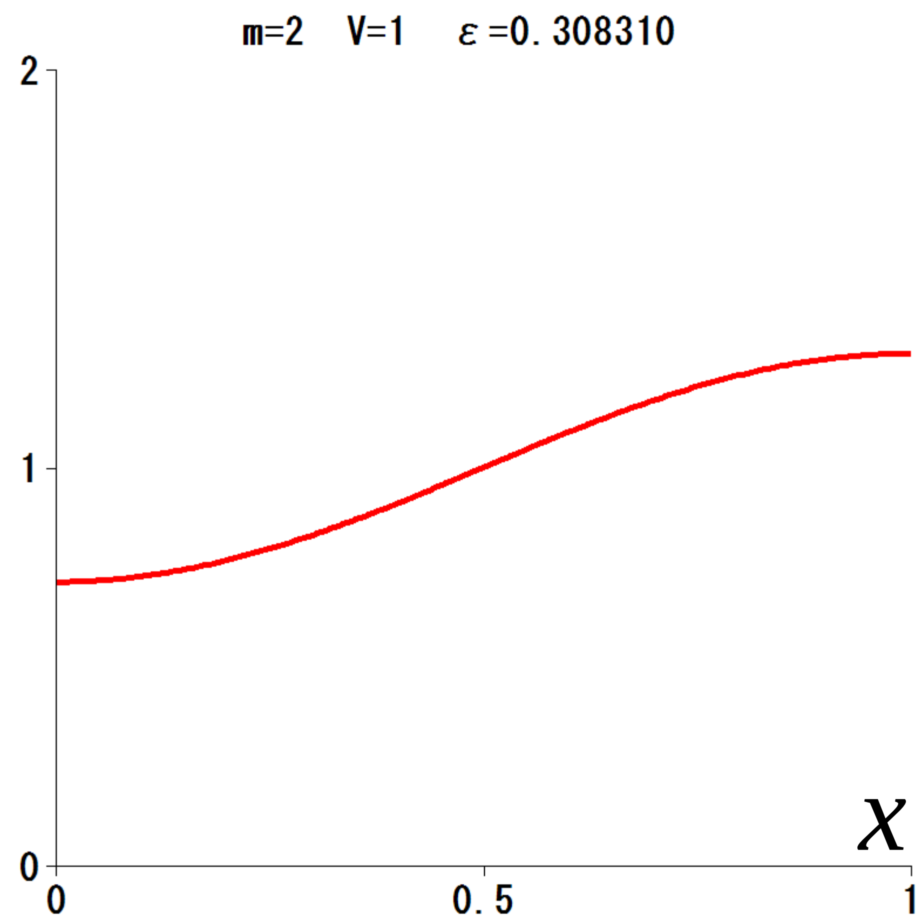
不安定

$\tilde{V} \to 0.5,\ \varepsilon^2 \to 0$ の形状

$$m = 2$$





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不安定

$\tilde{V} \to 0.5,\ \varepsilon^2 \to 0$ の形状

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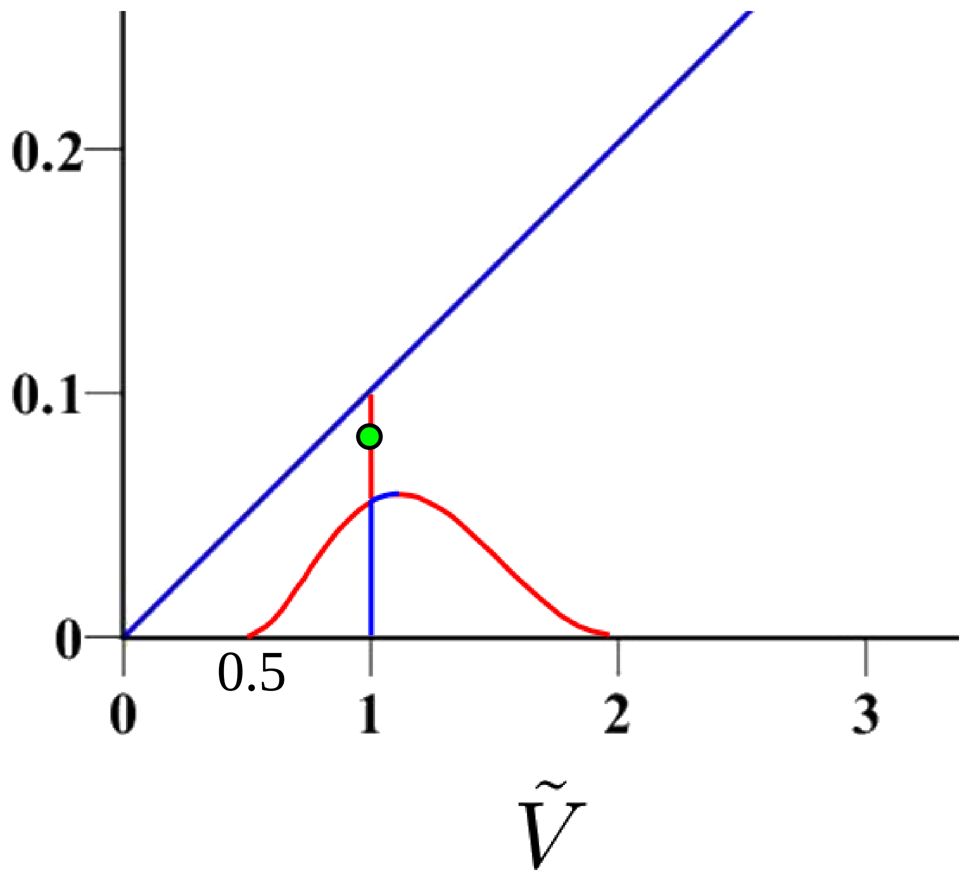

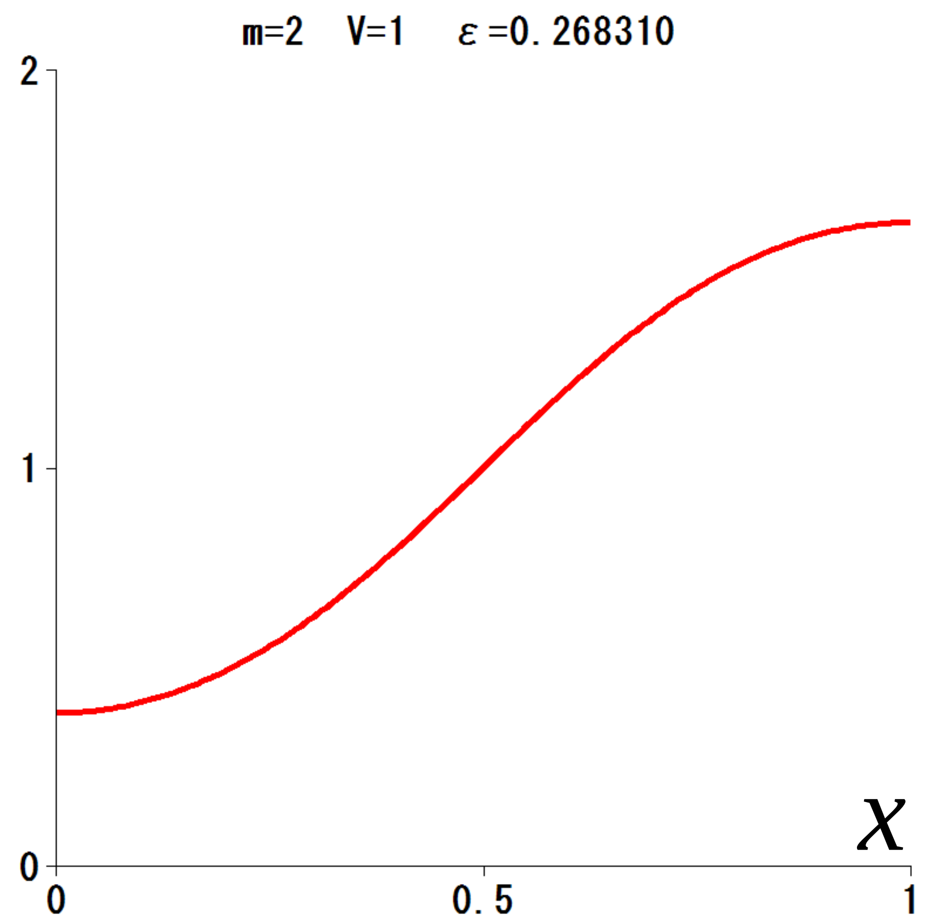

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不安定

$\tilde{V} \to 0.5,\ \varepsilon^2 \to 0$ の形状

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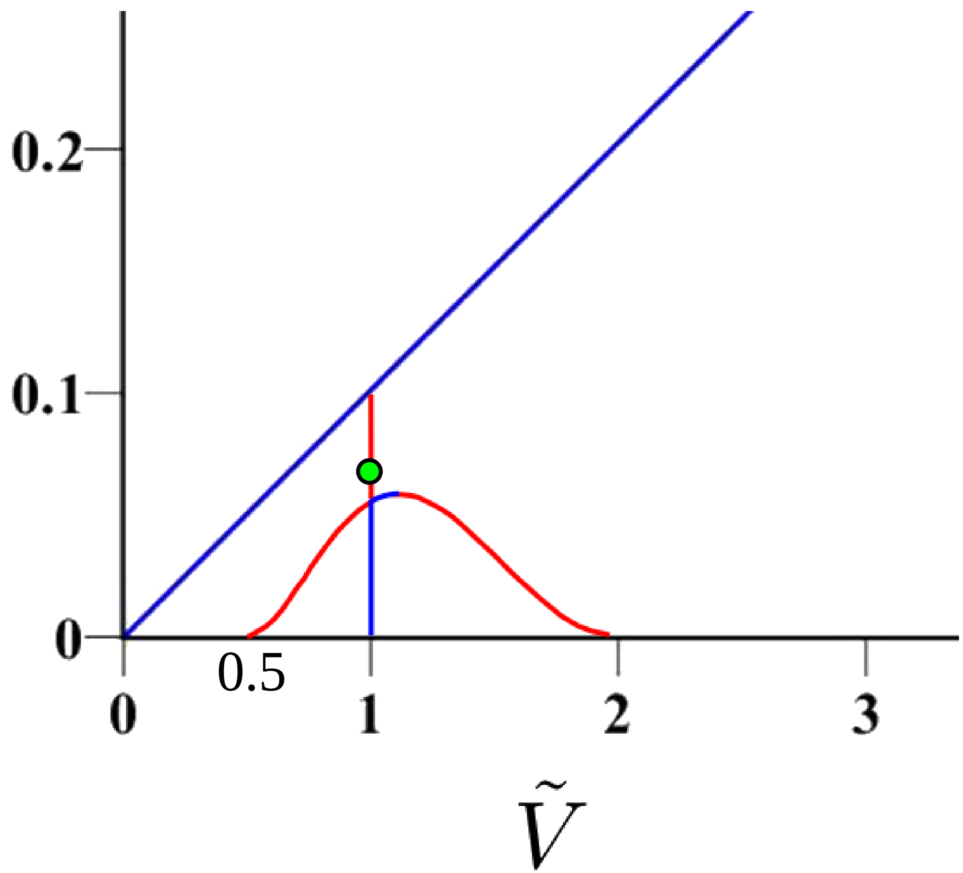

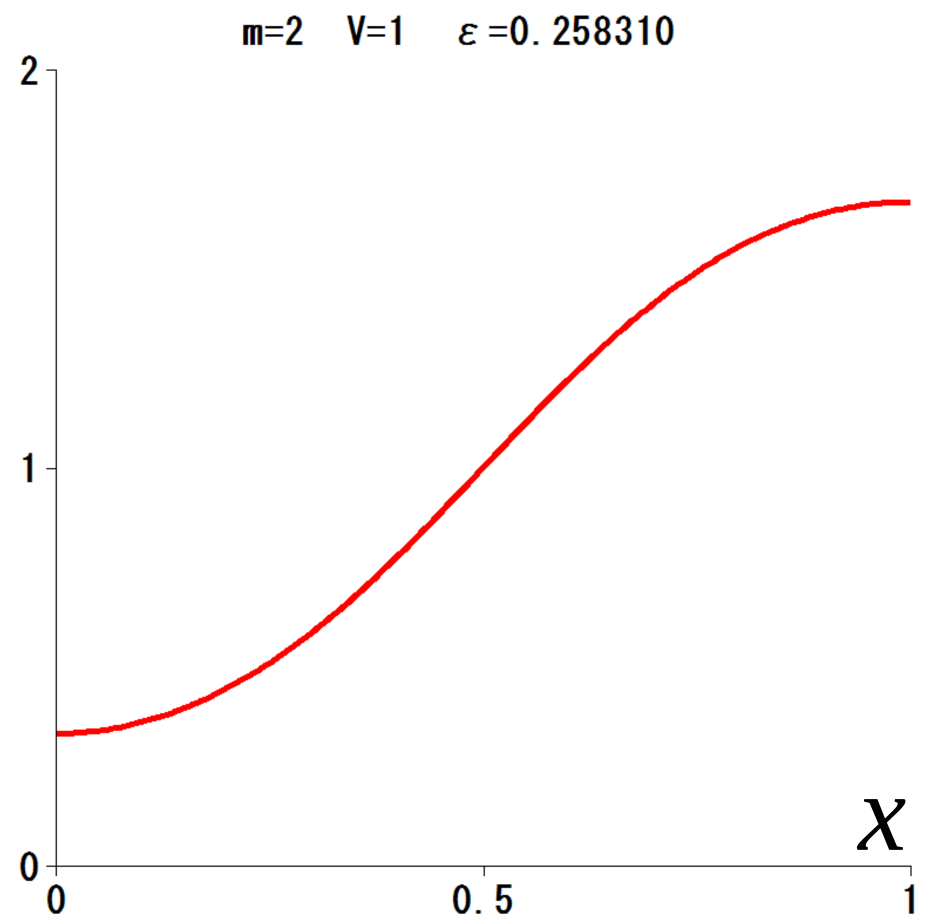

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安定

$\tilde{V} \to 0.5,\ \varepsilon^2 \to 0$ の形状

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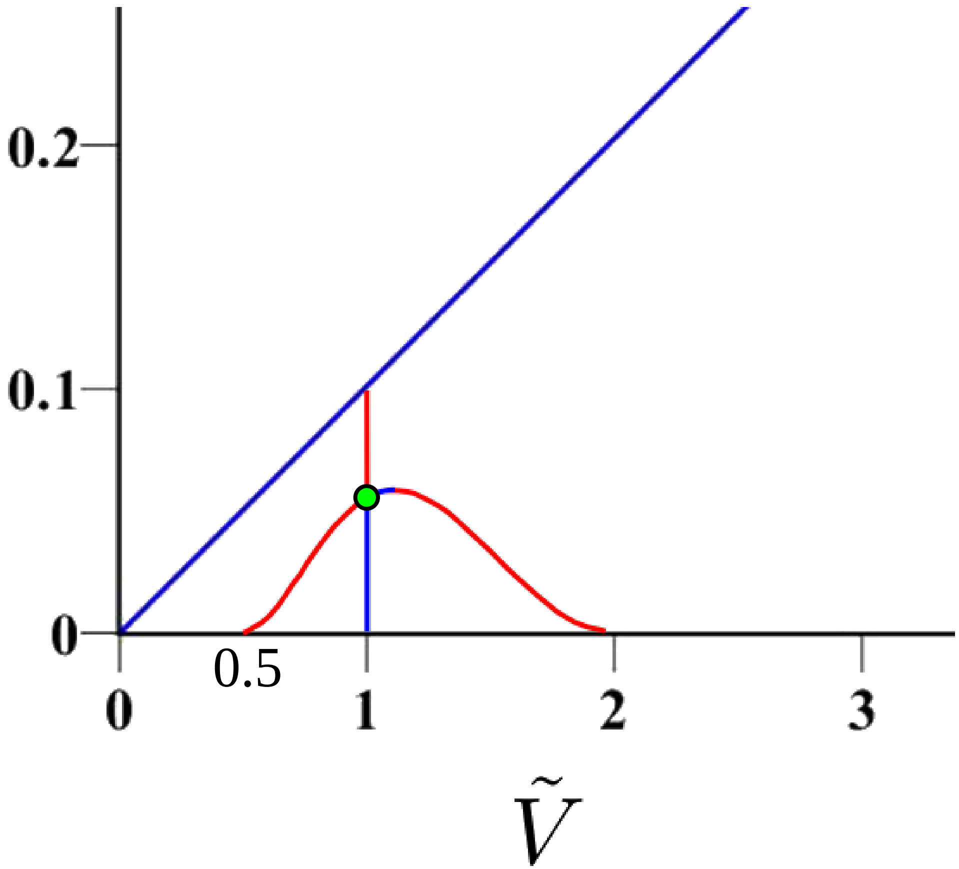

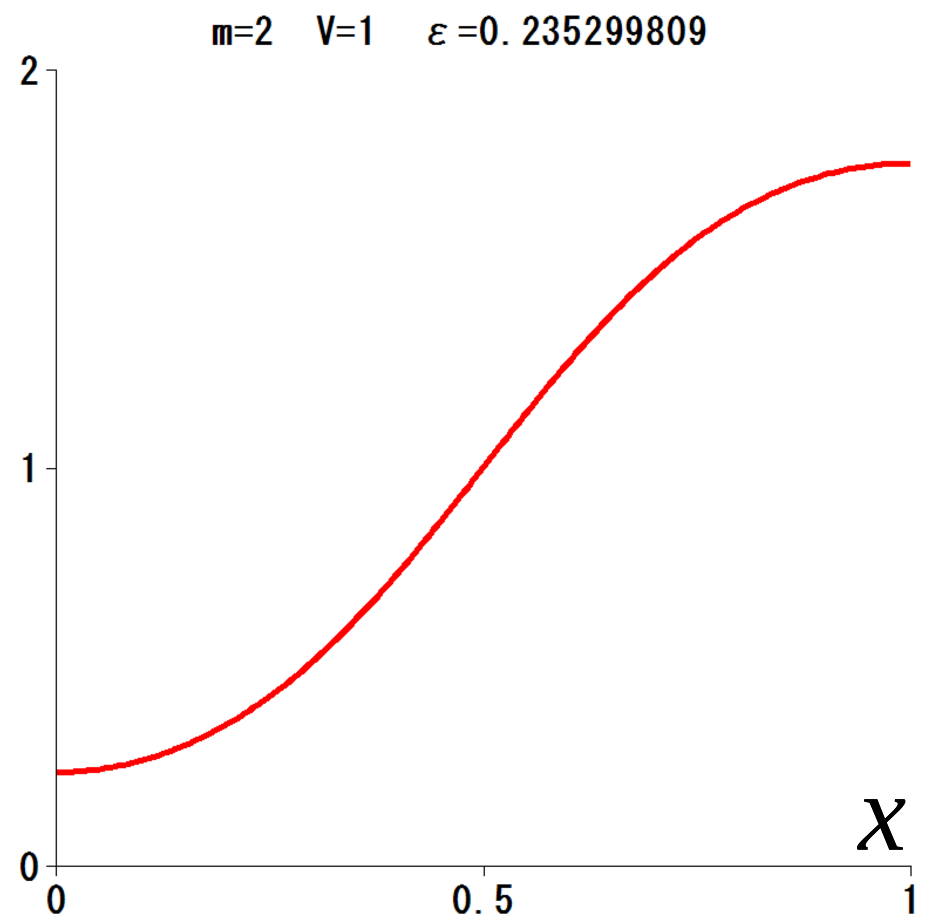

２次分岐点

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不安定

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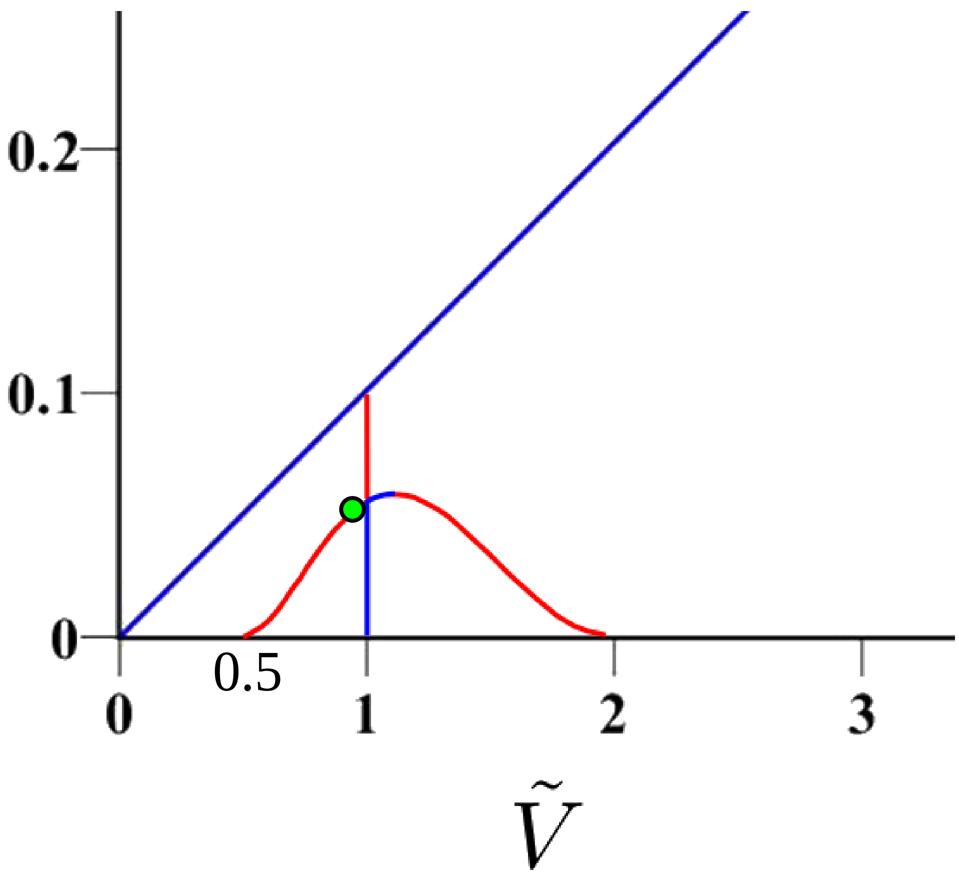

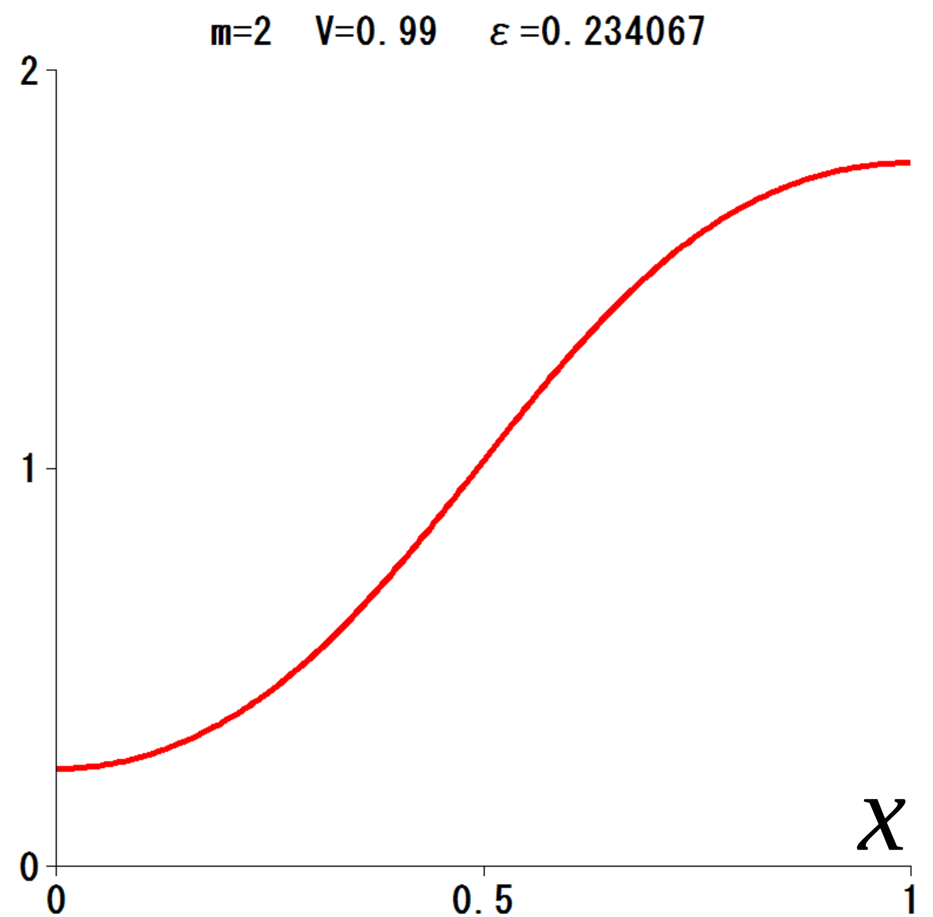

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不安定

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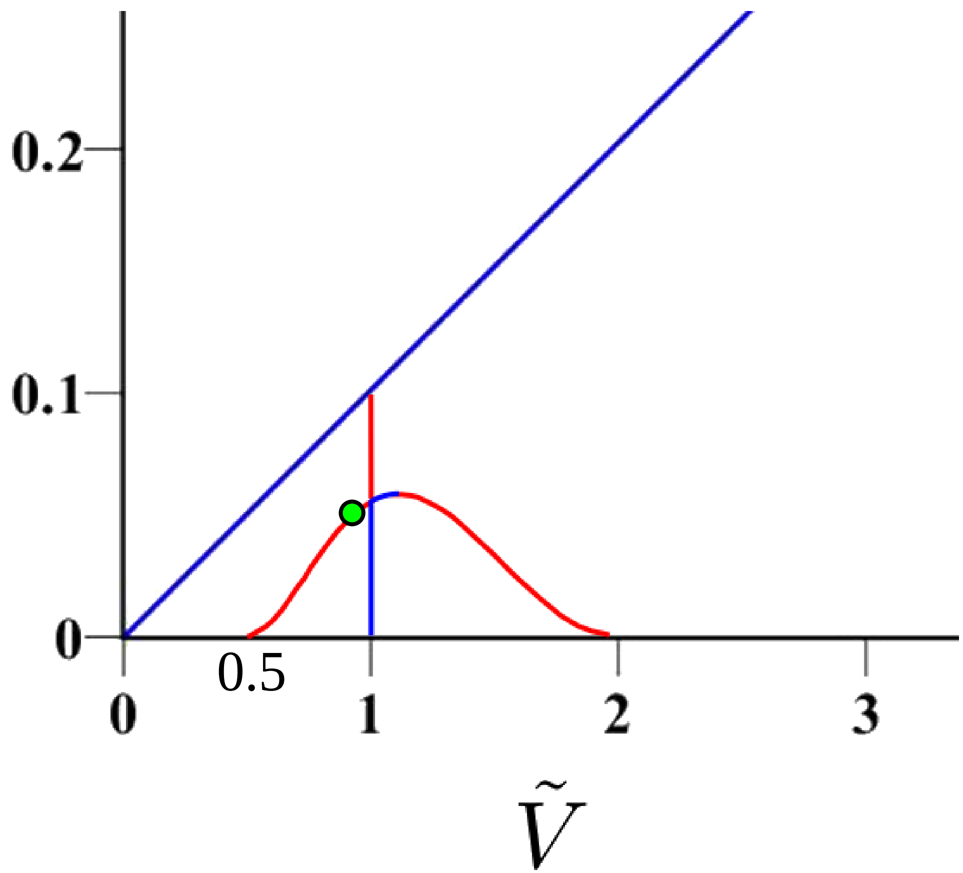

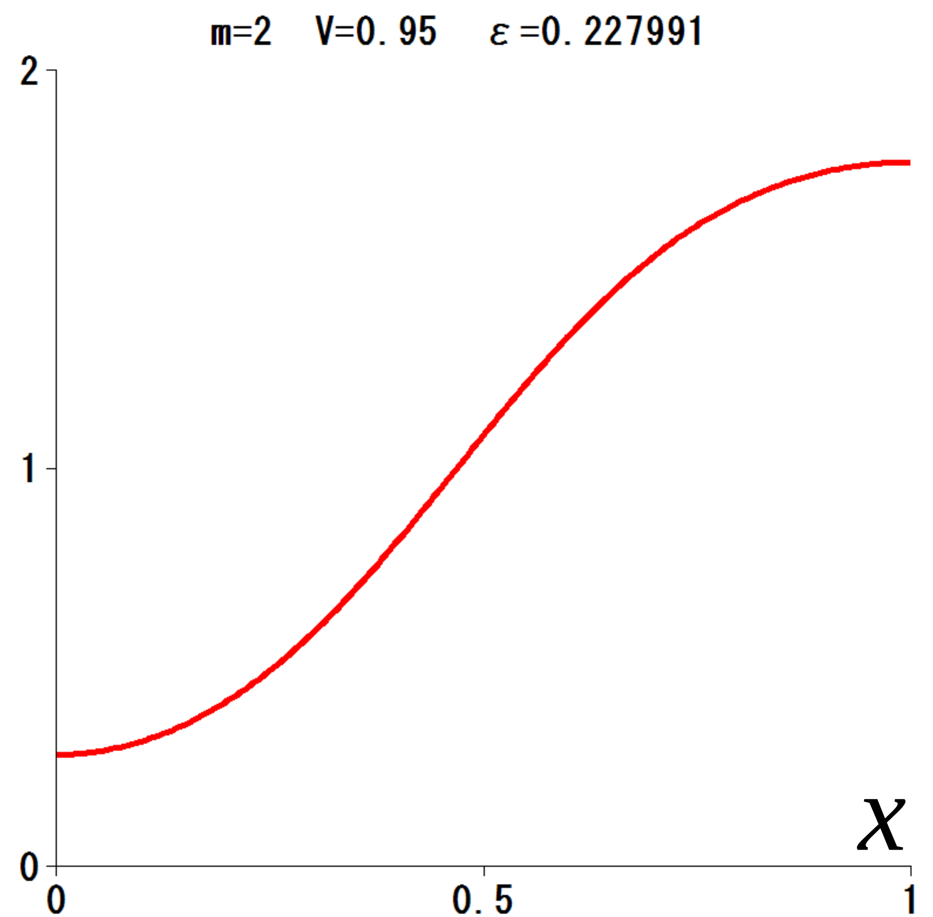

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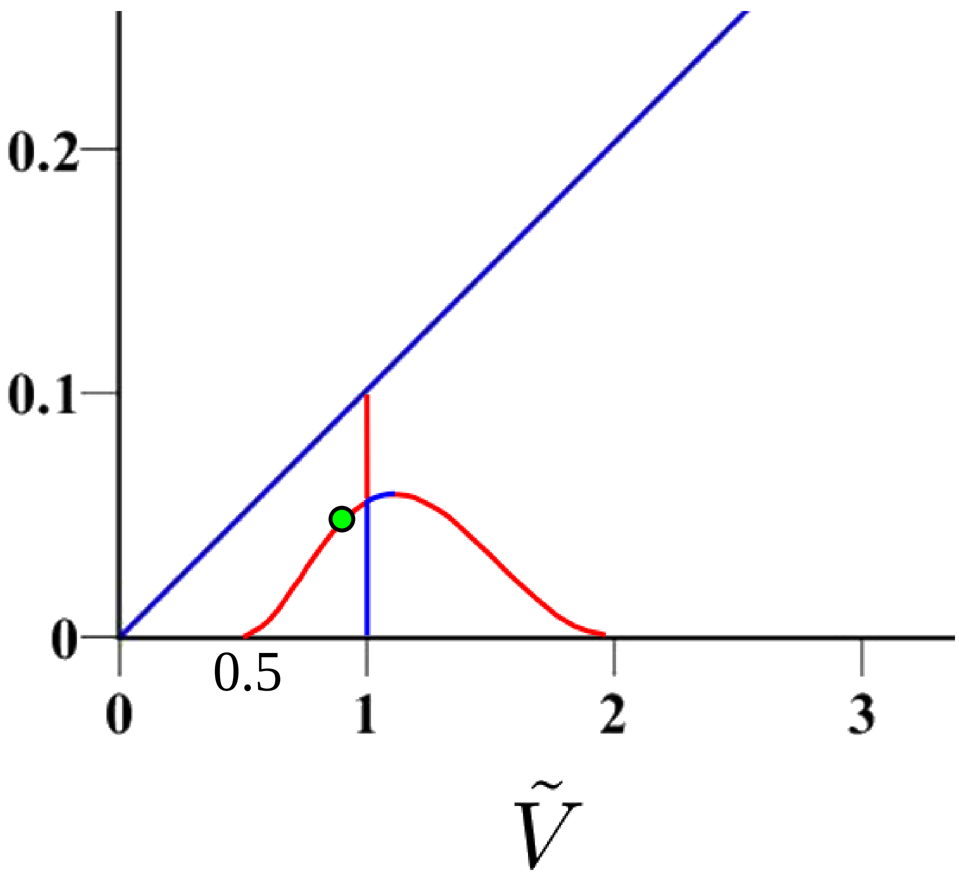

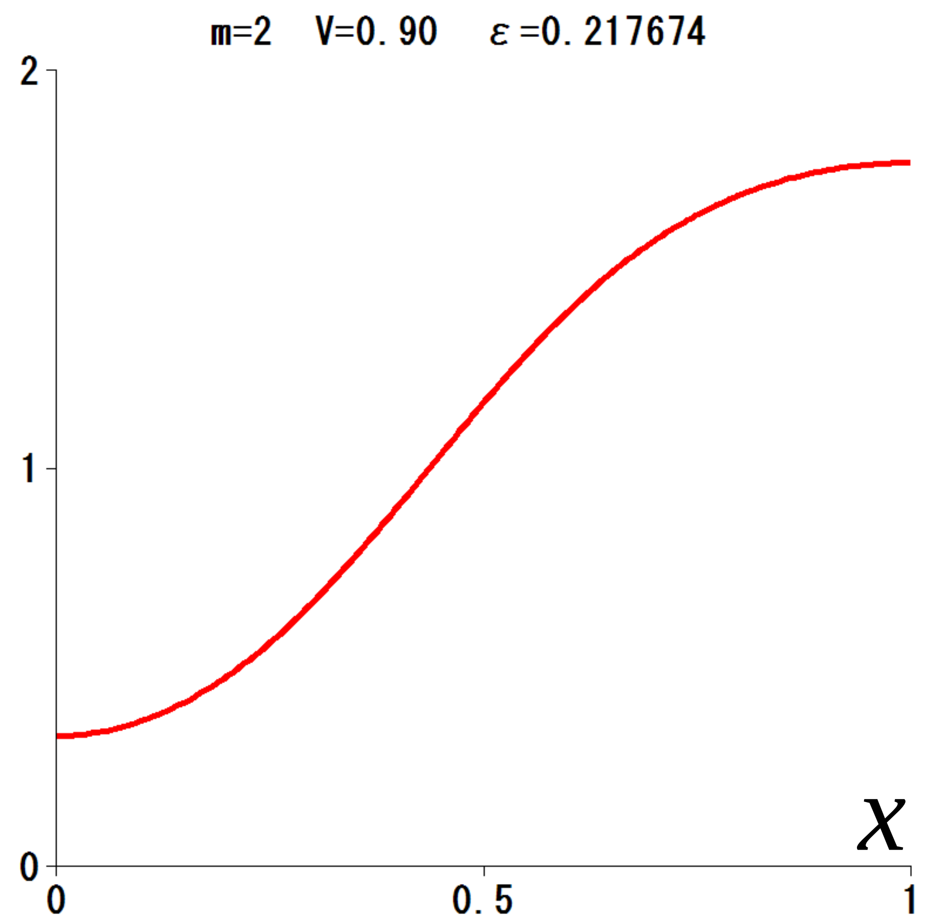

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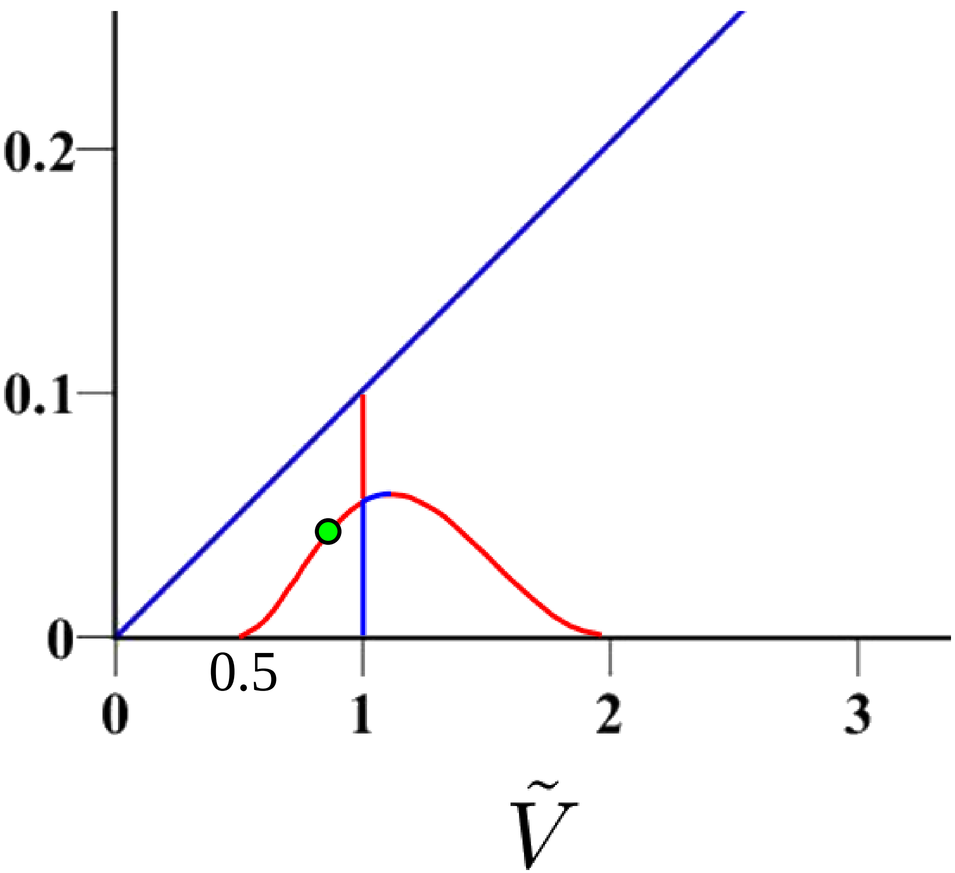

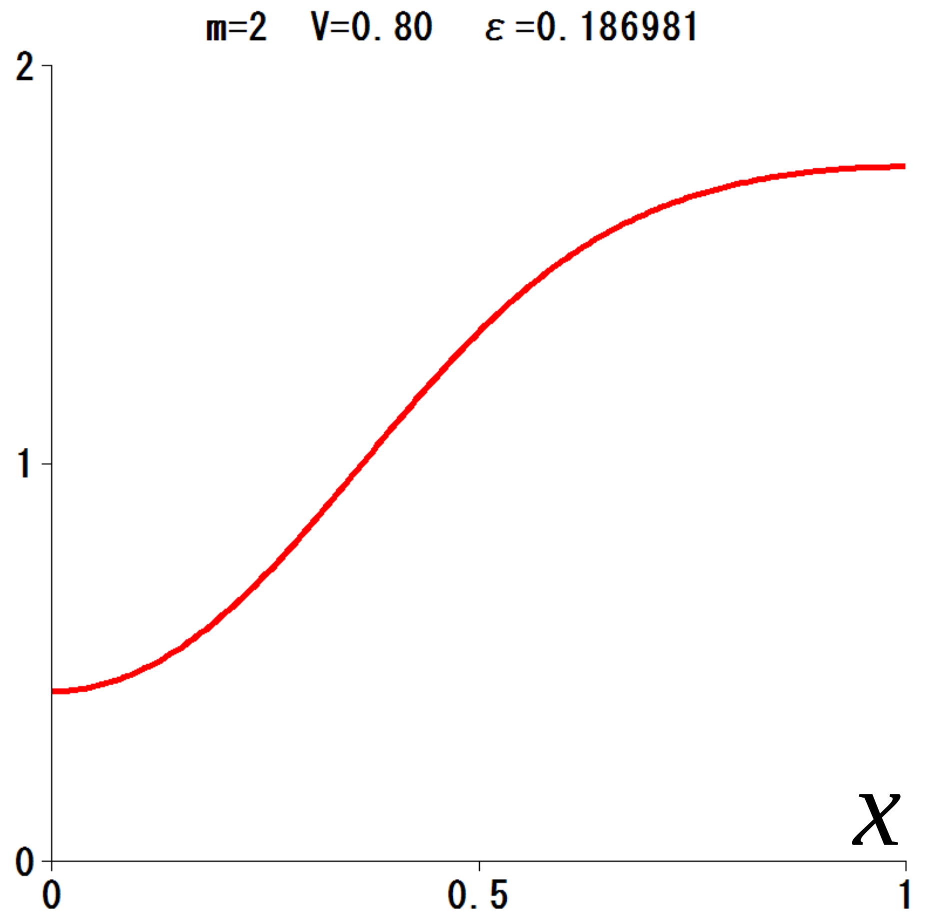

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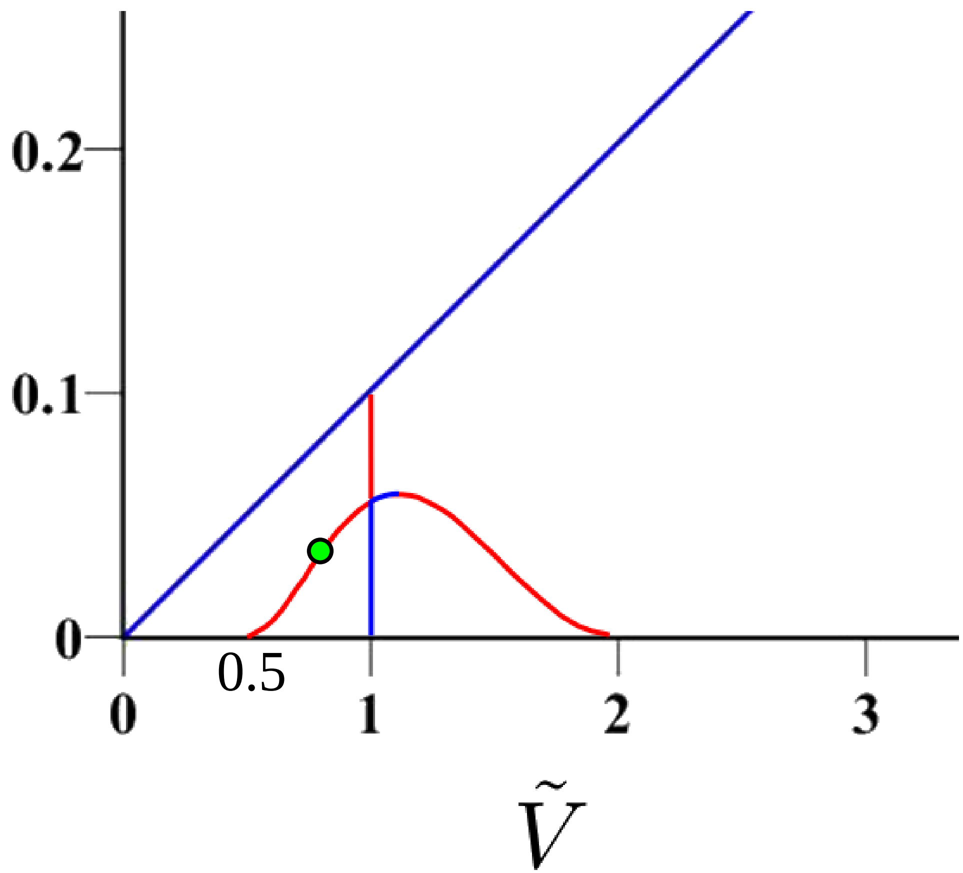

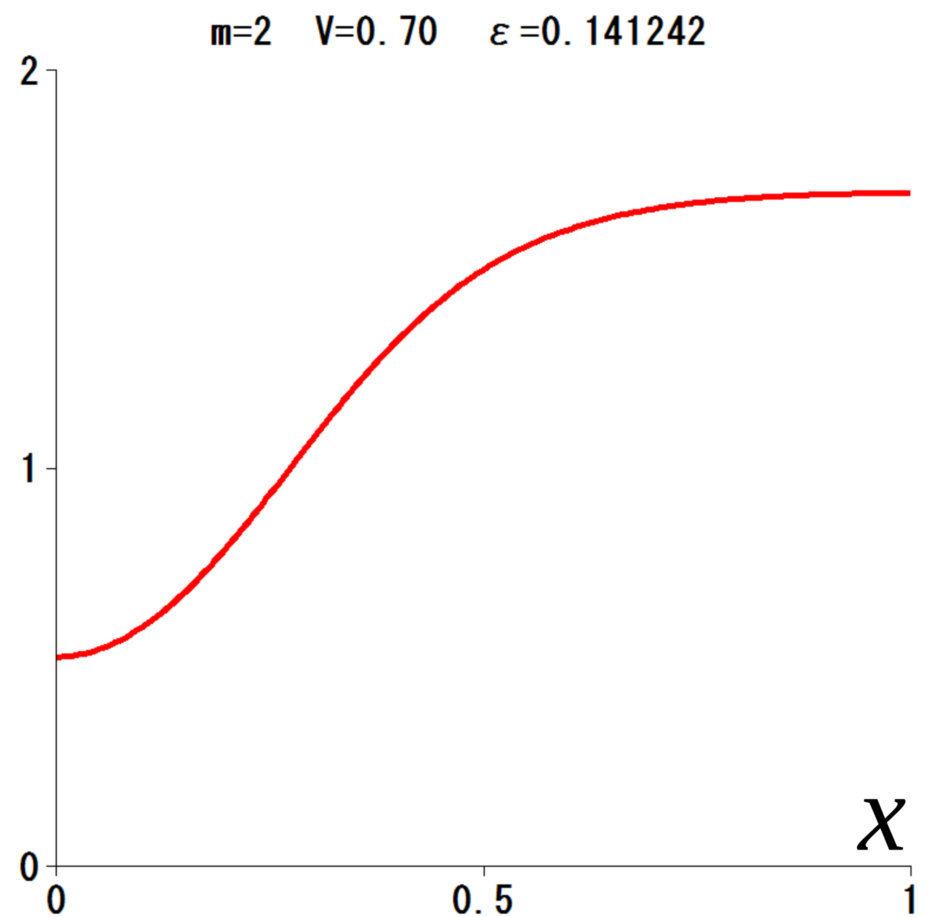

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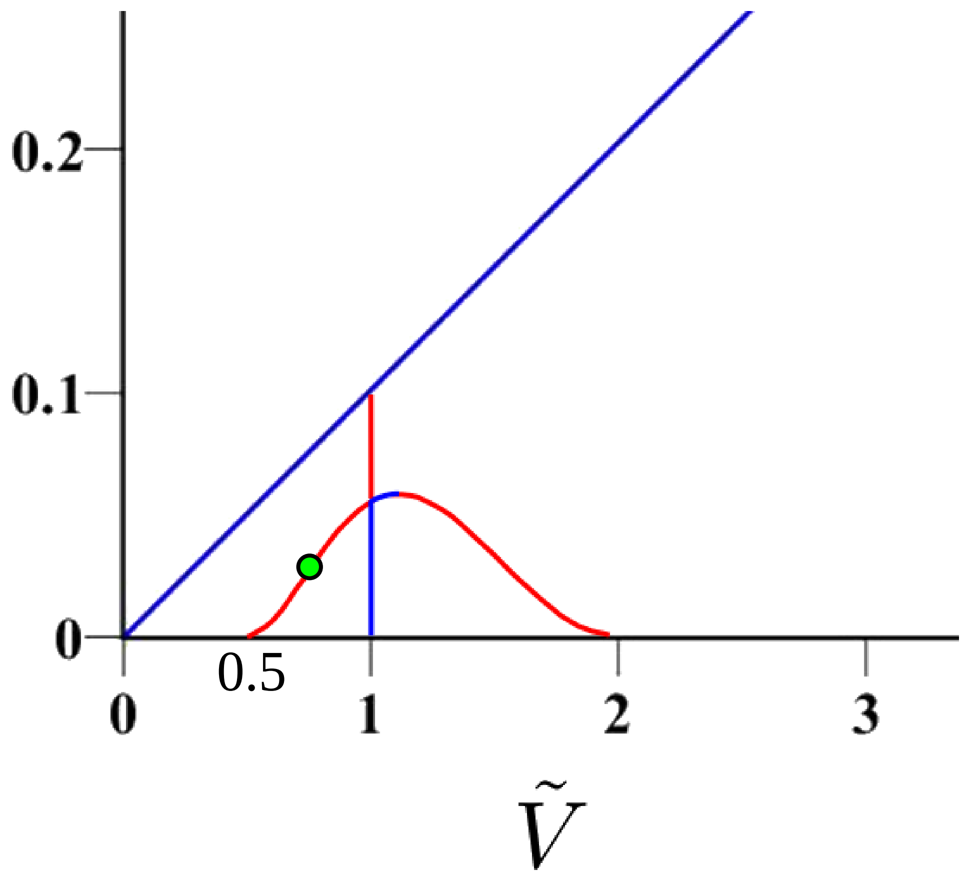

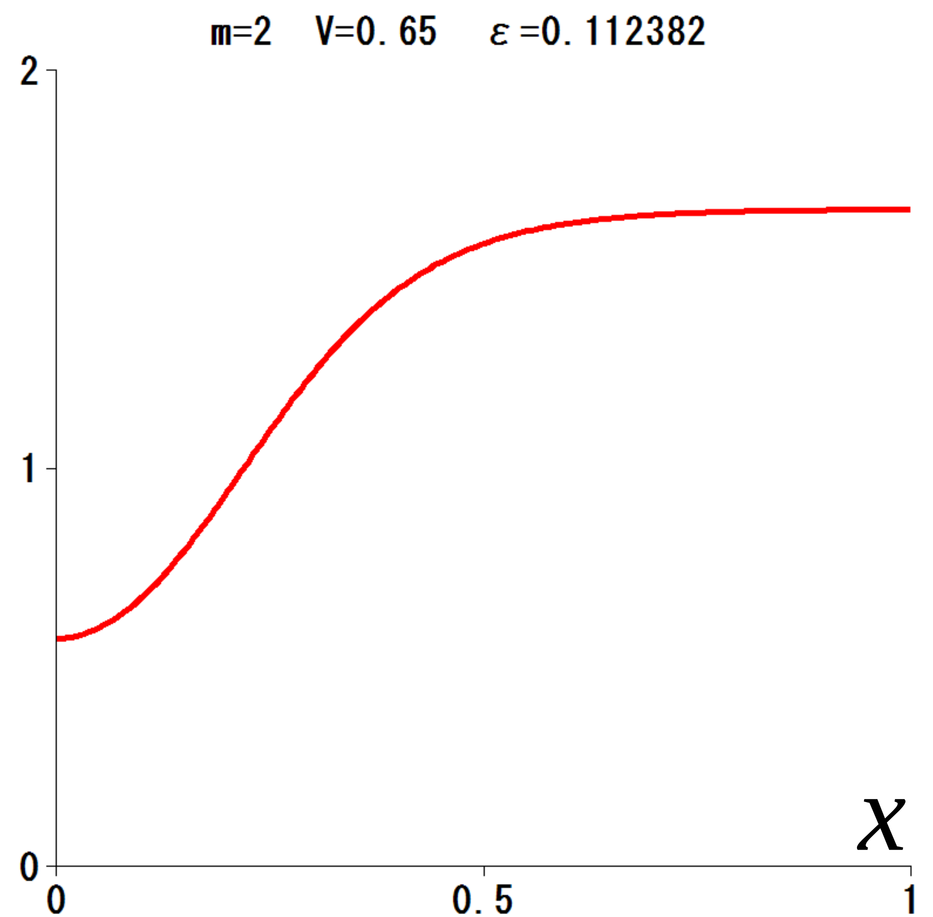

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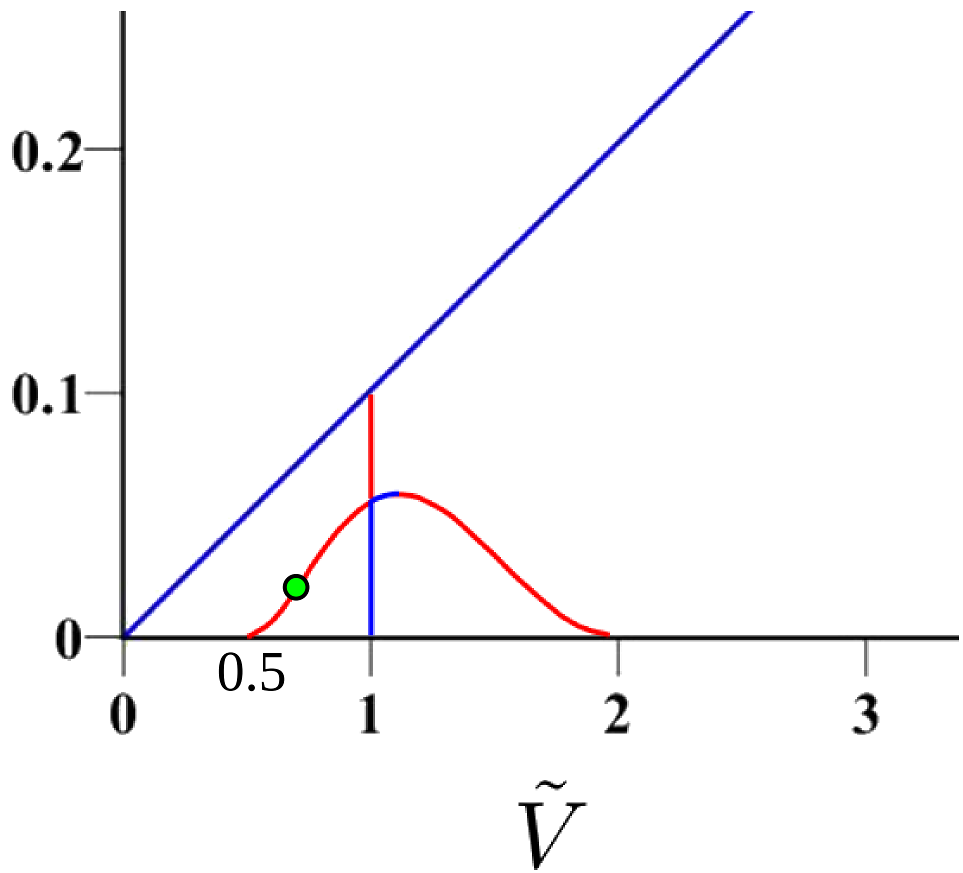

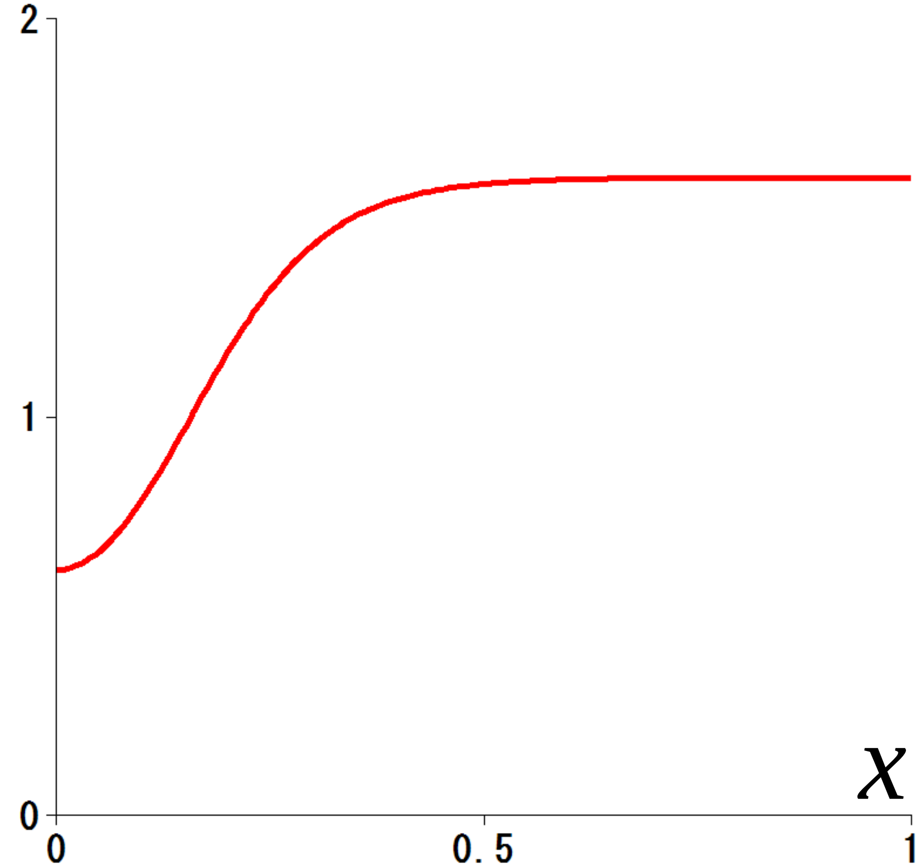

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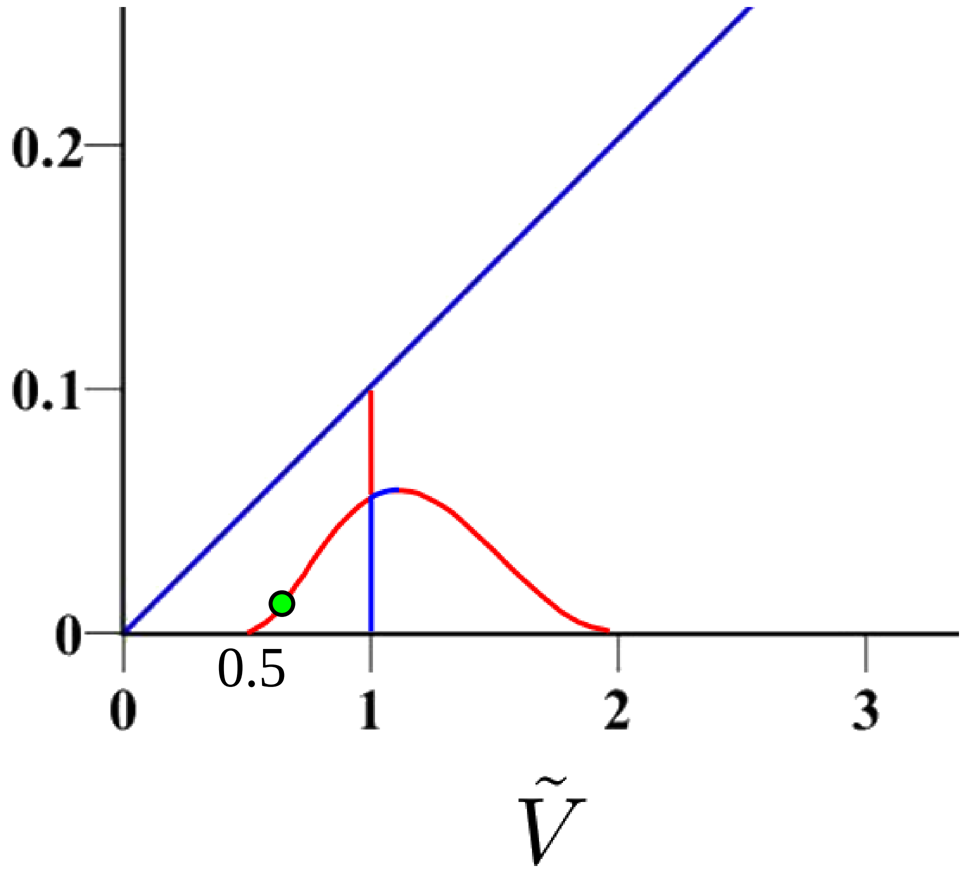

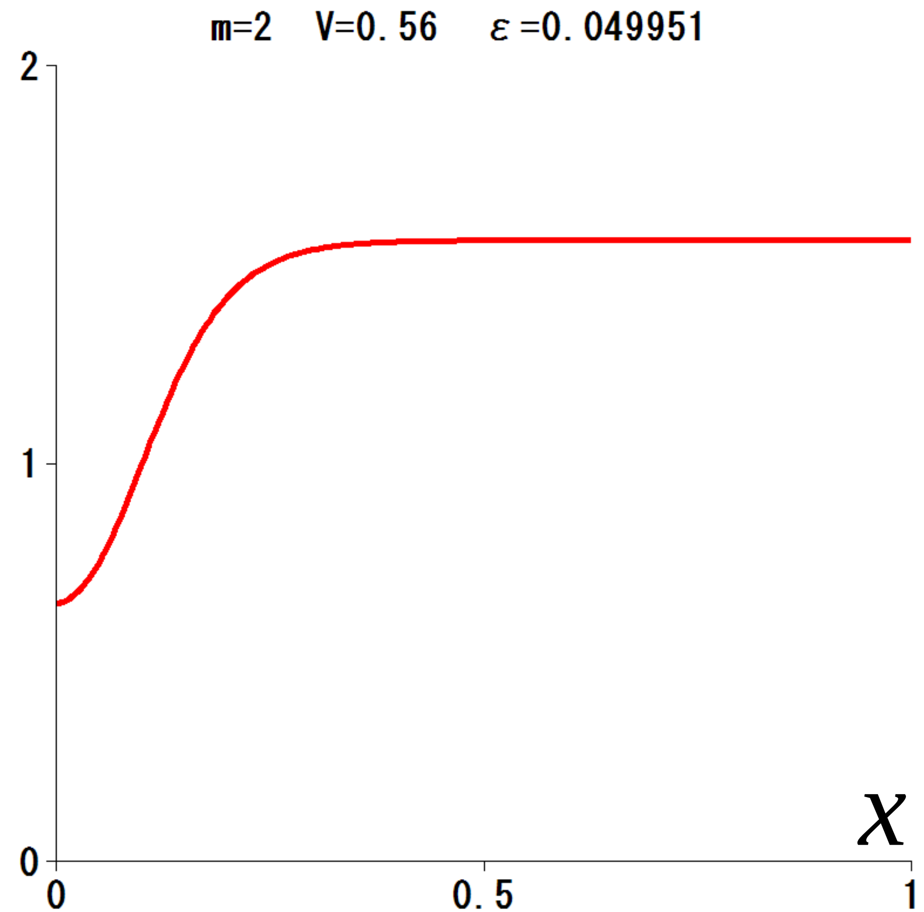

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$\tilde{V} \to 0.5,\ \varepsilon^2 \to 0$ の形状

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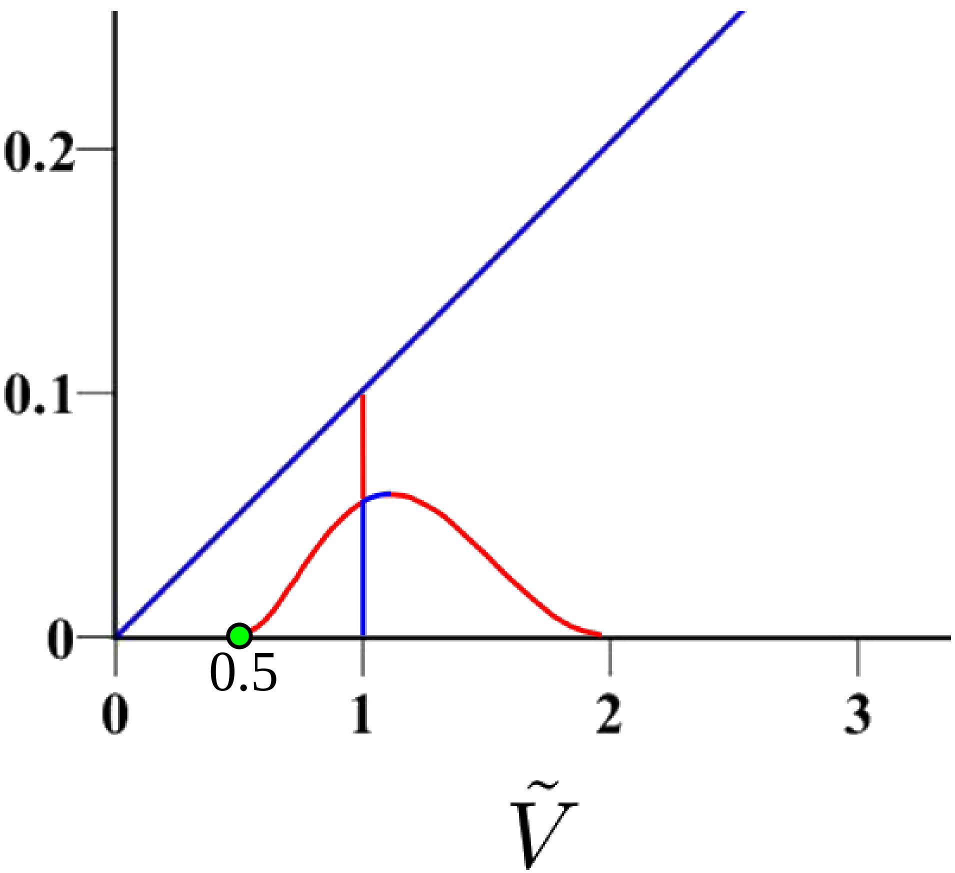
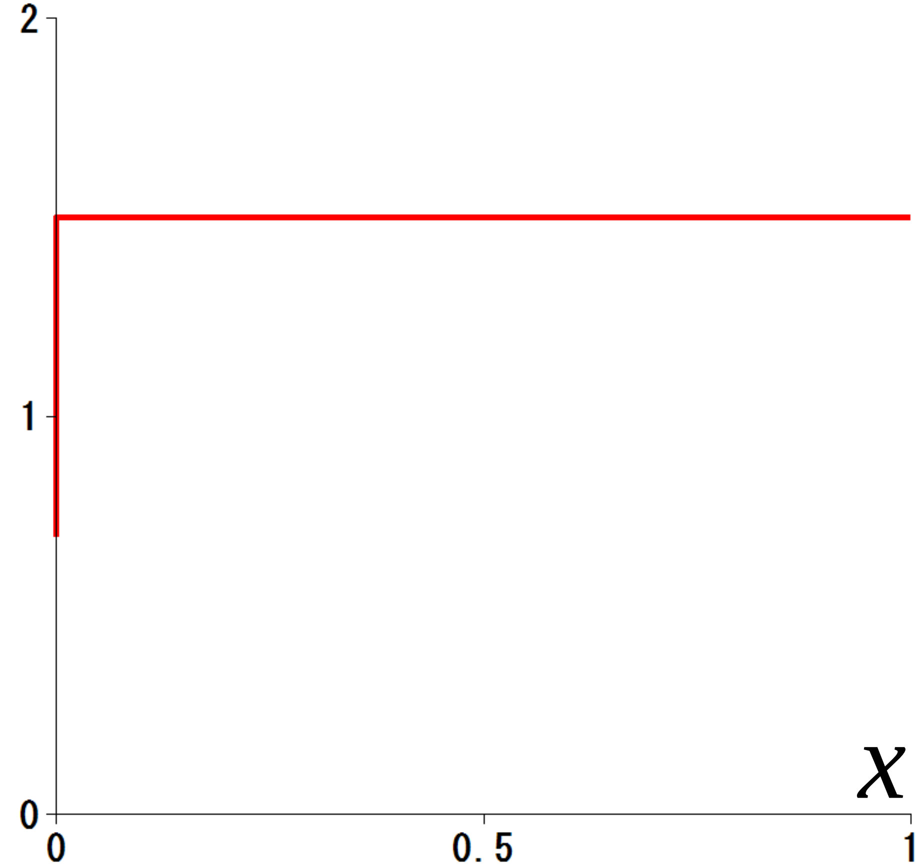


m=2 $\tilde{V} \to 0.5 \quad \varepsilon \to 0$



問題 正数 m が与えられたとき,

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(SLP)を満たす $(\tilde{V}, \varepsilon^2)$ および解 $W(x)$ をすべて求めよ.

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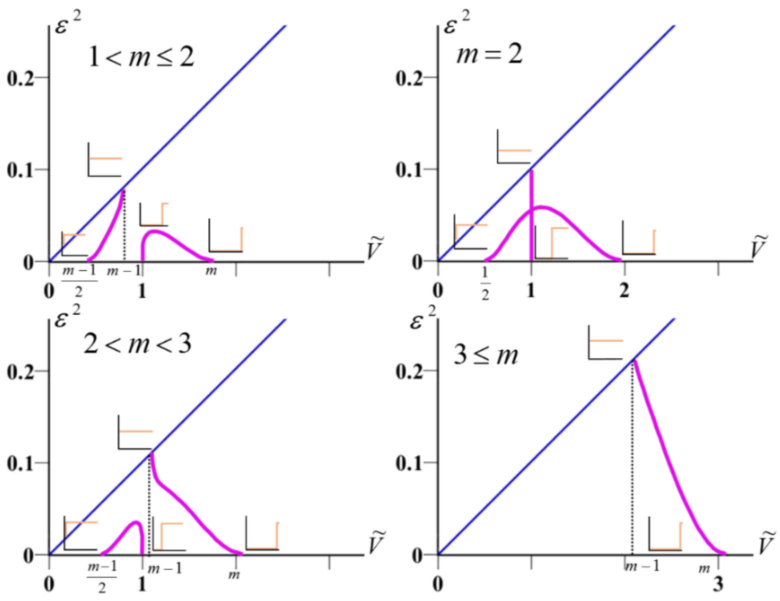
Numerical computation

Y.Mori, A. Jilkine and L. Edelstein-Keshet,
SIAM J, Appl. Math 71(2011), 1401-1427.



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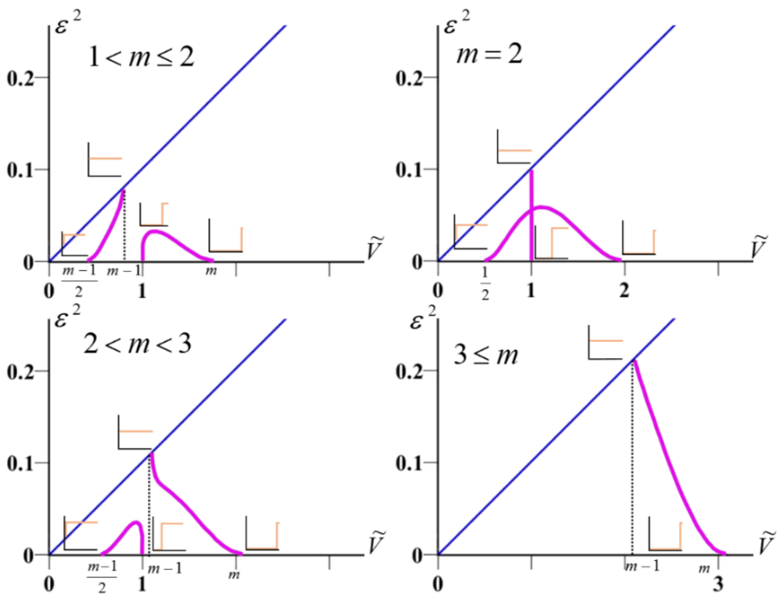

Mathematical results

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K.Kuto and T.Tsujikawa,
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some partial results

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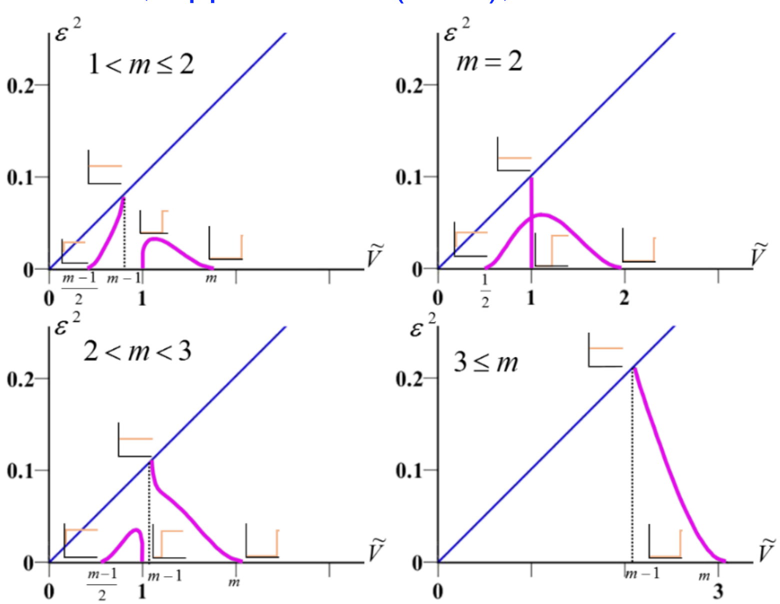

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Exact multiplicity　no results!

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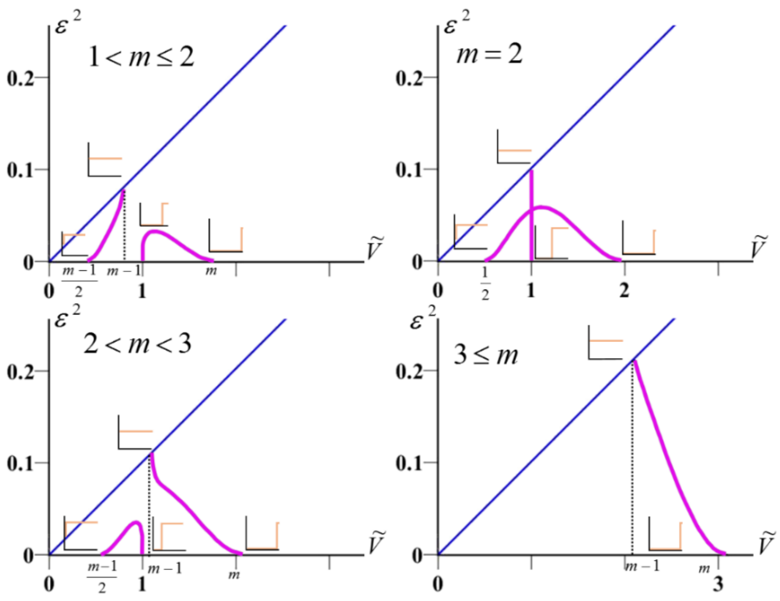

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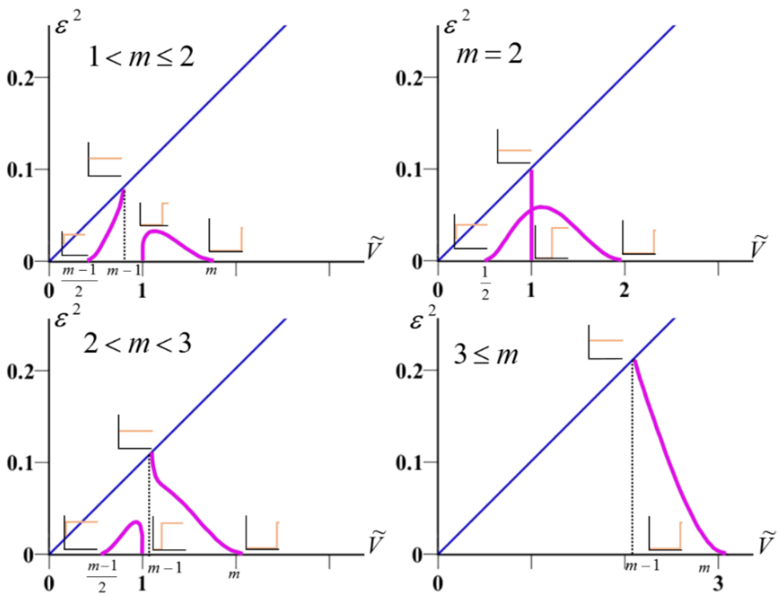

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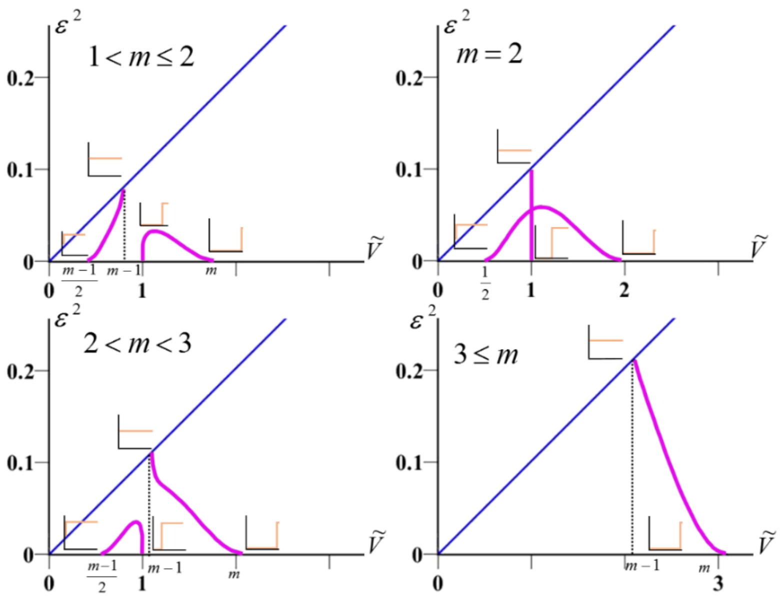
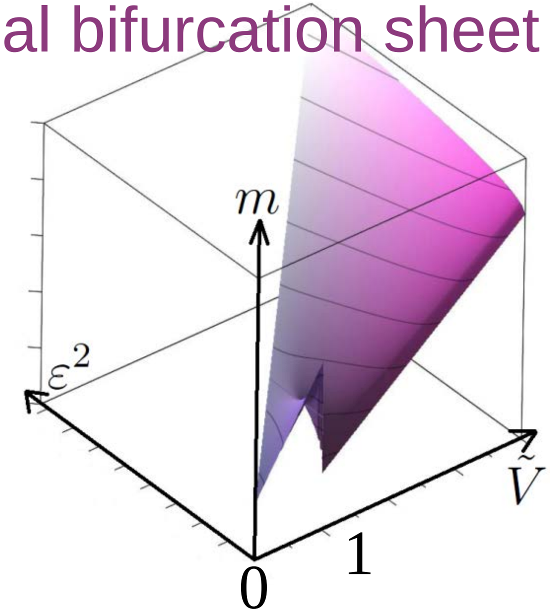


Exact multiplicity

global bifurcation sheet　の表示式

global bifurcation sheet



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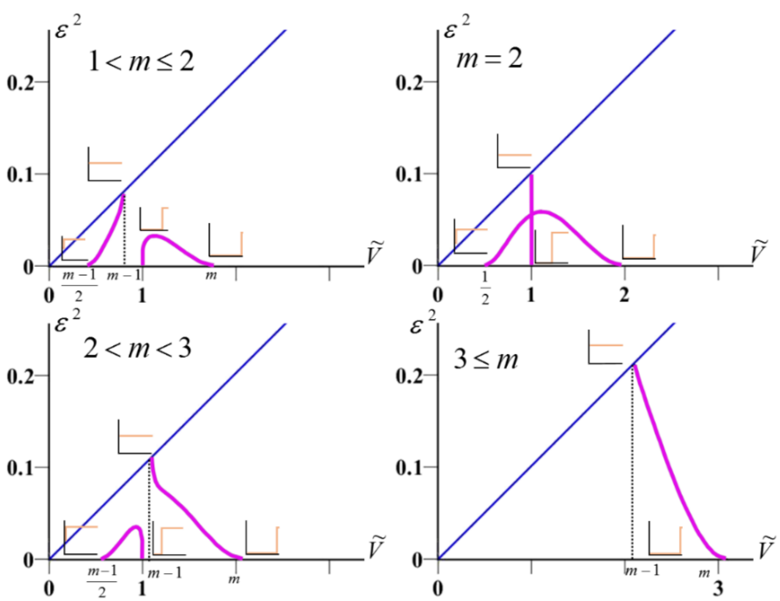
<u>Mathematical results</u>



Exact multiplicity

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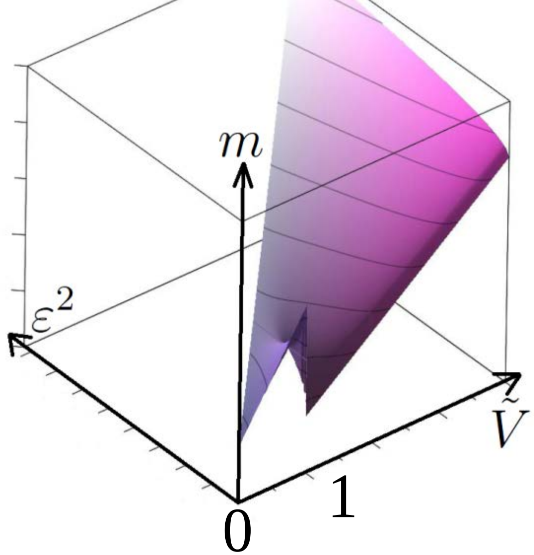

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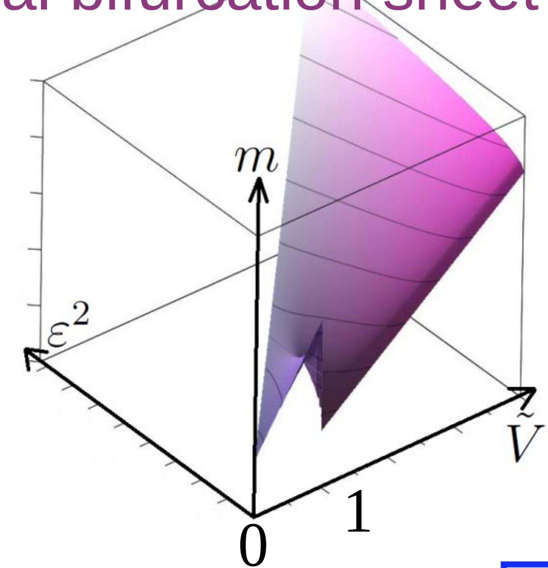
global bifurcation sheet



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$\varepsilon, \tilde{V}$: 与えられた正数,　　$W(x)$ = 未知関数

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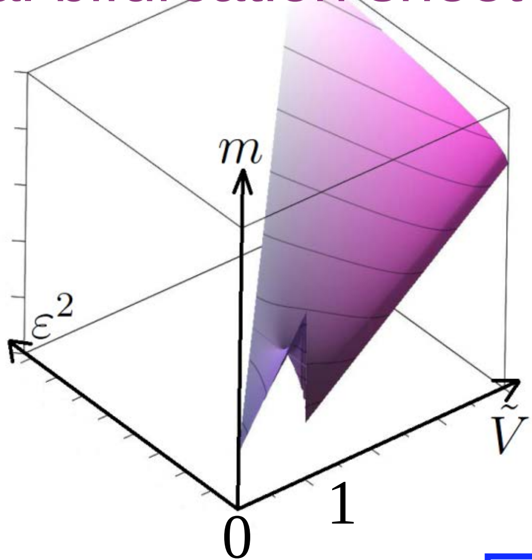

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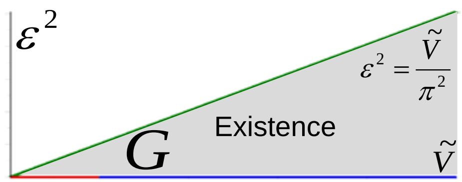
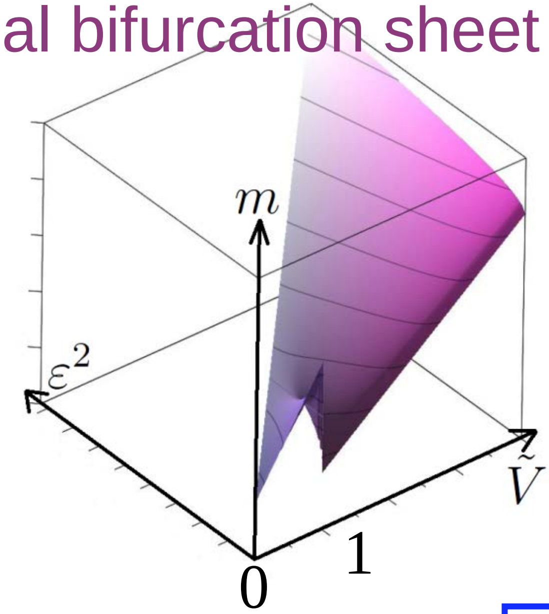


$$G := \left\{(\tilde{V}, \varepsilon^2) : 0 < \varepsilon^2 < \frac{\tilde{V}}{\pi^2}\right\}.$$

・この問題が解を持つ ⇔ $(\tilde{V}, \varepsilon^2) \in G$. さらに解は一意である. これを $W(x;\tilde{V},\varepsilon^2)$ とかく

global bifurcation sheet



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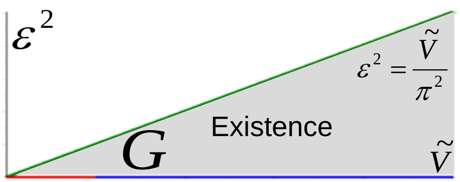

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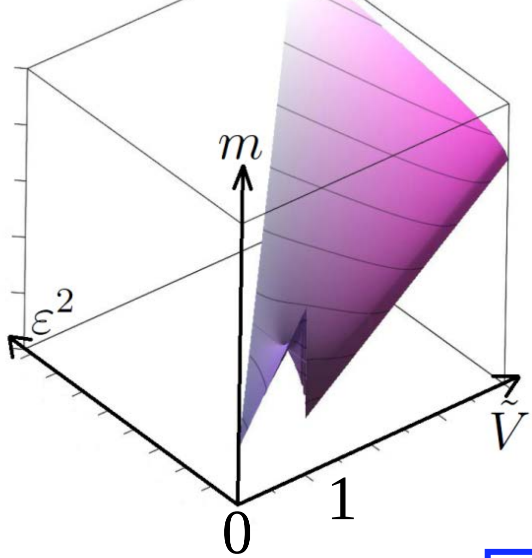
定義 : $m(\tilde{V},\varepsilon^2) := \int_0^1 W(x;\tilde{V},\varepsilon^2)\,dx + \tilde{V}$

global bifurcation sheet



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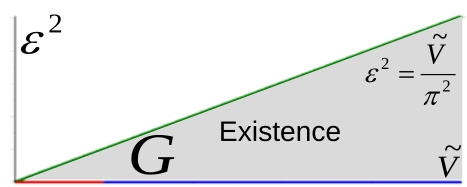

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定義：$m(\tilde V, \varepsilon^2) := \int_0^1 W(x;\tilde V,\varepsilon^2)\,dx + \tilde V$

定義：(SLP)の global bifurcation sheet $:= \left\{ \left(\tilde V, \varepsilon^2, m(\tilde V, \varepsilon^2)\right) : (\tilde V, \varepsilon^2) \in G \right\}$

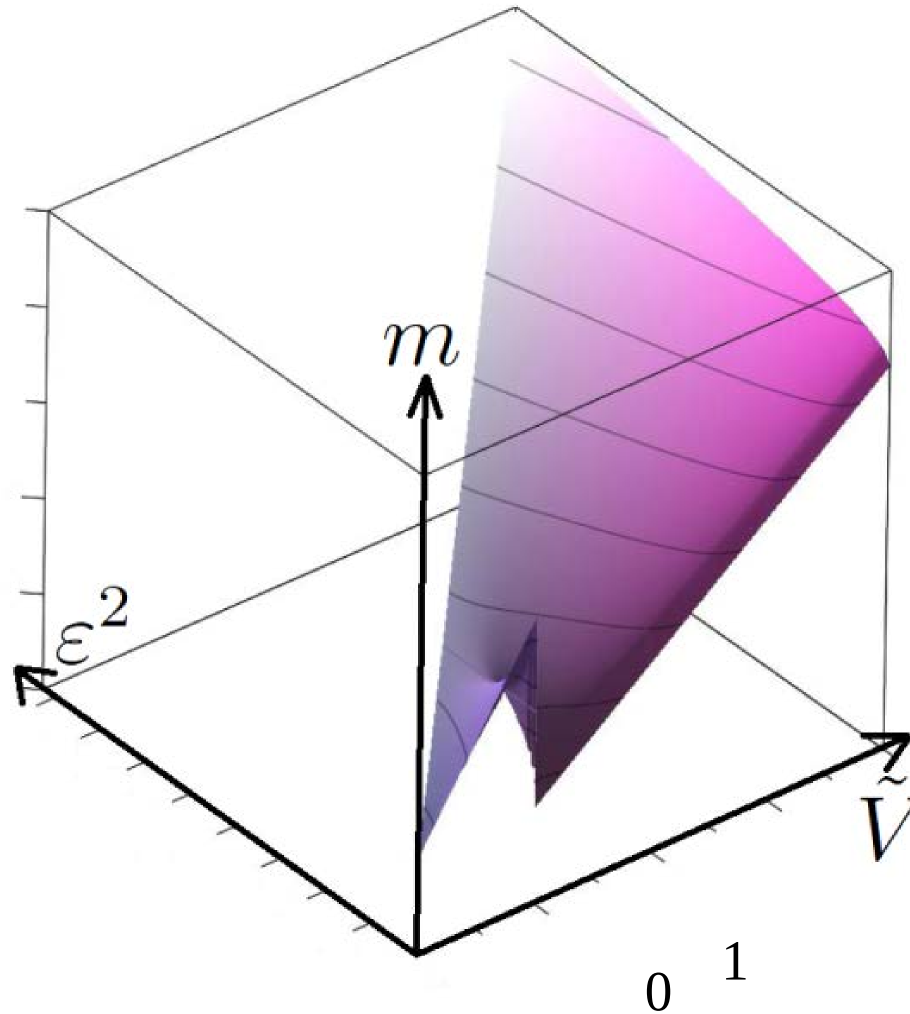
Remark. *Let* $W(x;\tilde{V},\varepsilon^2)$ *be the unique solution of* $(AP;\tilde{V})$, *and*

$$m(\tilde{V},\varepsilon^2) := \int_0^1 W(x;\tilde{V},\varepsilon^2)\,dx + \tilde{V},$$

then

$$m(\tilde{V},\varepsilon^2) = 2\tilde{V} + 2 - \tilde{V}\cdot m\left(\frac{1}{\tilde{V}}, \frac{\varepsilon^2}{\tilde{V}^2}\right) \qquad \text{for any} \quad \tilde{V} > 0, \ \ \varepsilon > 0.$$

In particular, $m(1,\varepsilon^2) = 2$ *for any* $\varepsilon > 0$.

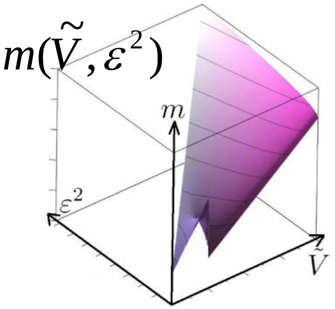


Theorem A （ $m(\tilde{V},\varepsilon^2)$ の表示式 ）.

T.Mori, K.Kuto, T.Tsujikawa, M.Nagayama and S.Yotsutani, Proc. of AIMS conf. 2014, submitted.

$m(\tilde{V},\varepsilon^2)$ *is represented by*

$$m(\tilde{V},\varepsilon^2) := \frac{4\tilde{V}+2}{3} + \frac{1}{\sqrt{3}} \cdot \sqrt{\tilde{V}^2+\tilde{V}+1} \cdot M(h,s),$$



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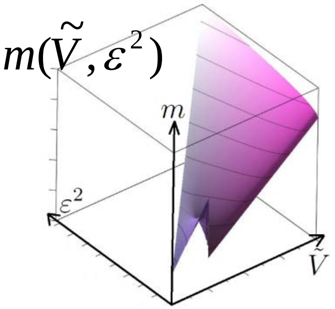

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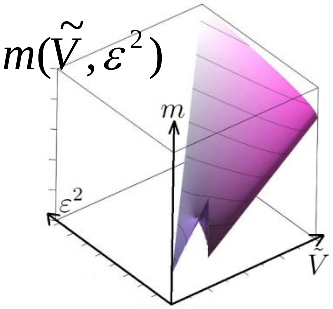

where $(h,\, s) = (h(\tilde{V},\, \varepsilon^2),\, s(\tilde{V},\, \varepsilon^2))$ *is the unique solution of the following system of transcendental equations*

$$\begin{cases} E(h,s) := \dfrac{\sqrt{2s(1-s)(1-sh)}\big/K(\sqrt{h})}{\sqrt{3h^2s^4 - 4(h^2+h)s^3 + (4h^2+2h+4)s^2 - 4(h+1)s + 3}} = \sqrt{3} \cdot \dfrac{\varepsilon}{\sqrt{\tilde{V}^2+\tilde{V}+1}}, \\[2ex] A(h,s) := \dfrac{2(hs^2 - sh + 1)(hs^2 - 2s + 1)(1 - hs^2)}{\sqrt{3h^2s^4 - 4(h^2+h)s^3 + (4h^2+2h+4)s^2 - 4(h+1)s + 3}^{\,3}} = \dfrac{1}{3\sqrt{3}} \cdot \dfrac{(1-\tilde{V})(2\tilde{V}+1)(\tilde{V}+2)}{\sqrt{\tilde{V}^2+\tilde{V}+1}^{\,3}}, \\[2ex] 0 < h < 1, \quad 0 < s < 1. \end{cases}$$

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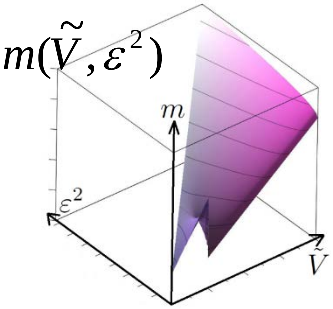
T.Mori, K.Kuto, T.Tsujikawa, M.Nagayama and S.Yotsutani, Proc. of AIMS conf. 2014, submitted.

$m(\tilde{V},\varepsilon^2)$ is represented by



$$m(\tilde{V},\varepsilon^2) := \frac{4\tilde{V}+2}{3} + \frac{1}{\sqrt{3}}\cdot\sqrt{\tilde{V}^2+\tilde{V}+1}\cdot M(h,s),$$

$$M(h,s) := \frac{-(hs^2-2(1+h)s+3)+4(1-s)(1-sh)\Pi(-sh,\sqrt{h})/K(\sqrt{h})}{\sqrt{3h^2s^4-4(h^2+h)s^3+(4h^2+2h+4)s^2-4(h+1)s+3}},$$

where $(h,s)=(h(\tilde{V},\varepsilon^2),\ s(\tilde{V},\varepsilon^2))$ *is the unique solution of the following system of transcendental equations*

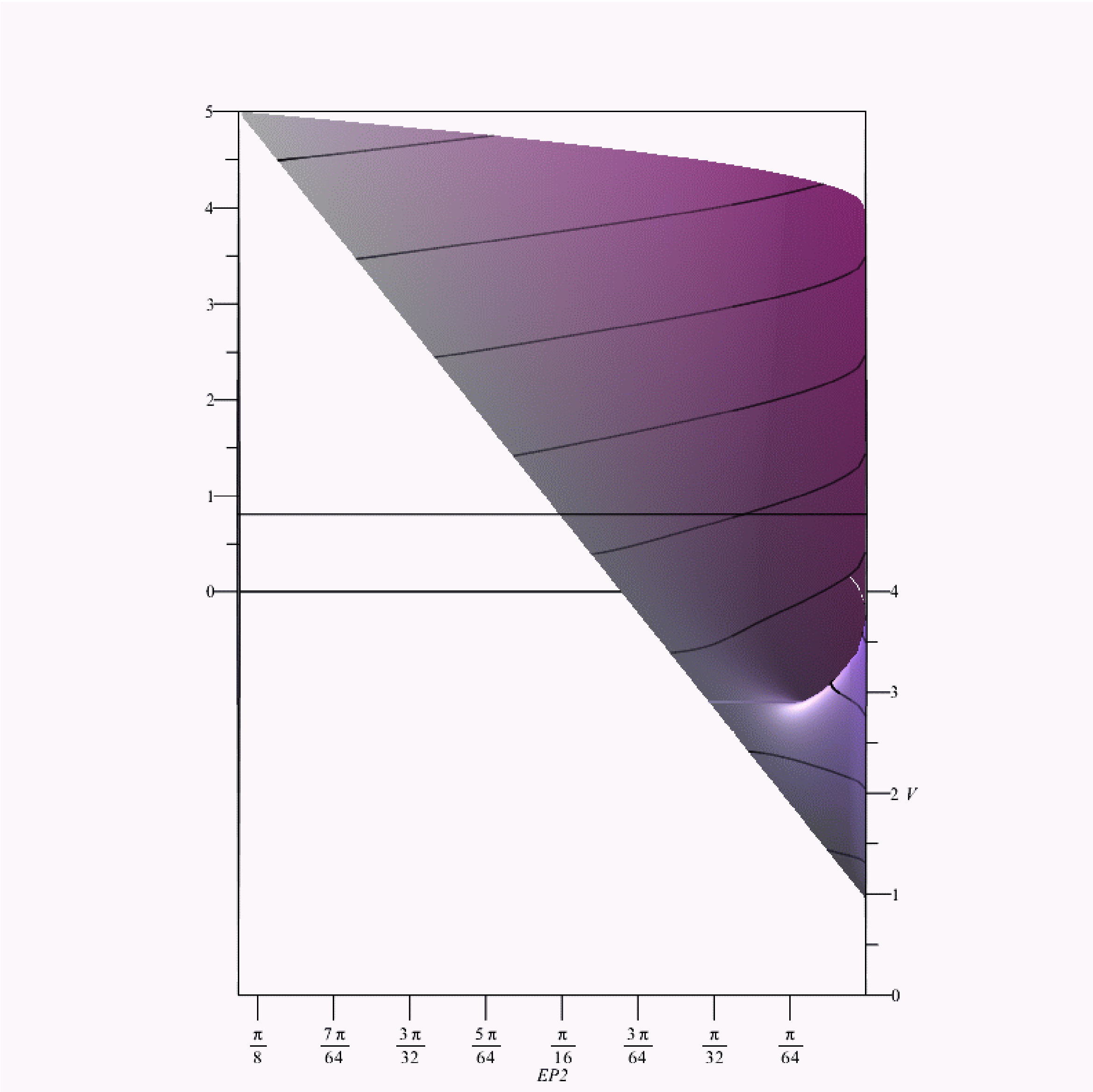
$$\begin{cases} E(h,s) := \dfrac{\sqrt{2s(1-s)(1-sh)}/K(\sqrt{h})}{\sqrt{3h^2s^4-4(h^2+h)s^3+(4h^2+2h+4)s^2-4(h+1)s+3}} = \sqrt{3}\cdot\dfrac{\varepsilon}{\sqrt{\tilde{V}^2+\tilde{V}+1}}, \\ A(h,s) := \dfrac{2(hs^2-sh+1)(hs^2-2s+1)(1-hs^2)}{\sqrt{3h^2s^4-4(h^2+h)s^3+(4h^2+2h+4)s^2-4(h+1)s+3}^{\,3}} = \dfrac{1}{3\sqrt{3}}\cdot\dfrac{(1-\tilde{V})(2\tilde{V}+1)(\tilde{V}+2)}{\sqrt{\tilde{V}^2+\tilde{V}+1}^{\,3}}, \\ 0<h<1,\quad 0<s<1. \end{cases}$$

Here, $K(\cdot)$ *is the complete elliptic integral of the first kind,*

$\Pi(\cdot,\cdot)$ *is the complete elliptic integral of the third kind.*

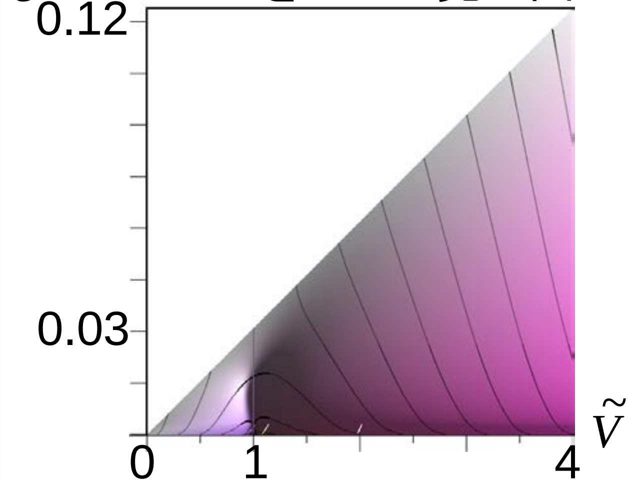
$$K(k) := \int_0^{\frac{\pi}{2}} \frac{d\varphi}{\sqrt{1-k^2\sin^2\varphi}}, \qquad \Pi(\mu,k) := \int_0^{\frac{\pi}{2}} \frac{d\varphi}{(1+\mu\sin^2\varphi)\sqrt{1-k^2\sin^2\varphi}}.$$

(SLP)の大域的分岐シート

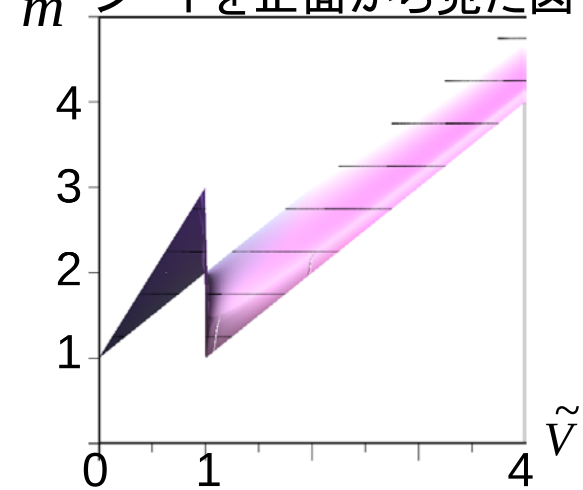


$$(SLP)\begin{cases} \varepsilon^2 W_{xx} + W(W-1)(\tilde{V}+1-W) = 0, \\ W_x(0) = W_x(1) = 0, \\ W'(x) > 0,\ \tilde{V} > 0, \\ \int_0^1 W(x)\,\mathrm{dx} + \tilde{V} = m. \end{cases}$$

シートを上から見た図



シートを正面から見た図

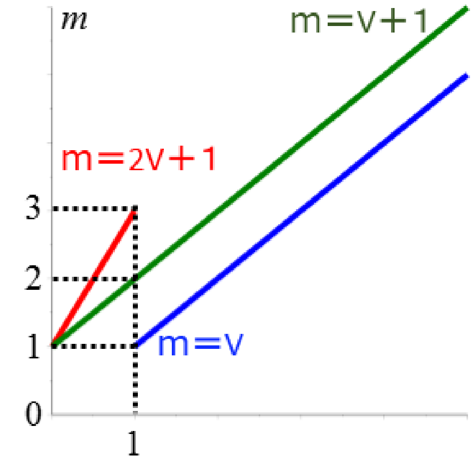


Theorem B *Let* $\tilde{V} > 0$ *be fixed. then*

(i) For $0 < \tilde{V} < 1$, $m(\tilde{V}, \varepsilon^2)$ *is decreasing in* $\varepsilon^2 \in (0, \tilde{V}/\pi^2)$, *and*

$$\lim_{\varepsilon^2 \downarrow 0} m(\tilde{V}, \varepsilon^2) = 2\tilde{V} + 1, \qquad \lim_{\varepsilon^2 \downarrow 0} W(\tilde{V}, \varepsilon^2) = \tilde{V} + 1 \text{ in } (0,1],$$

$$\lim_{\varepsilon^2 \uparrow \tilde{V}/\pi^2} m(\tilde{V}, \varepsilon^2) = \tilde{V} + 1, \qquad \lim_{\varepsilon^2 \uparrow \tilde{V}/\pi^2} W(\tilde{V}, \varepsilon^2) = 1, \text{ in } [0,1].$$



(ii) For $1 < \tilde{V}$, $m(\tilde{V}, \varepsilon^2)$ *is increasing in* $\varepsilon^2 \in (0, \tilde{V}/\pi^2)$, *and*

$$\lim_{\varepsilon^2 \downarrow 0} m(\tilde{V}, \varepsilon^2) = \tilde{V}, \qquad \lim_{\varepsilon^2 \downarrow 0} W(\tilde{V}, \varepsilon^2) = 0 \quad \text{in } [0,1),$$

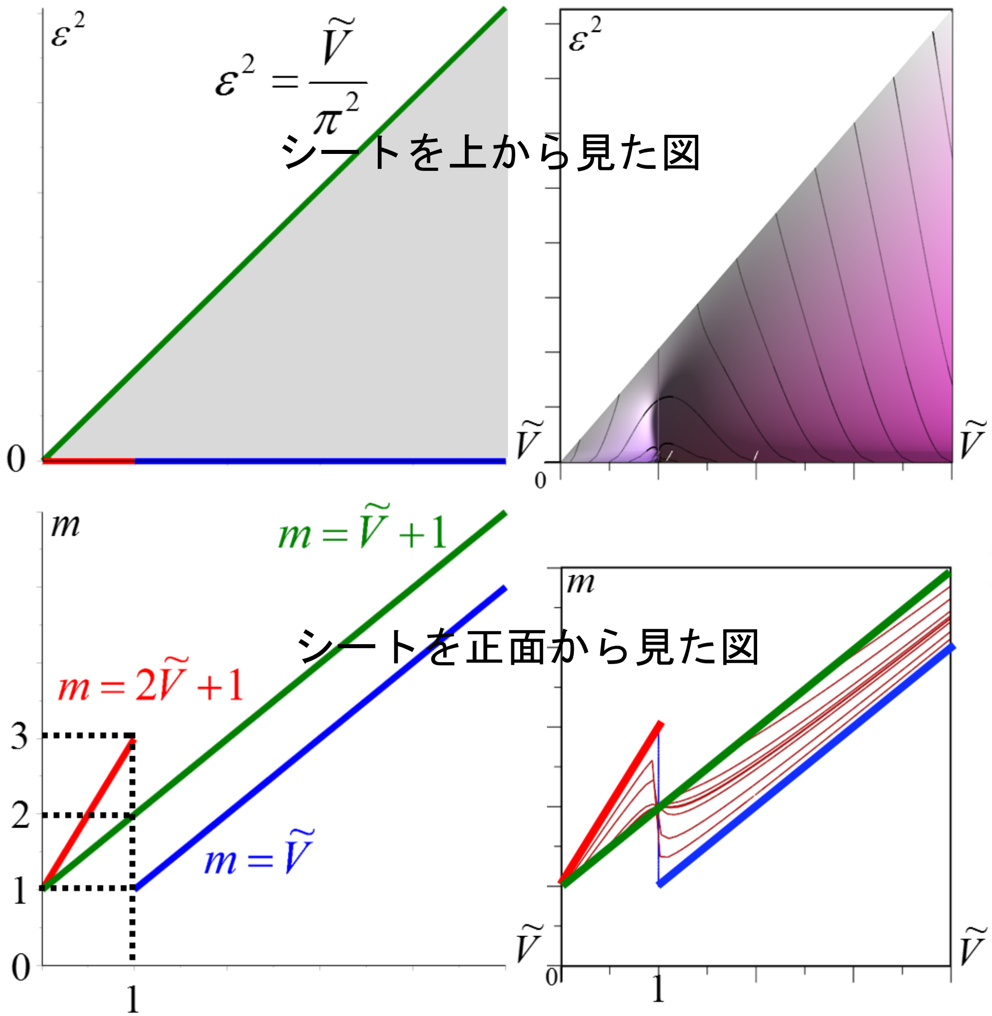
$$\lim_{\varepsilon^2 \uparrow \tilde{V}/\pi^2} m(\tilde{V}, \varepsilon^2) = \tilde{V} + 1. \qquad \lim_{\varepsilon^2 \uparrow \tilde{V}/\pi^2} W(\tilde{V}, \varepsilon^2) = 1 \text{ in } [0,1].$$

Remark

$$m(1, \varepsilon^2) = 2 \text{ for } \varepsilon^2 \in \left(0, \sqrt{\tilde{V}}/\pi^2\right),$$

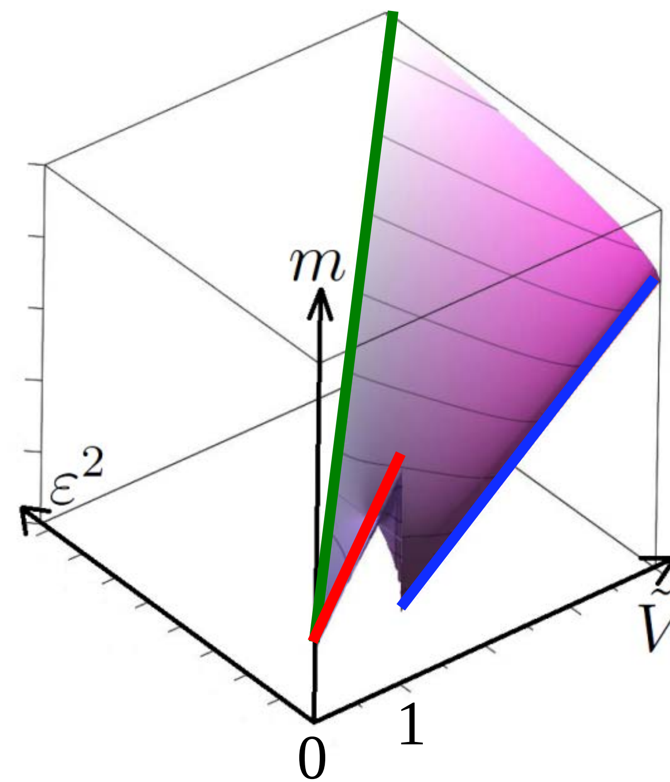
$$\lim_{\varepsilon^2 \downarrow 0} W(x; 1, \varepsilon^2) = 0 \ \left(0 \le x < 1/2\right), 1 \ (x = 1/2), \ 2 \ \left(1/2 < x \le 1\right).$$

Theorem B. （ $m(\tilde{V},\varepsilon^2)$ の端点での極限値 ）.



global bifurcation sheet

$$\left\{\left(\tilde{V},\ \varepsilon^2,\ m(\tilde{V},\varepsilon^2)\right) : (\tilde{V},\ \varepsilon^2) \in G\right\}$$

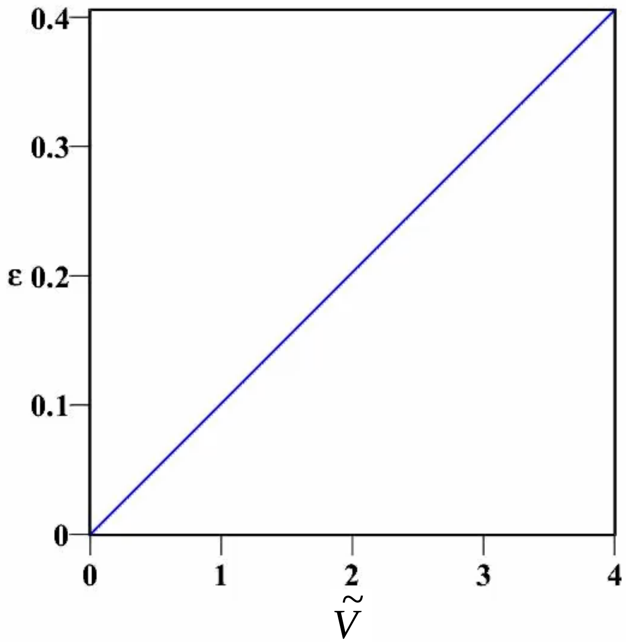


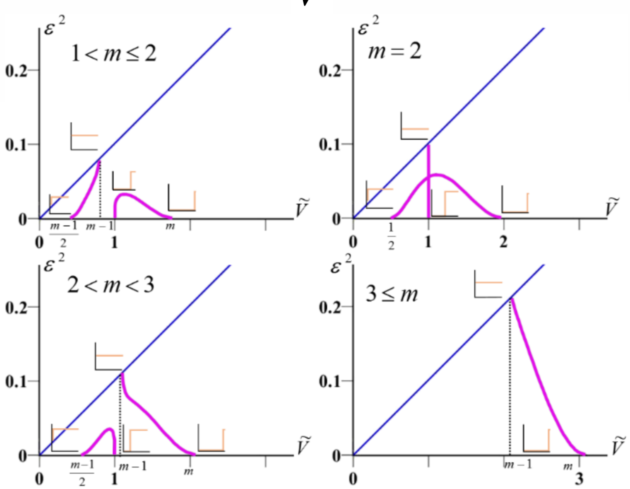
Bifurcation curve

Movie
bifurcation curve for each m

Move m Increasing 1 to 4

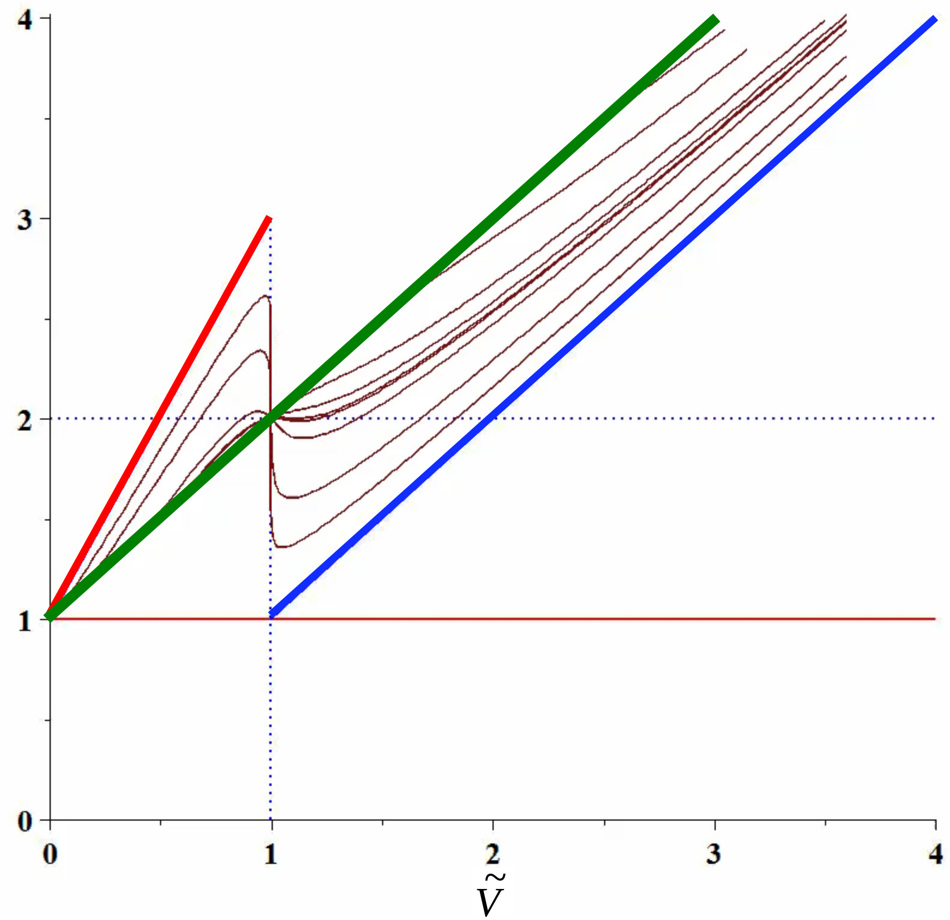
m=1.000





Front view of the sheet

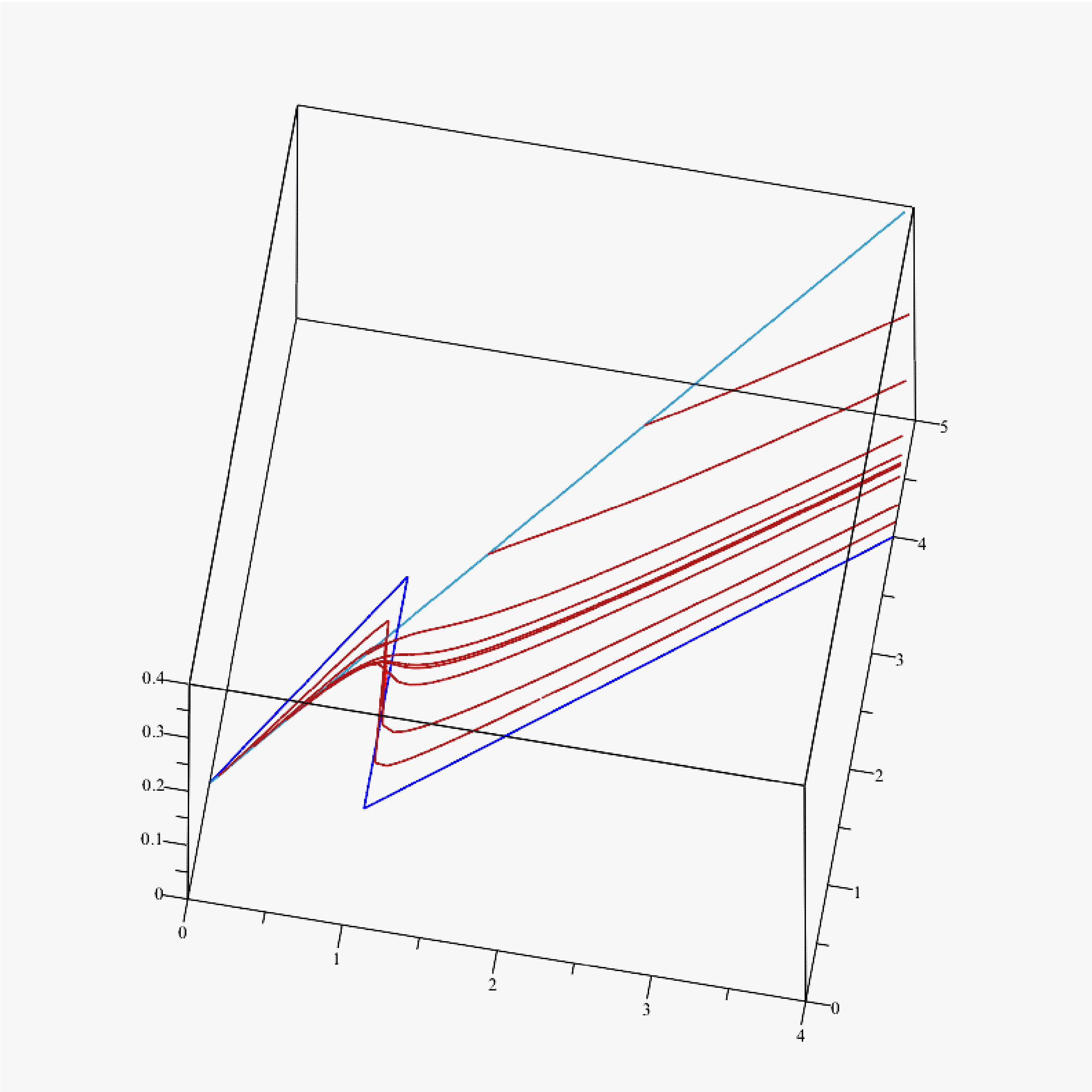
m=1.000



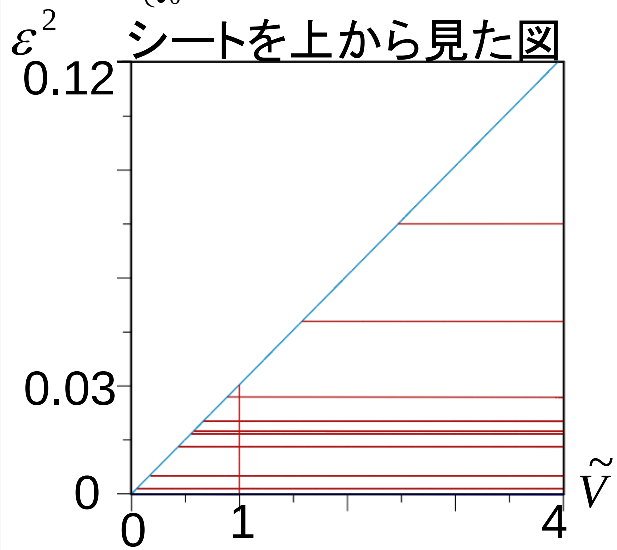
$$
(SLP)\begin{cases}
\varepsilon^2 W_{xx} + W(W-1)(\tilde{V}+1-W) = 0, \\
W_x(0) = W_x(1) = 0, \\
W'(x) > 0, \\
\int_0^1 W(x)\,\mathrm{d}x + \tilde{V} = m.
\end{cases}
$$

(SLP)の大域的分岐シート

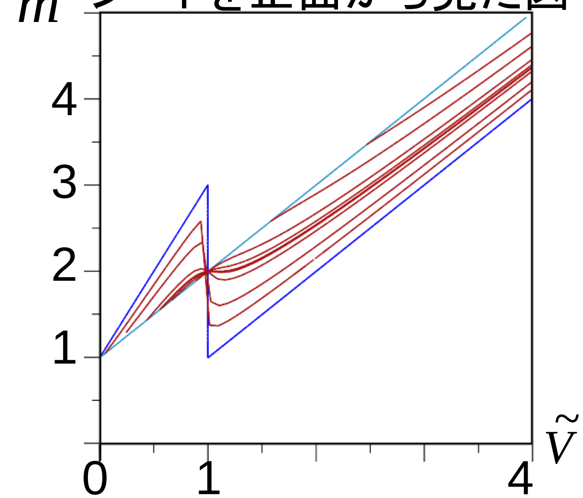
$$(SLP)\begin{cases} \varepsilon^2 W_{xx} + W(W-1)(\tilde{V}+1-W)=0, \\ W_x(0)=W_x(1)=0, \\ W'(x)>0, \\ \int_0^1 W(x)\,\mathrm{dx} + \tilde{V} = m. \end{cases}$$



シートを上から見た図



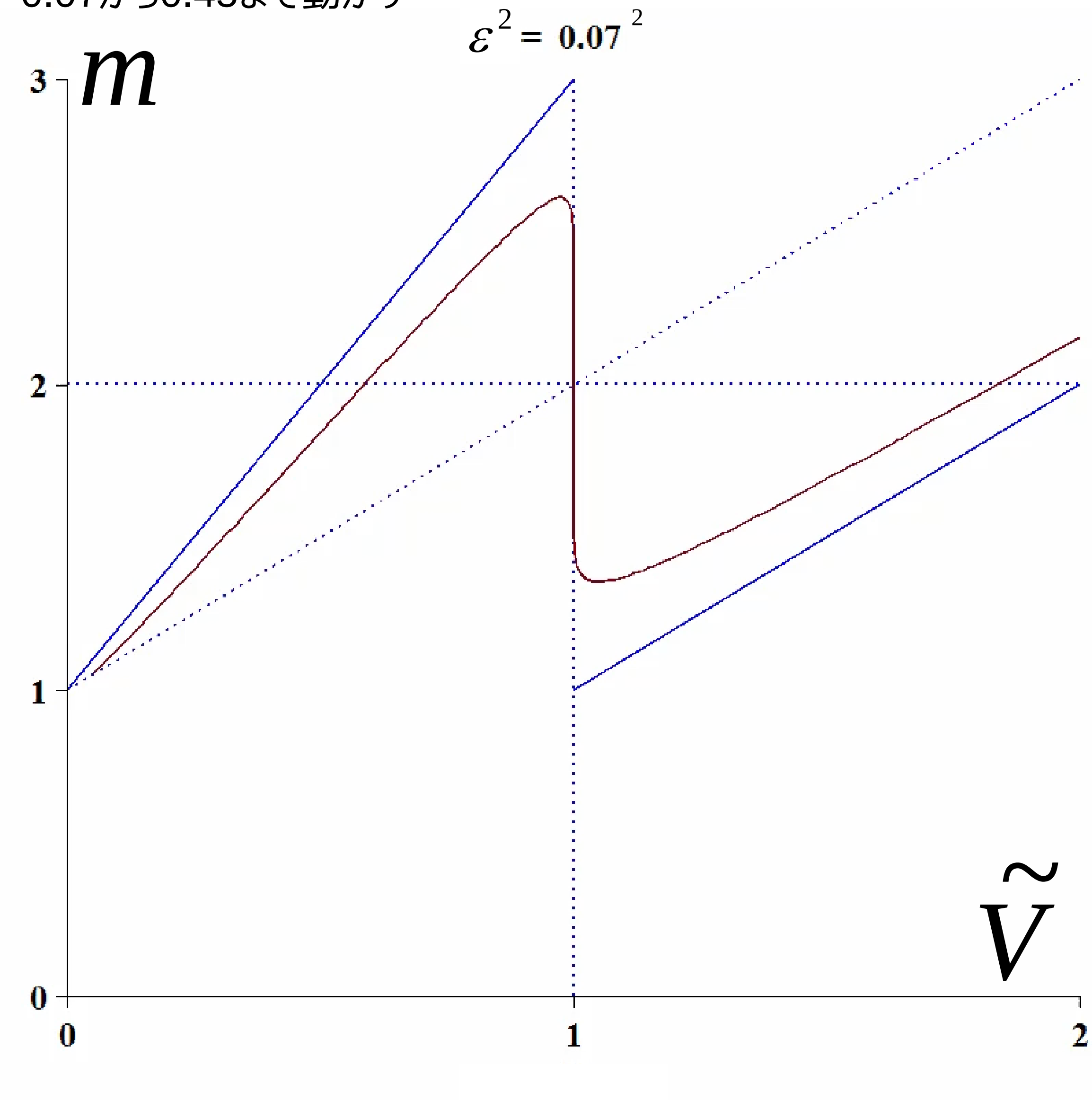
シートを正面から見た図

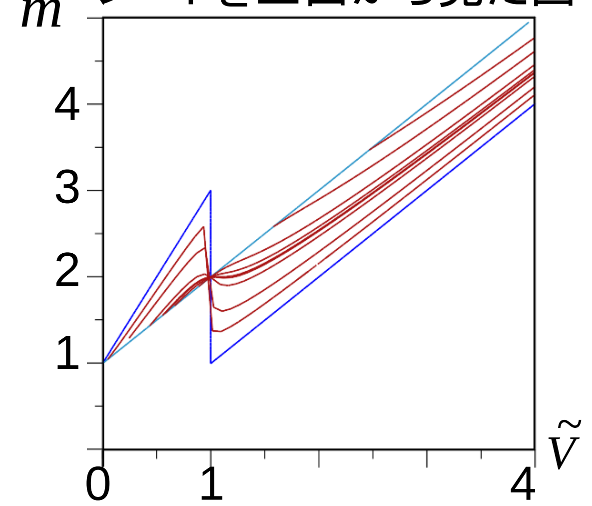


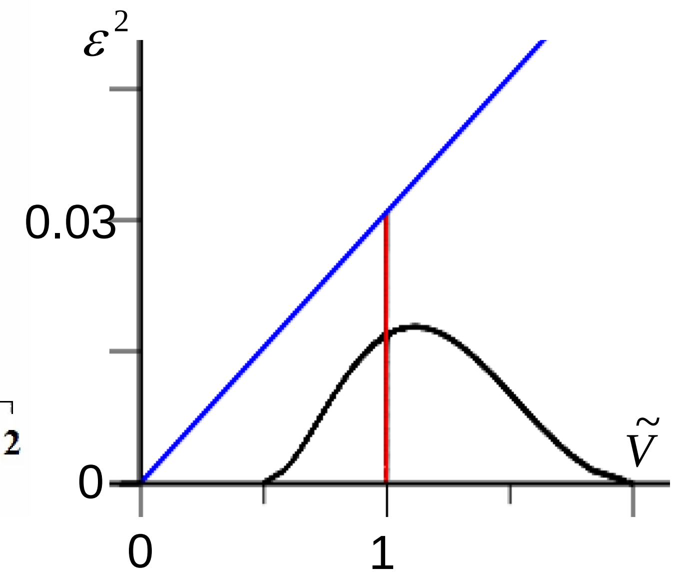
(SLP)の大域的分岐シート

各 ε ごとに分岐シートを正面から見た動画
0.07から0.45まで動かす

$$(SLP)\begin{cases}\varepsilon^2 W_{xx} + W(W-1)(\tilde{V}+1-W) = 0,\\ W_x(0) = W_x(1) = 0,\\ W'(x) > 0,\\ \int_0^1 W(x)\,\mathrm{dx} + \tilde{V} = m.\end{cases}$$







(SLP)の大域的分岐シート

各 ε ごとに分岐シートを正面から見た動画
0.07から0.45まで動かす

$$
(SLP)\begin{cases}
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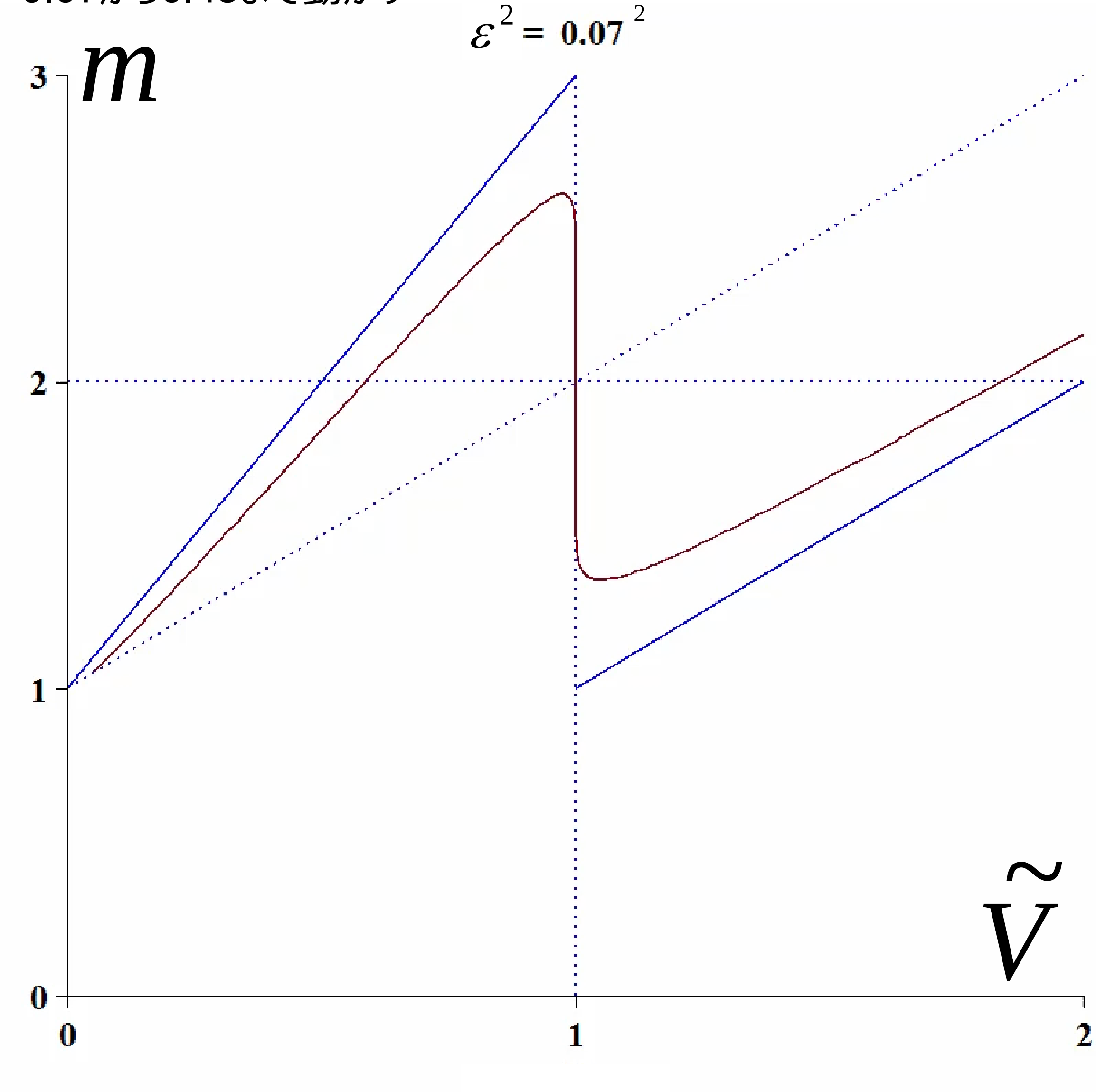
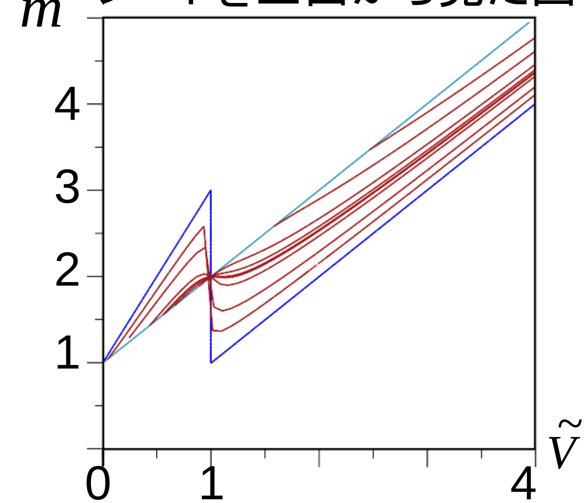
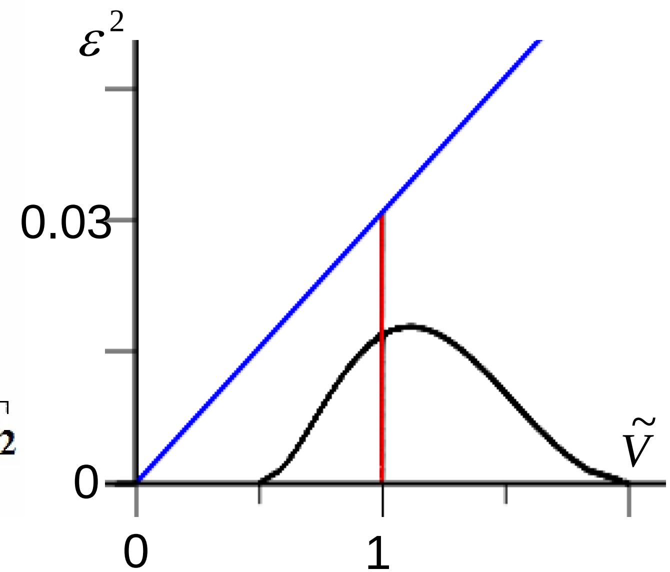
W_x(0) = W_x(1) = 0, \\
W'(x) > 0, \\
\int_0^1 W(x)\,\mathrm{dx} + \tilde{V} = m.
\end{cases}
$$



シートを正面から見た図

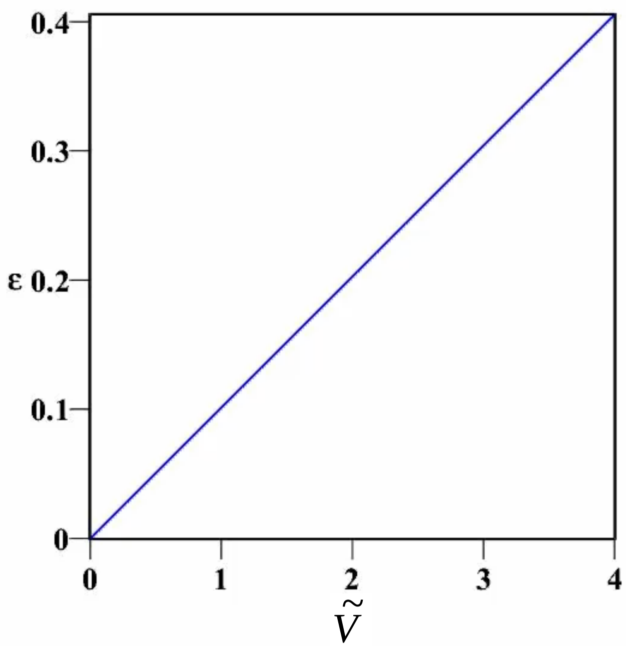


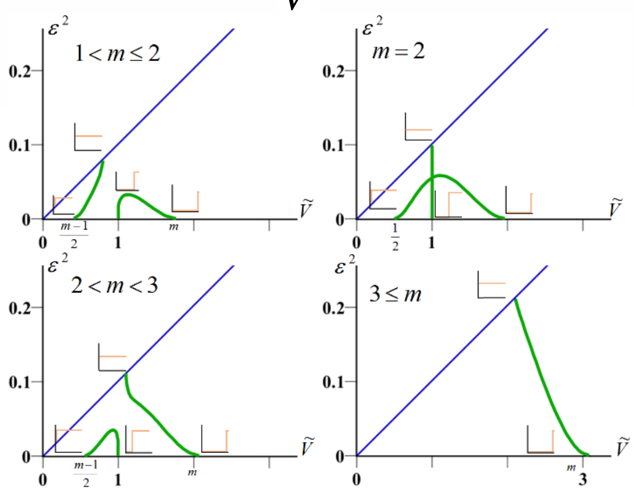


分岐曲線

m=1.000

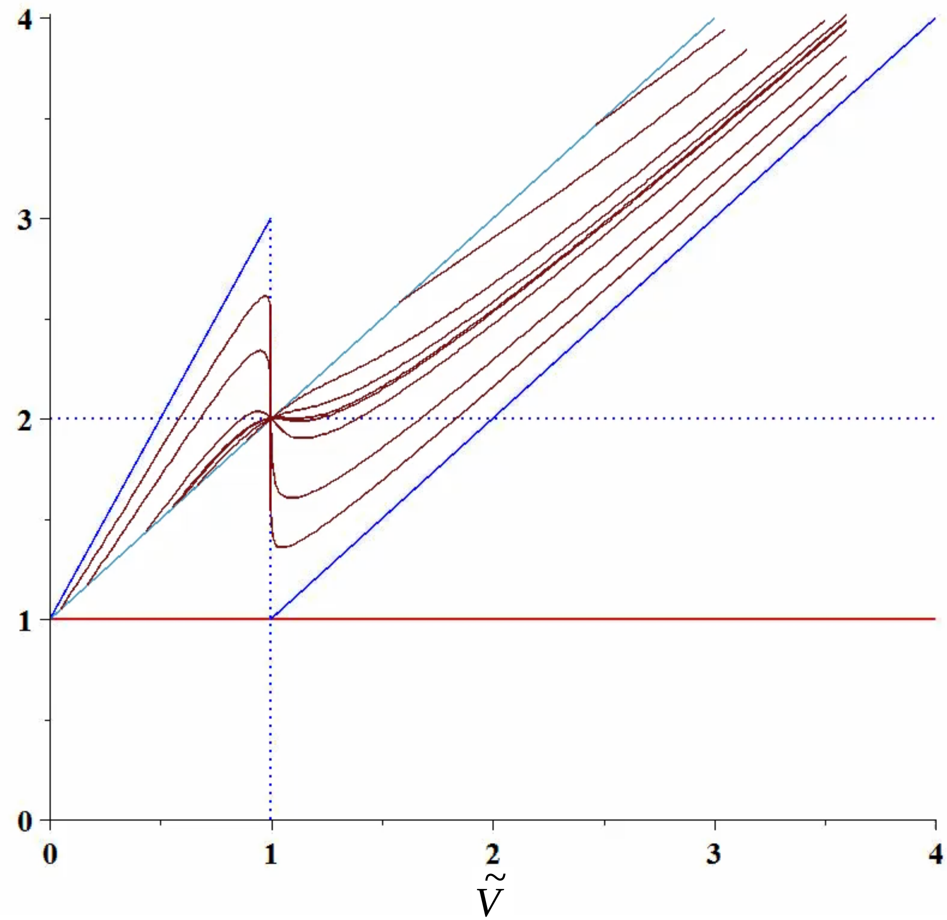
各 m ごとの分曲線図の動画
m を1から4まで動かす





分岐シートの正面

m=1.000



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(SLP)\begin{cases}
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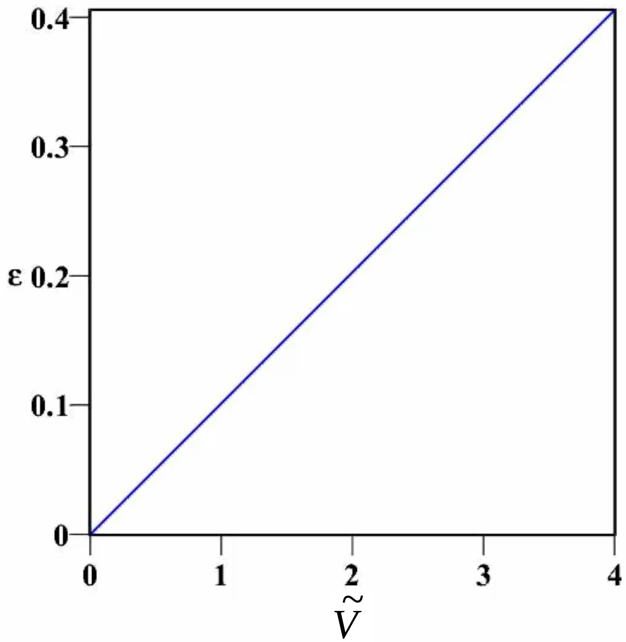
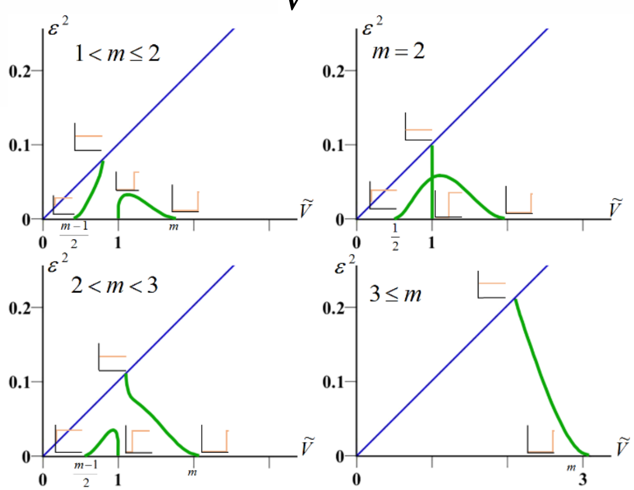
W_x(0) = W_x(1) = 0, \\
W'(x) > 0, \\
\int_0^1 W(x)\,\mathrm{dx} + \tilde{V} = m.
\end{cases}
$$

分岐曲線

m=1.000

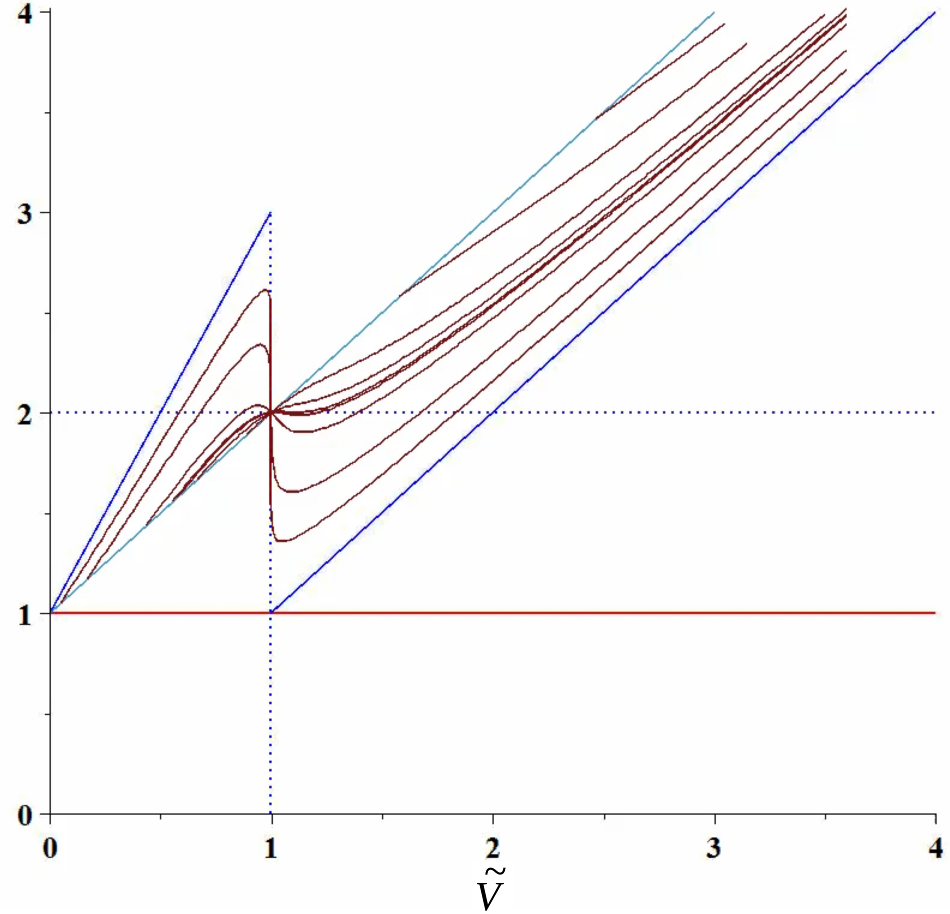
各 m ごとの分曲線図の動画
m を1から4まで動かす





分岐シートの正面

m=1.000



$$
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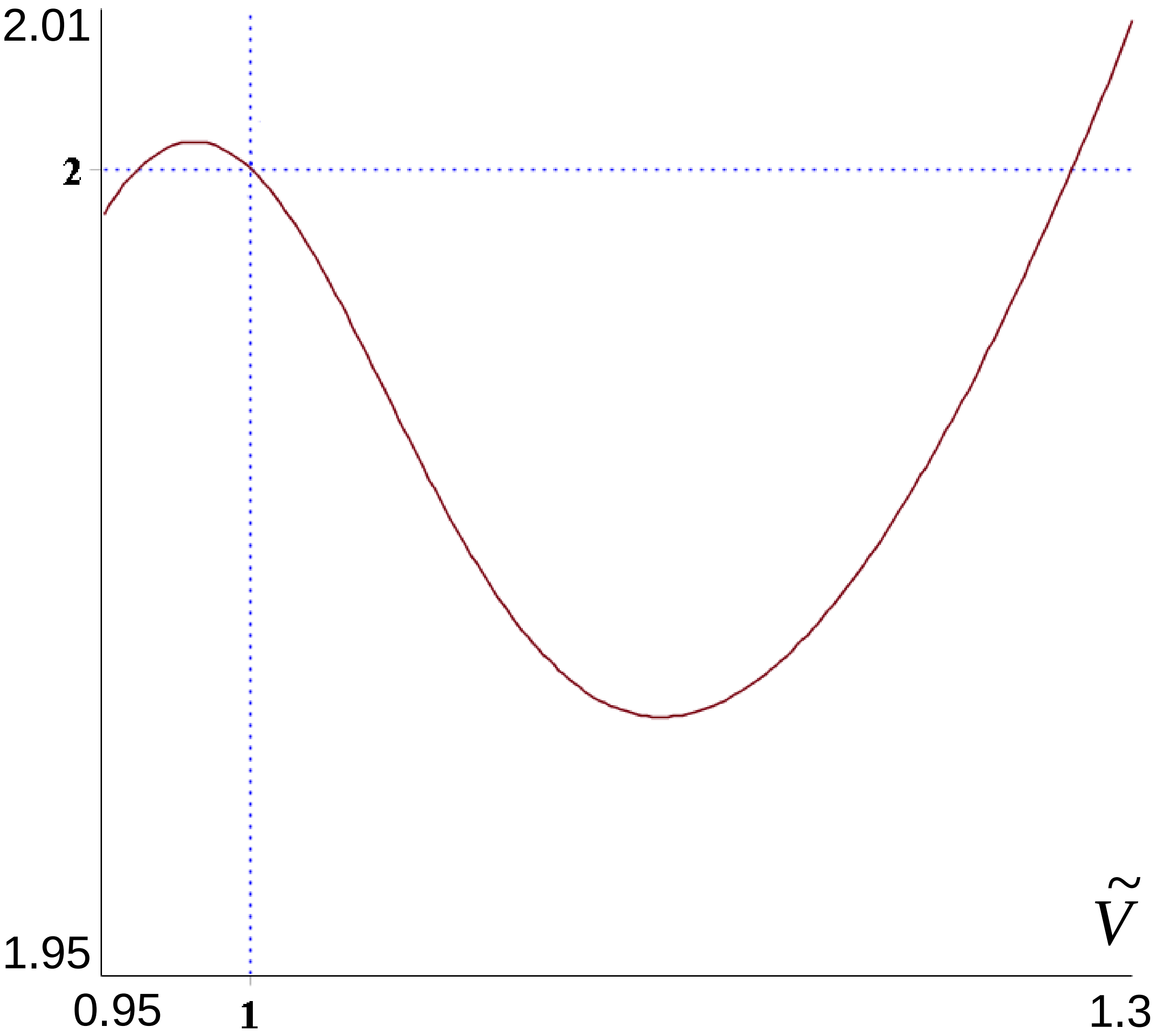
W_x(0) = W_x(1) = 0, \\
W'(x) > 0, \\
\int_0^1 W(x)\,\mathrm{dx} + \tilde{V} = m.
\end{cases}
$$

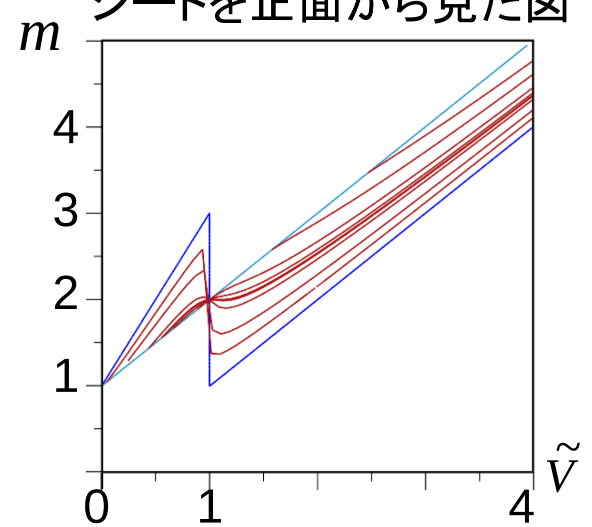
(SLP)の大域的分岐シート

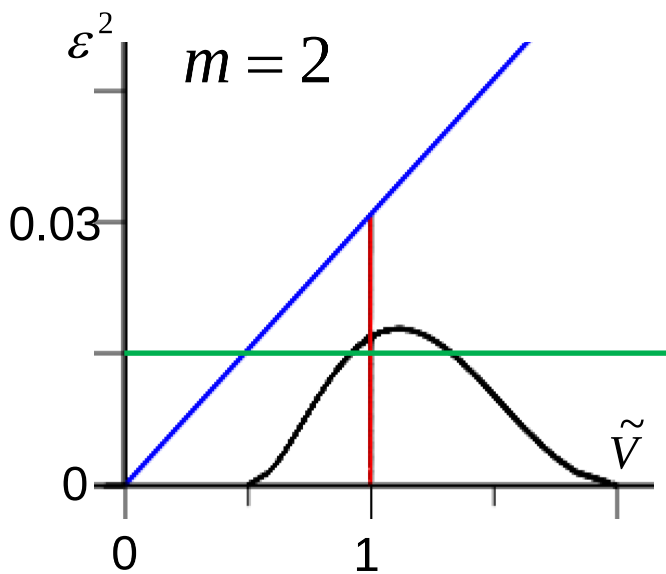
Zoom up $(\tilde{V}, m) \in [0.95, 1.3] \times [1.95, 2.01]$

Fix $\varepsilon^2 = 0.23^2$



シートを正面から見た図

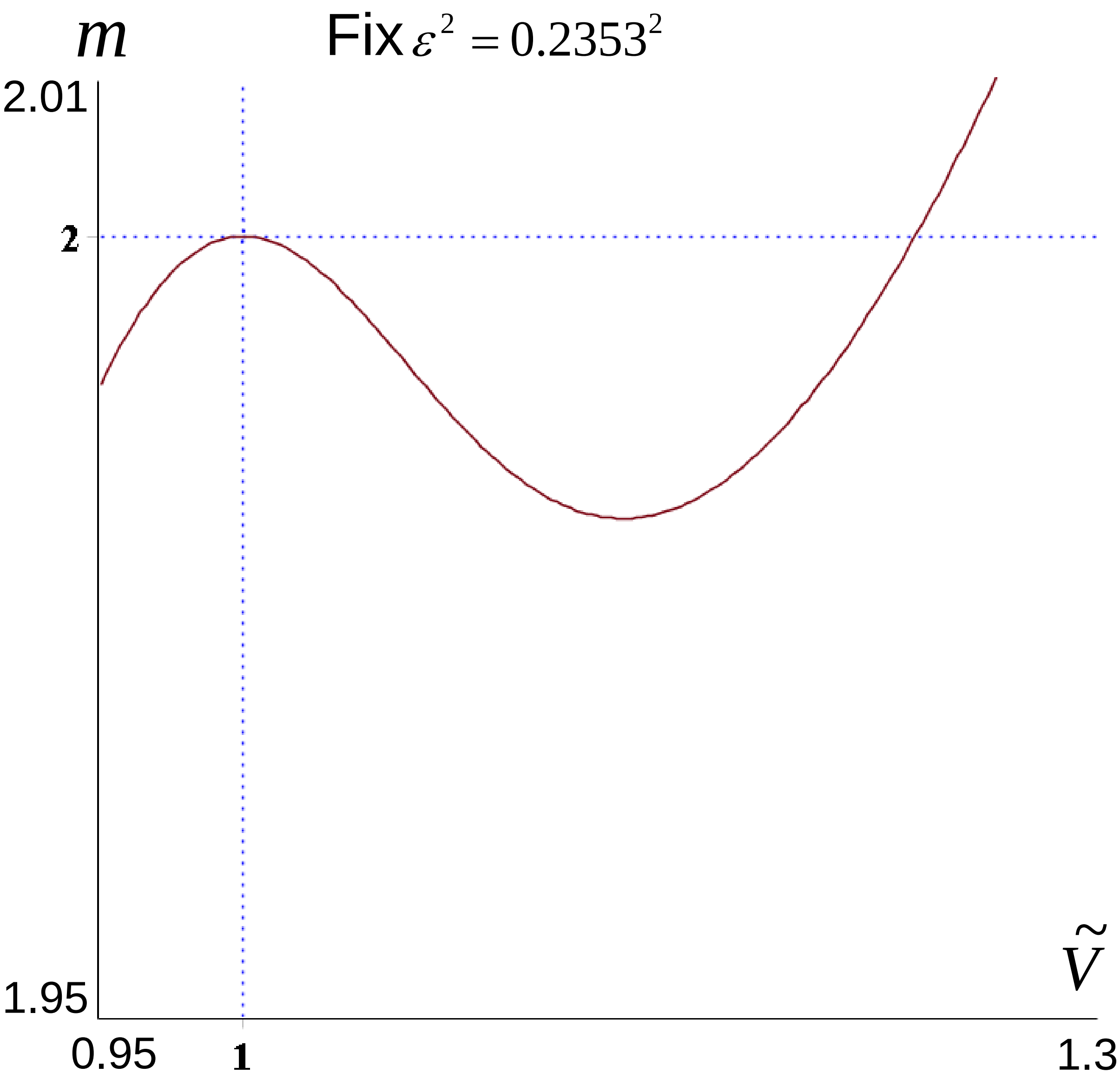




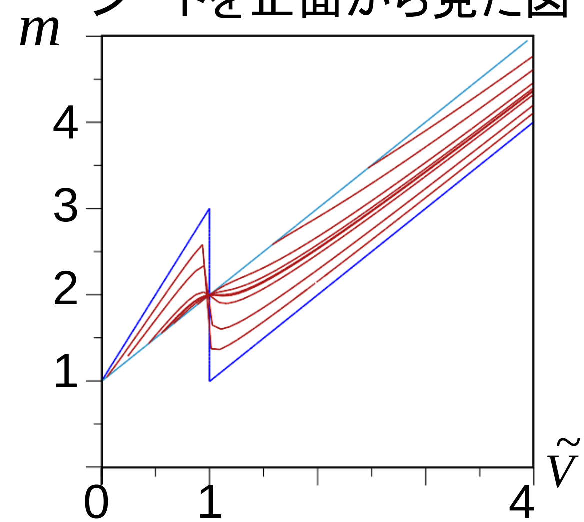
(SLP)の大域的分岐シート

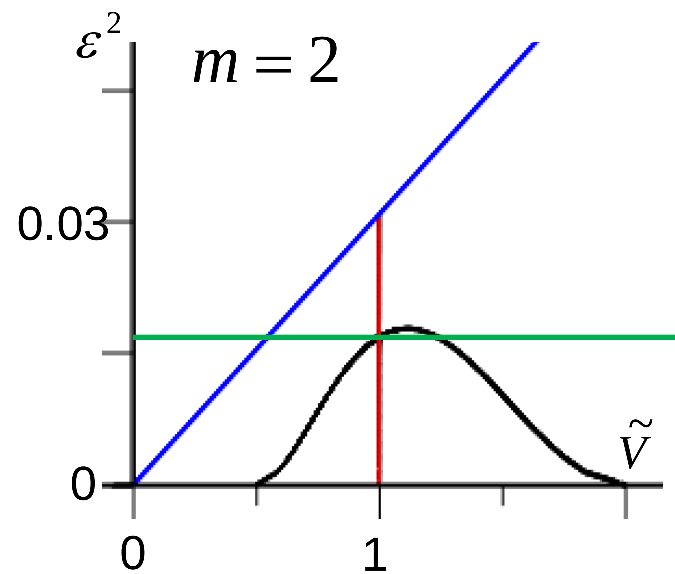
Zoom up $\left(\tilde{V}, m\right) \in [0.95, 1.3] \times [1.95, 2.01]$

Fix $\varepsilon^2 = 0.2353^2$



シートを正面から見た図

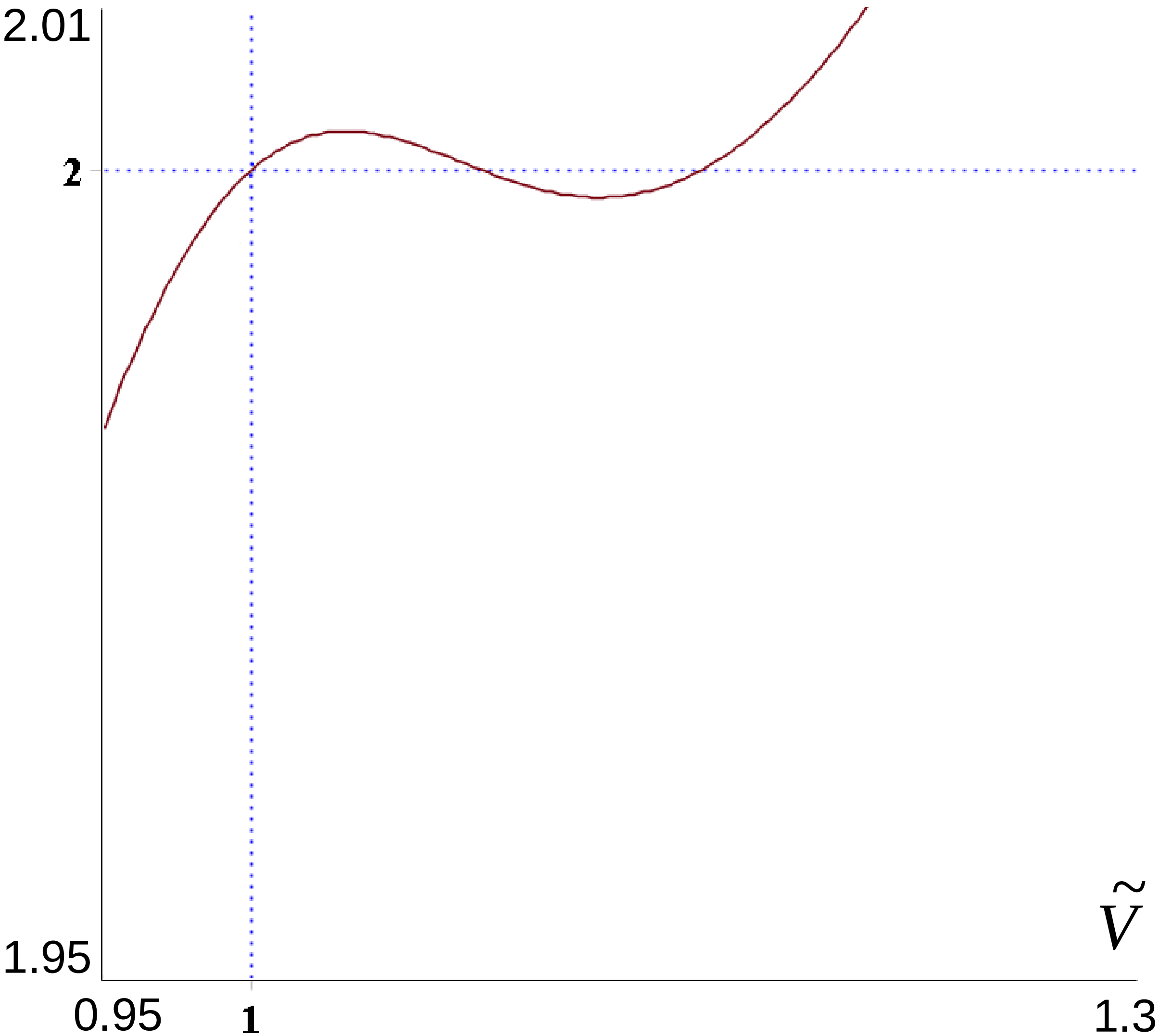




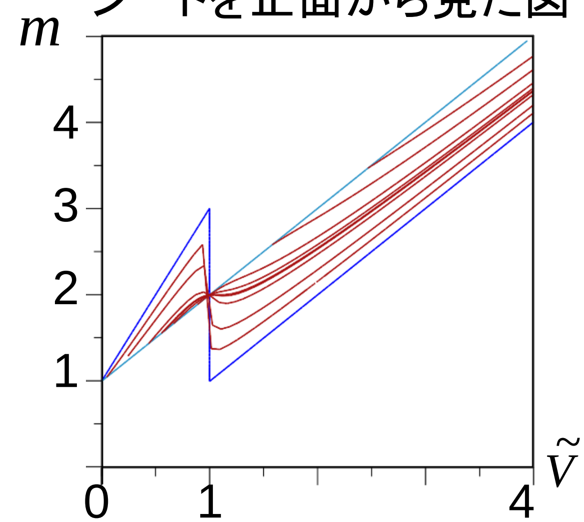
(SLP)の大域的分岐シート

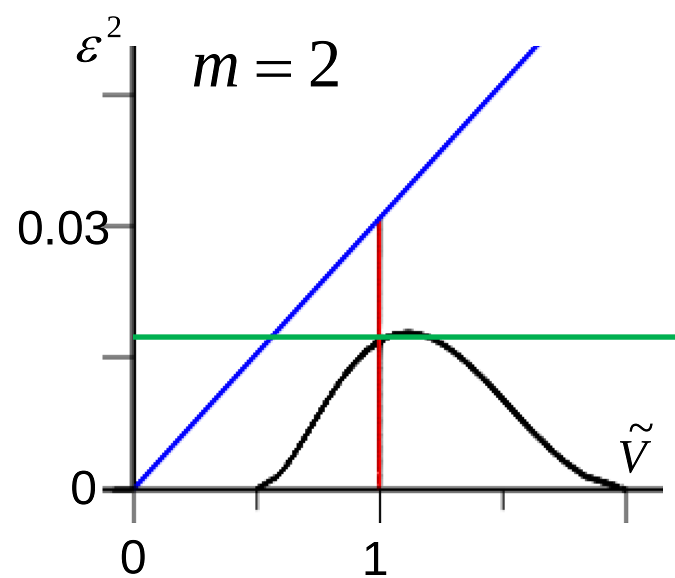
Zoom up $(\tilde{V}, m) \in [0.95, 1.3] \times [1.95, 2.01]$

Fix $\varepsilon^2 = 0.2412^2$



シートを正面から見た図

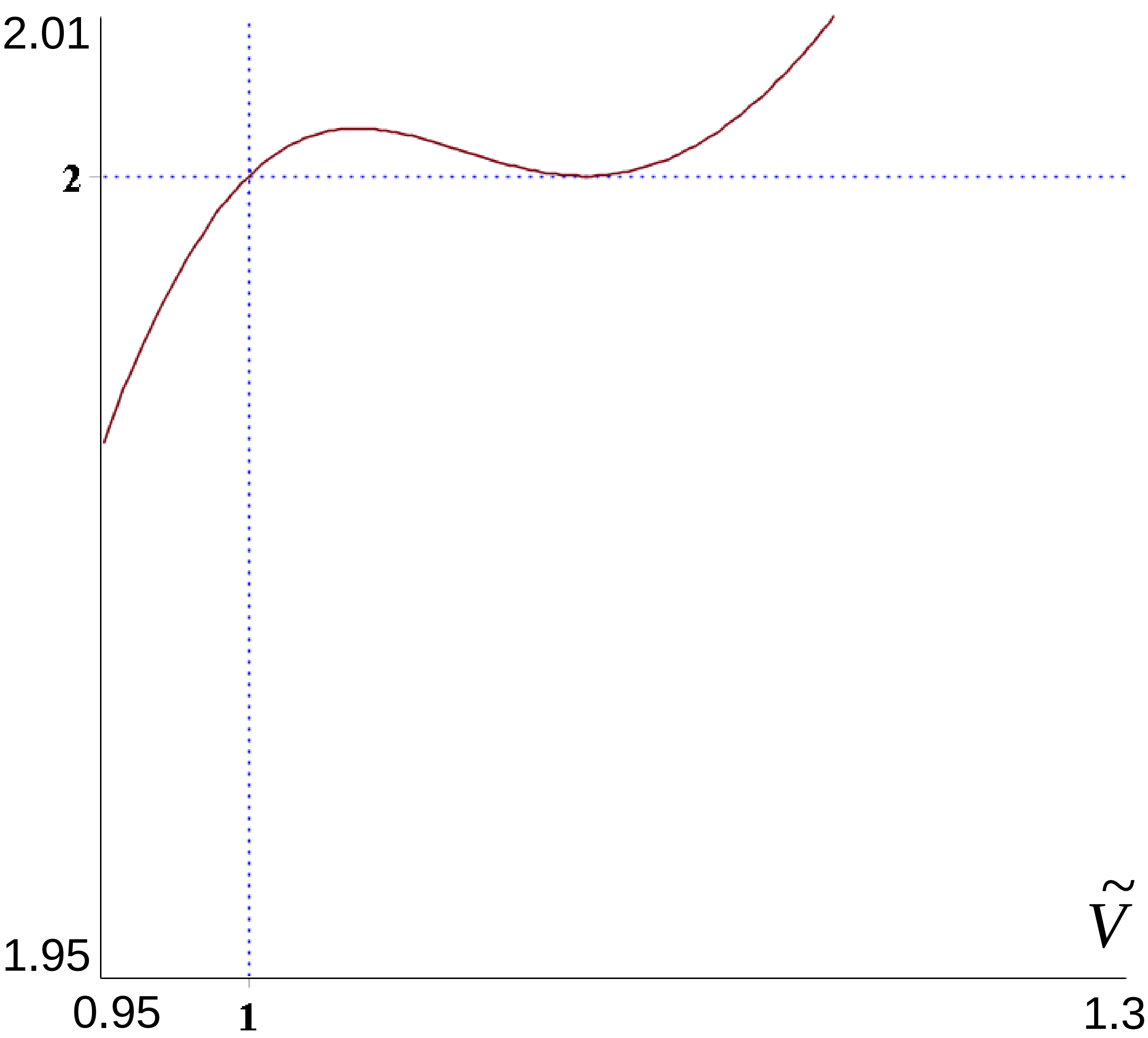


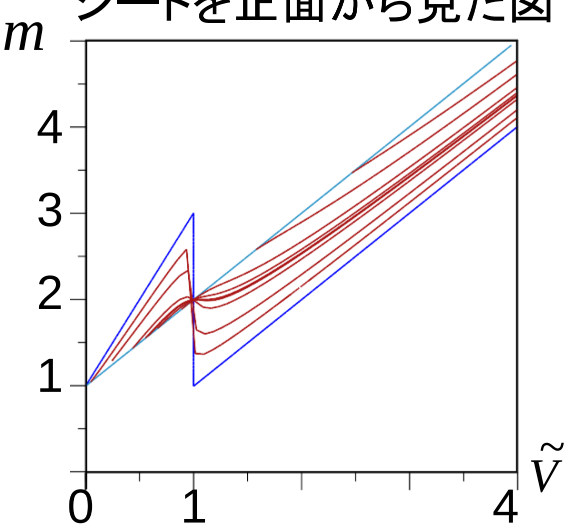


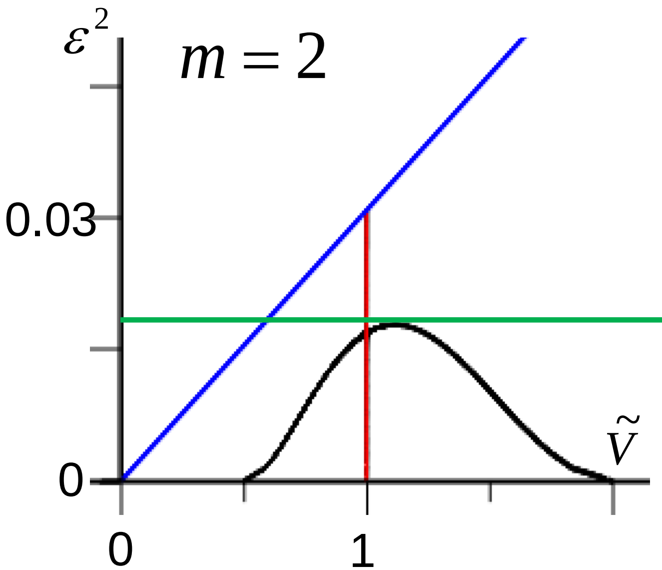
(SLP)の大域的分岐シート

Zoom up $(\tilde{V}, m) \in [0.95, 1.3] \times [1.95, 2.01]$

Fix $\varepsilon^2 = 0.241863750^2$



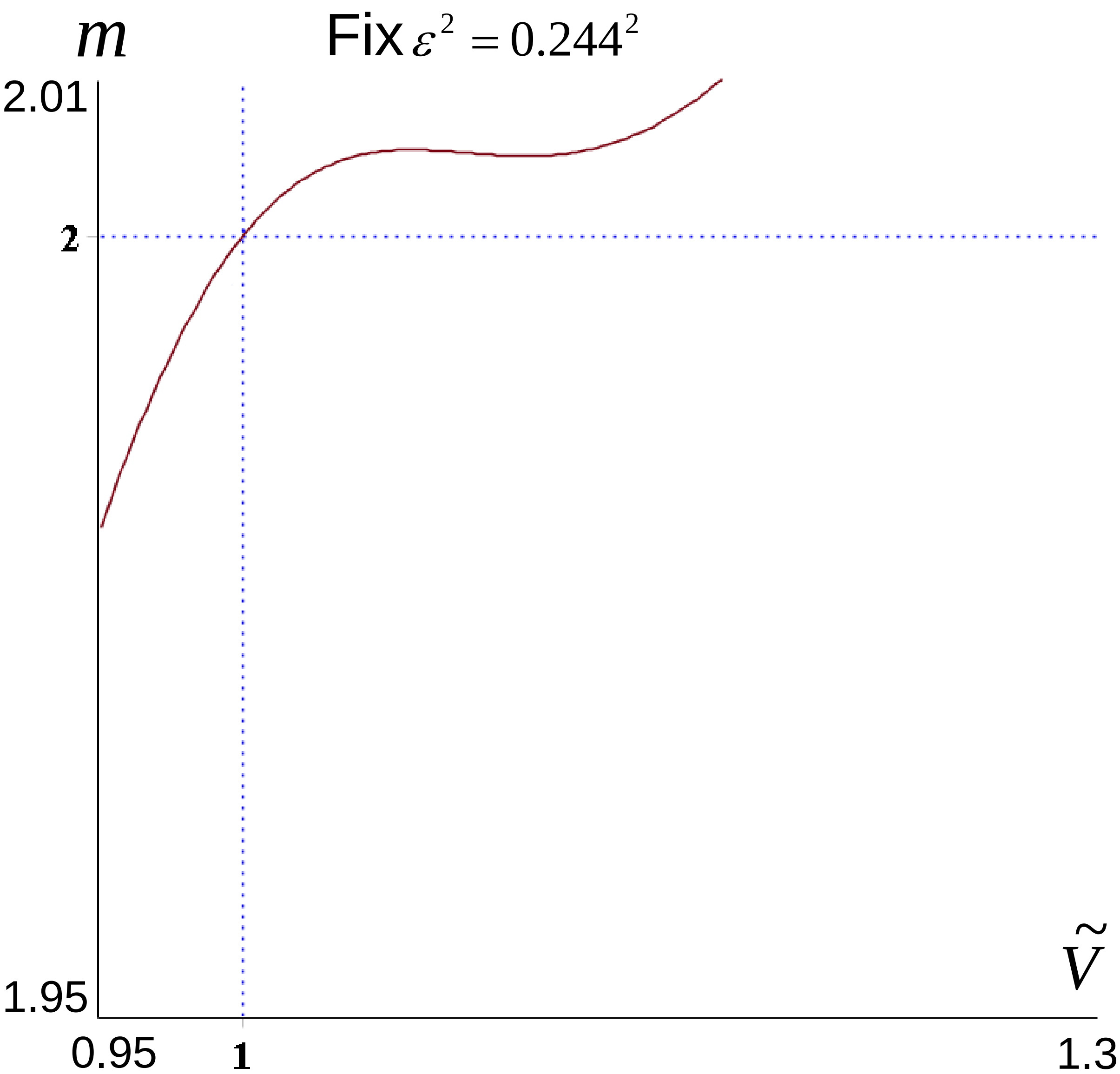




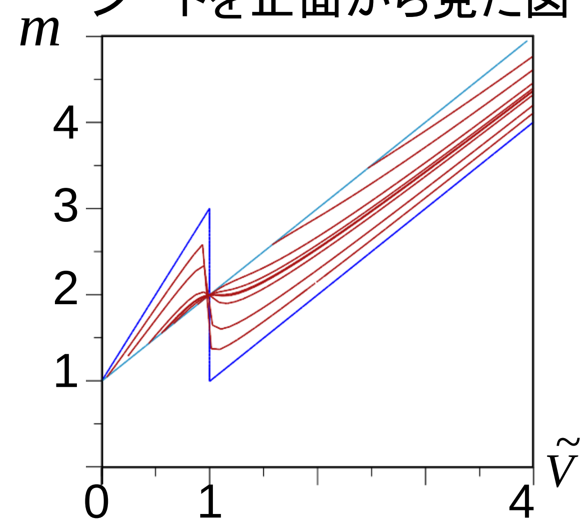
(SLP)の大域的分岐シート

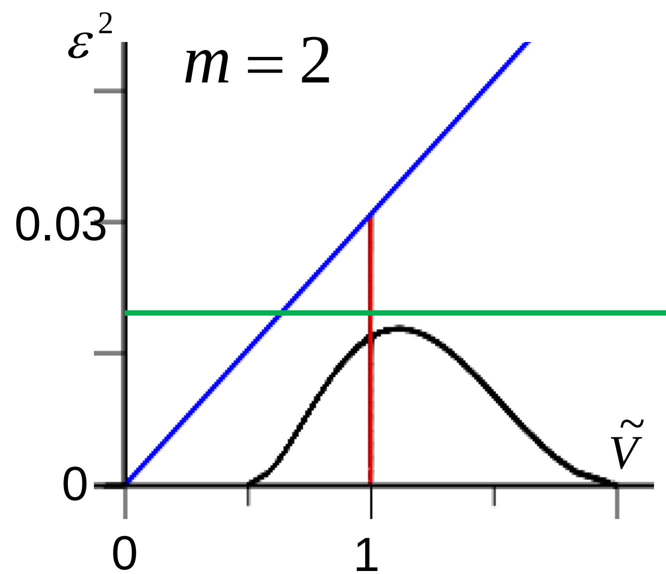
Zoom up $(\tilde{V}, m) \in [0.95, 1.3]\times[1.95, 2.01]$

Fix $\varepsilon^2 = 0.244^2$



シートを正面から見た図

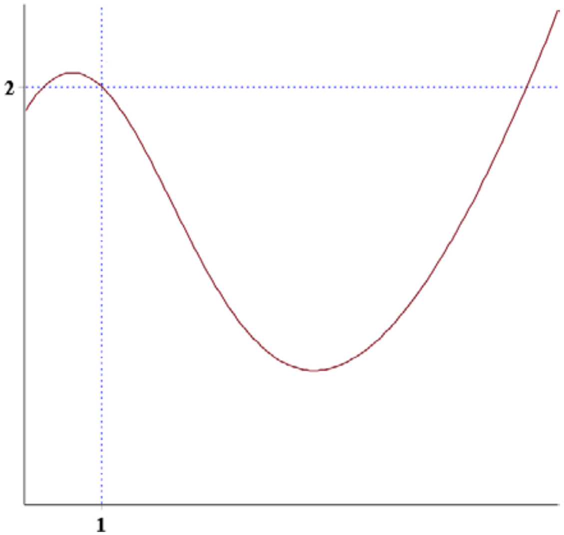




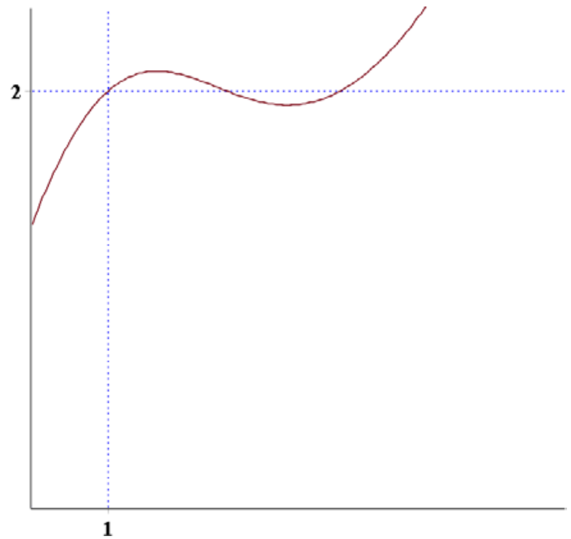
(SLP)の大域的分岐シート

Zoom up

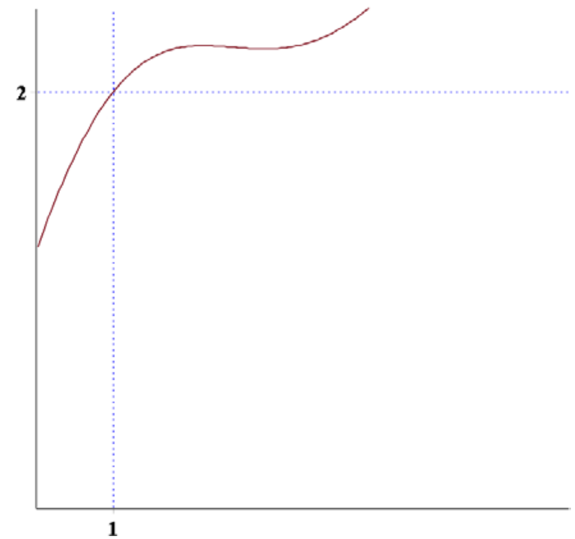
$$\varepsilon^2 = 0.23^2$$



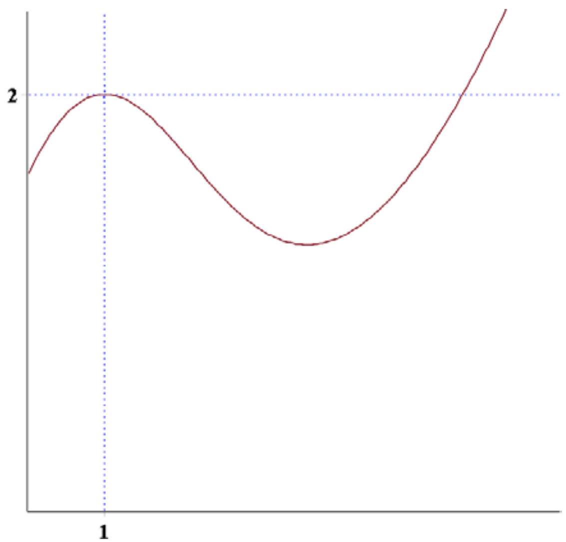
$$\varepsilon^2 = 0.2412^2$$



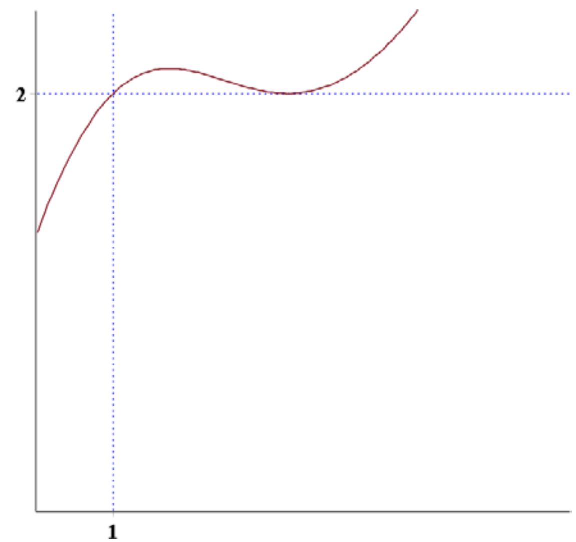
$$\varepsilon^2 = 0.244^2$$



$$\varepsilon^2 = 0.2353^2$$

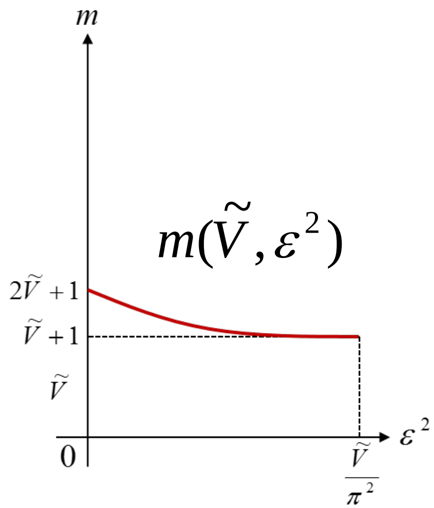


$$\varepsilon^2 = 0.241863750^2$$



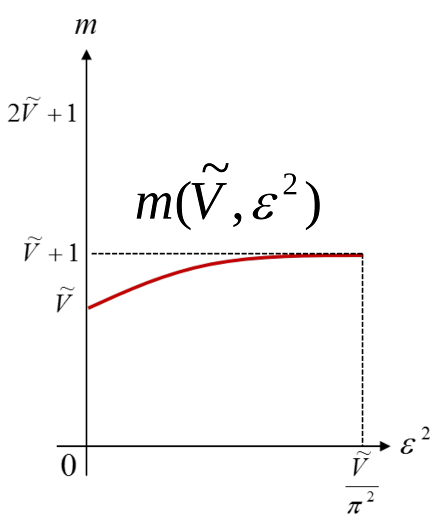
Theorem C *Let $\tilde{V} > 0$ be fixed. It holds that*

$$\frac{\partial m(\tilde{V}, \varepsilon^2)}{\partial \varepsilon} < 0 \quad \textit{for} \quad \varepsilon^2 \in \left(0, \frac{\tilde{V}}{\pi^2}\right) \textit{ with } \tilde{V} \in (0, 1),$$

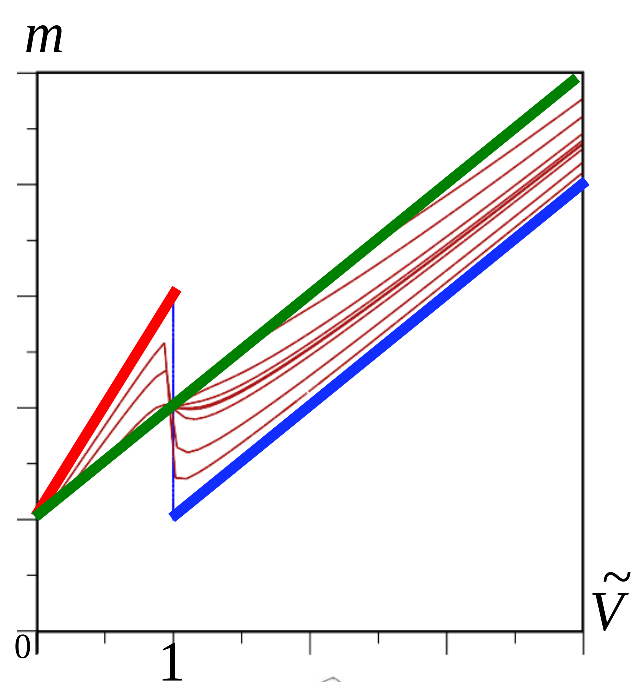


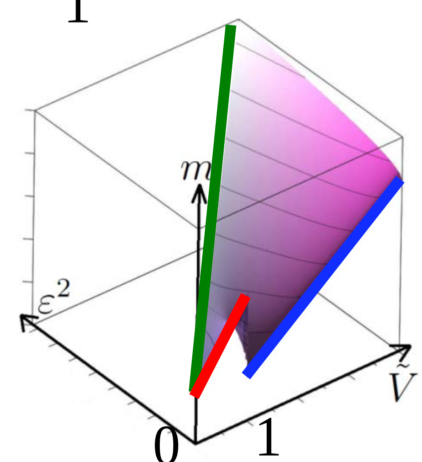
$\tilde{V} \in (0, 1)$ *be the fixed.*

$$\frac{\partial m(\tilde{V}, \varepsilon^2)}{\partial \varepsilon} > 0 \quad \textit{for} \quad \varepsilon^2 \in \left(0, \frac{\tilde{V}}{\pi^2}\right) \textit{ with } \tilde{V} \in (1, \infty).$$



$\tilde{V} \in (1, \infty)$ *be the fixed.*

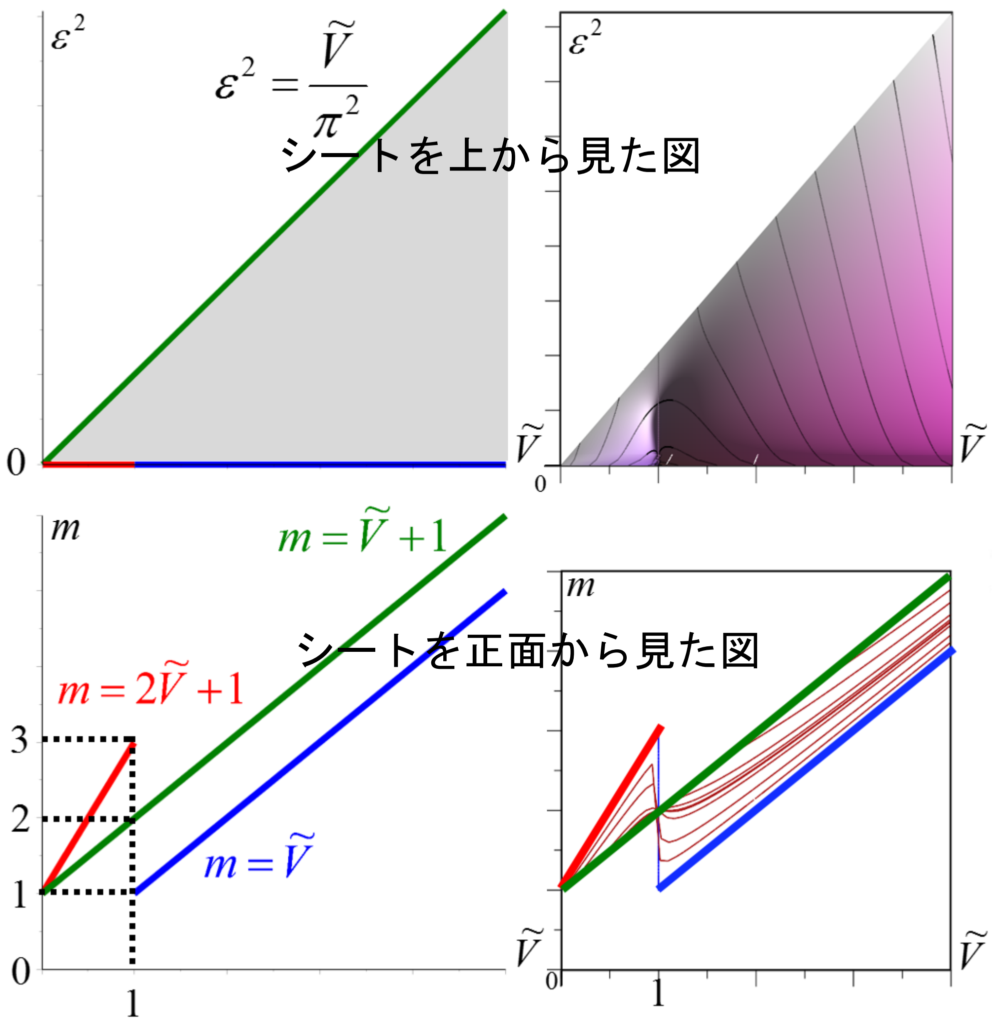




Theorem C. *Let* $\tilde{V} > 0$ *be fixed. It holds that*

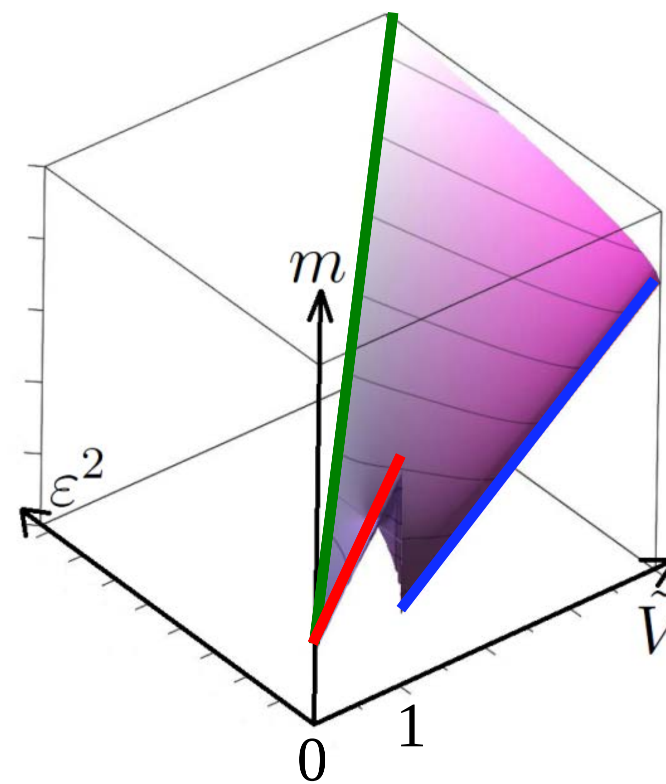
$$\frac{\partial m(\tilde{V},\varepsilon^2)}{\partial \varepsilon} < 0 \text{ for } 0 < \tilde{V} < 1,\ 0 < \varepsilon^2 < \frac{\tilde{V}}{\pi^2}. \qquad \frac{\partial m(\tilde{V},\varepsilon^2)}{\partial \varepsilon} > 0 \text{ for } 1 < \tilde{V},\ 0 < \varepsilon^2 < \frac{\tilde{V}}{\pi^2}.$$

Theorem B. （ $m(\tilde{V},\varepsilon^2)$ の端点での極限値 ）.



global bifurcation sheet

$$\left\{\left(\tilde{V},\ \varepsilon^2,\ m(\tilde{V},\varepsilon^2)\right) : (\tilde{V},\ \varepsilon^2) \in G\right\}$$

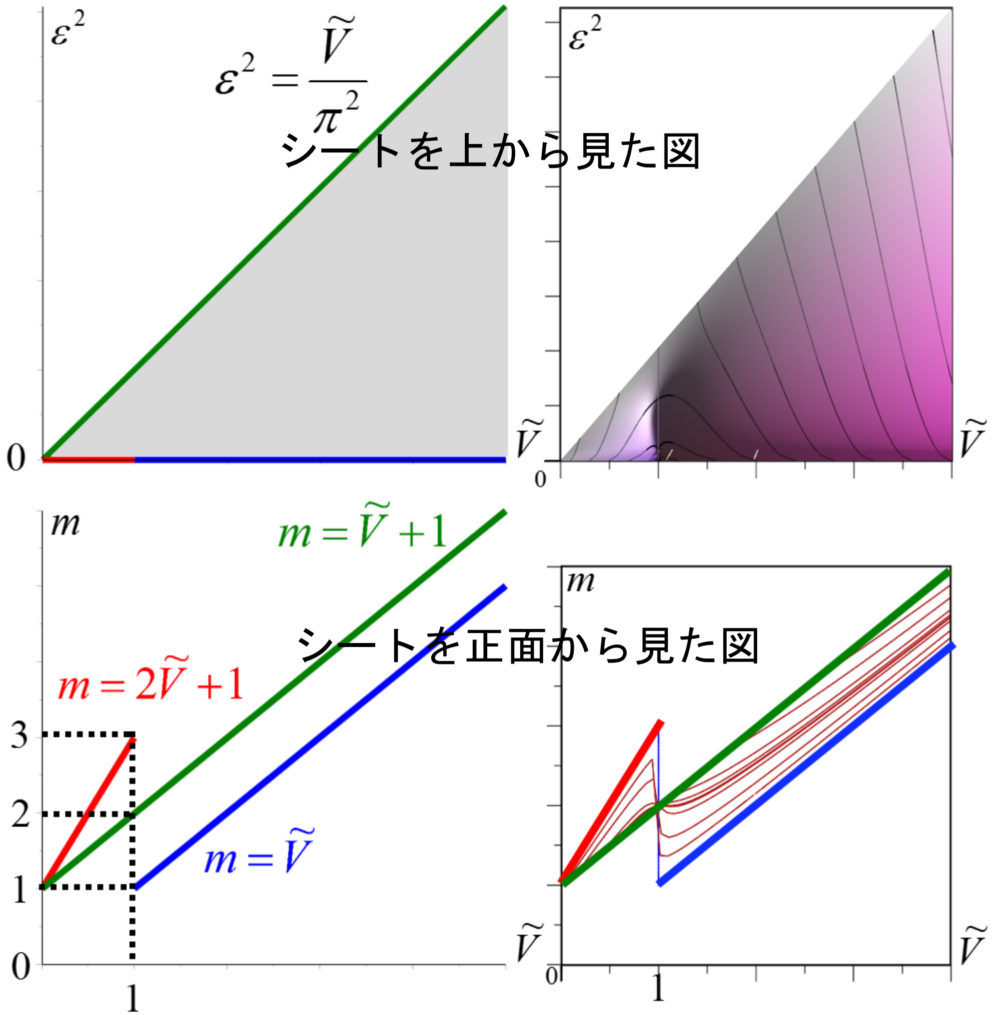


Theorem C. *Let* $\tilde{V} > 0$ *be fixed. It holds that*

$$\frac{\partial m(\tilde{V},\varepsilon^2)}{\partial \varepsilon} < 0 \text{ for } 0 < \tilde{V} < 1,\ 0 < \varepsilon^2 < \frac{\tilde{V}}{\pi^2}. \qquad \frac{\partial m(\tilde{V},\varepsilon^2)}{\partial \varepsilon} > 0 \text{ for } 1 < \tilde{V},\ 0 < \varepsilon^2 < \frac{\tilde{V}}{\pi^2}.$$

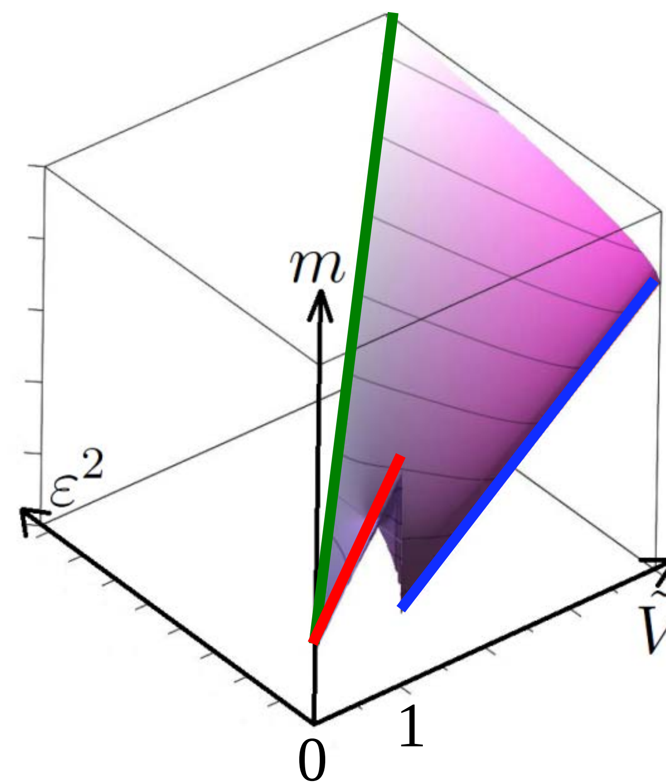
これらより, 以下の Th. 1 - 5 に示すように, Exact multiplicityと大域的分岐曲線の存在・非存在・挙動がわかる.

Theorem B. ($m(\tilde{V},\varepsilon^2)$ の端点での極限値).

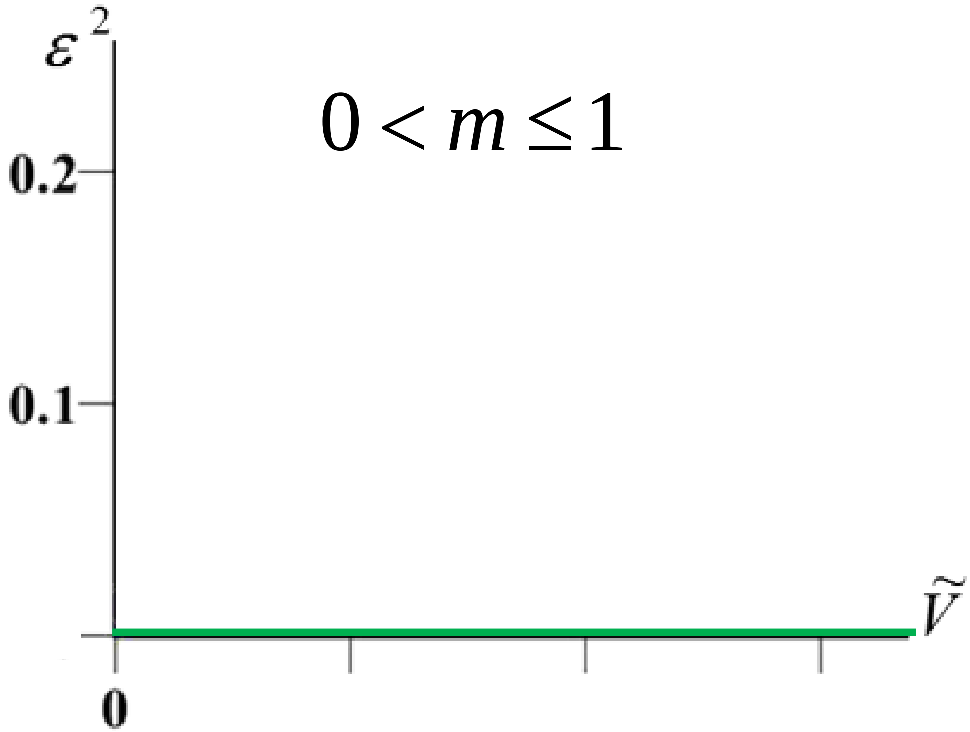


global bifurcation sheet

$$\left\{ \left(\tilde{V},\ \varepsilon^2,\ m(\tilde{V},\varepsilon^2) \right) : (\tilde{V},\ \varepsilon^2) \in G \right\}$$



Theorem 1. *Let $0 < m \leq 1$ be given. There exists no solution of (SLP).*



Theorem 2. *Let $1 < m < 2$ be given. The followings hold.*

(i) For $\tilde{V} \in \left(0, \frac{m-1}{2}\right] \cup [m-1, 1] \cup [m, \infty)$, *there exists no solution of (SLP).*

(ii) For $\tilde{V} \in \left(\frac{m-1}{2}, m-1\right)$, *there exists the unique* $\varepsilon^2(\tilde{V}) \in \left(0, \frac{\tilde{V}}{\pi^2}\right)$

such that $W(x; \tilde{V}, \varepsilon^2(\tilde{V}))$ *is a solution of (SLP).*

Moreover, $\varepsilon(\tilde{V})$ *is continuous in* $\left(\frac{m-1}{2}, m-1\right)$,

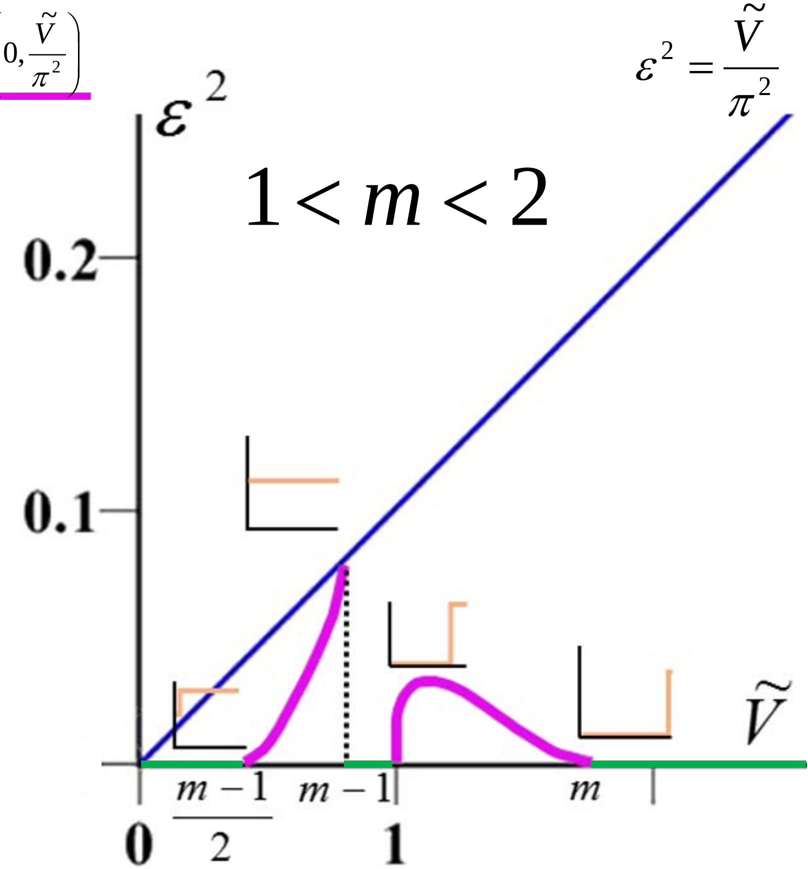
$$\varepsilon^2(\tilde{V}) \to 0 \text{ as } \tilde{V} \downarrow \frac{m-1}{2}, \quad \varepsilon^2(\tilde{V}) \to \frac{m-1}{\pi^2} \text{ as } \tilde{V} \uparrow m-1.$$

(iii) For $\tilde{V} \in (1, m)$, *there exists the unique* $\varepsilon^2(\tilde{V}) \in \left(0, \frac{\tilde{V}}{\pi^2}\right)$

such that $W(x; \tilde{V}, \varepsilon^2(\tilde{V}))$ *is a solution of (SLP).*

Moreover, $\varepsilon(\tilde{V})$ *is continuous in* $(1, m)$,

$$\varepsilon^2(\tilde{V}) \to 0 \text{ as } \tilde{V} \downarrow 1, \quad \varepsilon^2(\tilde{V}) \to 0 \text{ as } \tilde{V} \uparrow m.$$



Theorem 3. *Let* $m=2$ *be given. The followings hold.*

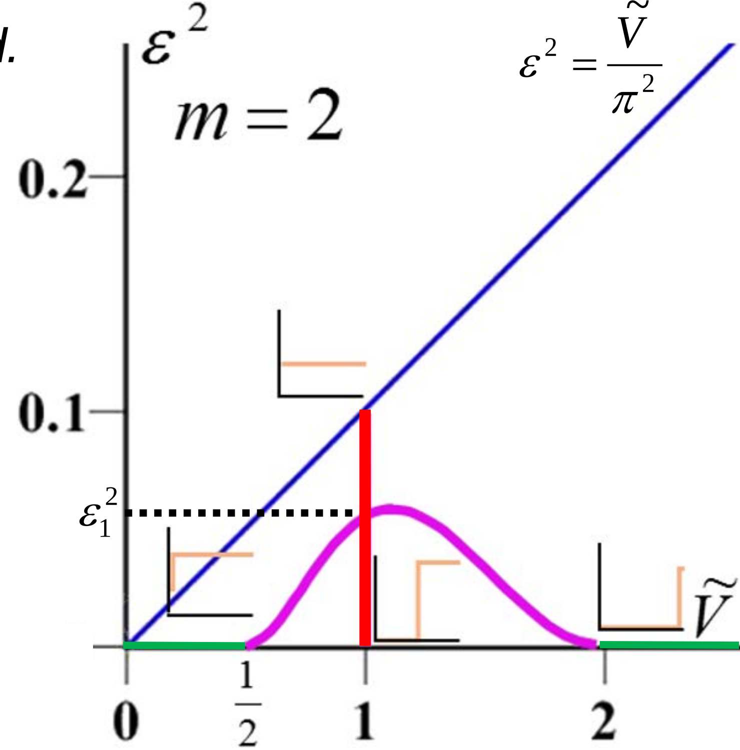
(i) For $\tilde{V} \in \left(0, \frac{1}{2}\right] \cup [2, \infty)$, *there exists no solution of (SLP).*

(ii) For $\tilde{V} \in \left(\frac{1}{2}, 1\right)$, *there exists the unique* $\varepsilon^2(\tilde{V}) \in \left(0, \frac{\tilde{V}}{\pi^2}\right)$

such that $W(x; \tilde{V}, \varepsilon^2(\tilde{V}))$ *is a solution of (SLP).*

Moreover, $\varepsilon(\tilde{V})$ *is continuous in* $\left(\frac{1}{2}, 1\right)$,

$\varepsilon^2(\tilde{V}) \to 0$ *as* $\tilde{V} \downarrow \frac{1}{2}$, $\varepsilon^2(\tilde{V}) \to \varepsilon_1^2 = 0.05536\cdots$ *as* $\tilde{V} \uparrow 1$.



(iii) For $\tilde{V} = 1$, *there exist infinity many solutions.*

More precisely, all solutions are given by $\left\{ W(x; 1, \varepsilon^2) : \varepsilon^2 \in \left(0, \frac{1}{\pi^2}\right) \right\}$.

(iv) For $\tilde{V} \in (1, 2)$ *there exists the unique* $\varepsilon^2(\tilde{V}) \in \left(0, \frac{\tilde{V}}{\pi^2}\right)$

such that $W(x; \tilde{V}, \varepsilon^2(\tilde{V}))$ *is a solution of (SLP).*

Moreover, $\varepsilon(\tilde{V})$ *is continuous in* $(1, 2)$,

$\varepsilon^2(\tilde{V}) \to \varepsilon_1^2$ *as* $\tilde{V} \downarrow 1$, $\varepsilon^2(\tilde{V}) \to 0$ *as* $\tilde{V} \uparrow 2$.

Theorem 4. *Let* $2 < m < 3$ *be given. The followings hold.*

(i) For $\tilde{V} \in \left(0, \frac{m-1}{2}\right] \cup [1, m-1] \cup [m, \infty)$, *there exists no solution of (SLP).*

(ii) For $\tilde{V} \in \left(\frac{m-1}{2}, 1\right)$, *there exists the unique* $\varepsilon^2(\tilde{V}) \in \left(0, \frac{\tilde{V}}{\pi^2}\right)$

such that $W(x; \tilde{V}, \varepsilon^2(\tilde{V}))$ *is a solution of (SLP).*

Moreover, $\varepsilon(\tilde{V})$ *is continuous in* $\left(\frac{m-1}{2}, 1\right)$,

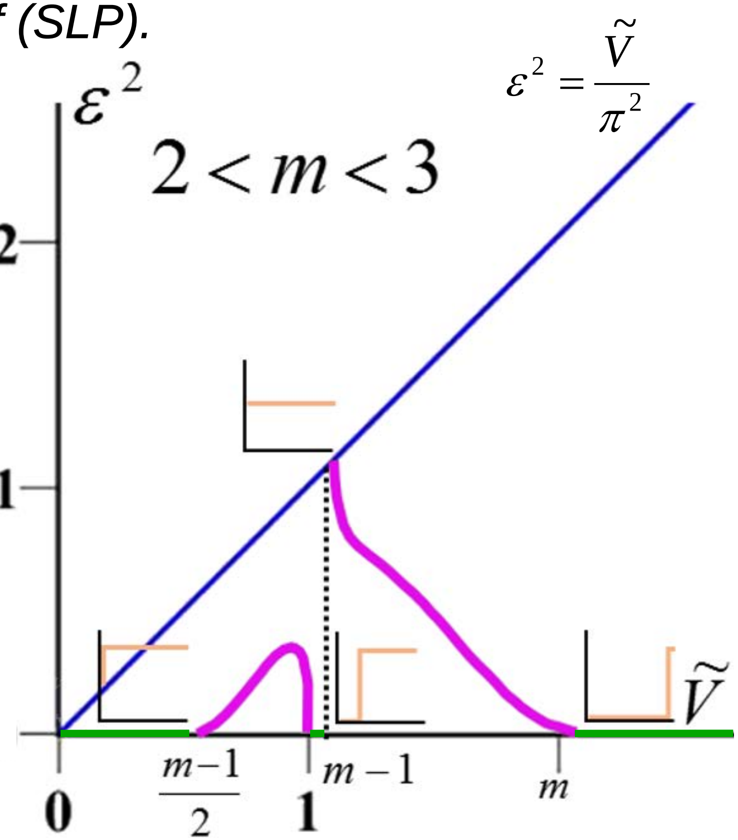
$\varepsilon^2(\tilde{V}) \to 0$ *as* $\tilde{V} \downarrow \frac{m-1}{2}$, $\quad \varepsilon^2(\tilde{V}) \to 0$ *as* $\tilde{V} \uparrow 1$.

(iii) For $\tilde{V} \in (m-1, 1)$, *there exists the unique* $\varepsilon^2(\tilde{V}) \in \left(0, \frac{\tilde{V}}{\pi^2}\right)$

such that $W(x; \tilde{V}, \varepsilon^2(\tilde{V}))$ *is a solution of (SLP).*

Moreover, $\varepsilon(\tilde{V})$ *is continuous in* $(m-1, m)$,

$\varepsilon^2(\tilde{V}) \to \frac{m-1}{\pi^2}$ *as* $\tilde{V} \downarrow m-1$, $\quad \varepsilon^2(\tilde{V}) \to 0$ *as* $\tilde{V} \uparrow m$.



Theorem 5. *Let* $m \geq 3$ *be given. The followings hold.*

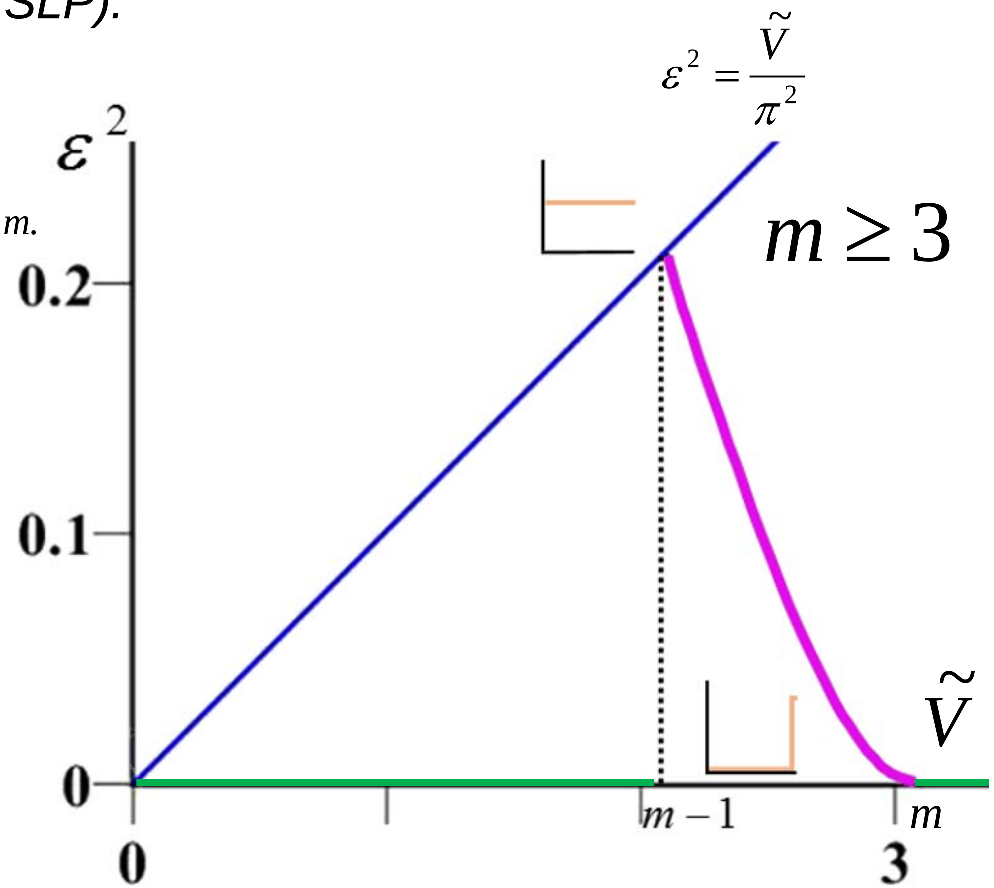
(i) For $\tilde{V} \in (0,\ m-1] \cup [m,\ \infty)$, *there exists no solution of (SLP).*

(ii) For $\tilde{V} \in (m-1,\ m)$, *there exists the unique* $\varepsilon^2(\tilde{V}) \in \left(0, \frac{\tilde{V}}{\pi^2}\right)$

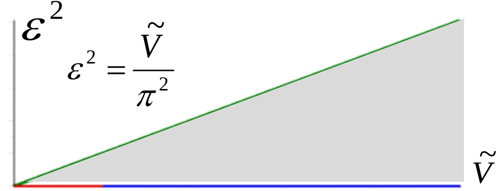
such that $W(x; \tilde{V}, \varepsilon^2(\tilde{V}))$ *is a solution of (SLP).*

Moreover, $\varepsilon(\tilde{V})$ *is continuous in* $(m-1,\ m)$,

$\varepsilon^2(\tilde{V}) \to \frac{m-1}{\pi^2}$ *as* $\tilde{V} \downarrow m-1$, $\varepsilon^2(\tilde{V}) \to 0$ *as* $\tilde{V} \uparrow m$.



Prop.

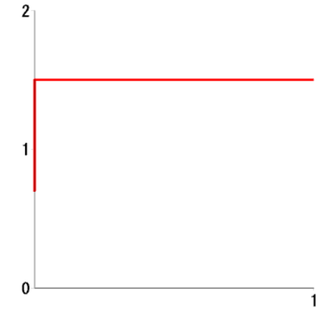


$$\begin{cases} \varepsilon^2 W_{xx} + W(W-1)(\tilde{V}+1-W) = 0, & x \in (0,1), \\ W_x(0) = W_x(1) = 0, & \\ W(x) > 0, \ W'(x) > 0, & x \in (0,1) \end{cases}$$

$$\lim_{\varepsilon^2 \uparrow \frac{V}{\pi^2}} W(x; \tilde{V}, \varepsilon^2) = 1,$$

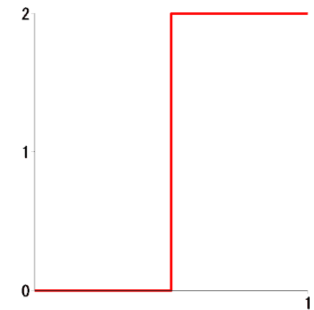
$$\lim_{\varepsilon^2 \downarrow 0} W(x; \tilde{V}, \varepsilon^2) = \begin{cases} \frac{\tilde{V}+2}{3} - \frac{(3s_0 - 1)\sqrt{\tilde{V}(\tilde{V}+1)}}{\sqrt{8 s_0}} & (x = 0), \\ V + 1 & (0 < x \le 1). \end{cases}$$

for $0 < \tilde{V} < 1$.



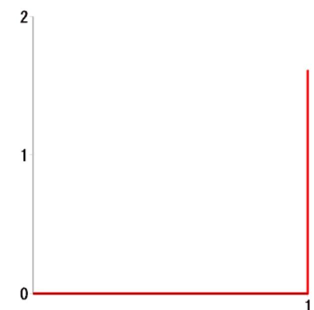
with $s_0 := \frac{1}{9} \cdot \frac{7\tilde{V}^2 + 7\tilde{V} + 4 - 2\sqrt{2}(2\tilde{V}+1)\sqrt{(\tilde{V}+2)(1-\tilde{V})}}{V(\tilde{V}+1)}.$

$$W\left(\frac{1}{2}; 1, \varepsilon^2\right) = 1, \qquad \lim_{\varepsilon^2 \downarrow 0} W(x; 1, \varepsilon^2) = \begin{cases} 0 & \left(0 \le x < \frac{1}{2}\right), \\ 2 & \left(\frac{1}{2} < x < 1\right). \end{cases}$$



$$\lim_{\varepsilon^2 \downarrow 0} W(x; \tilde{V}, \varepsilon^2) = \begin{cases} 0 & (0 \le x < 1), \\ \frac{\tilde{V}+2}{3} + \frac{\sqrt{2}}{4} \cdot \frac{(3s_1 - 1)\sqrt{\tilde{V}+1}}{\sqrt{s_1}} & (x = 1). \end{cases}$$

for $1 < \tilde{V}$.



with $s_1 := \frac{1}{9} \cdot \frac{4\tilde{V}^2 + 7\tilde{V} + 7 - 2\sqrt{2}(\tilde{V}+2)\sqrt{(2\tilde{V}+1)(\tilde{V}-1)}}{\tilde{V}+1}.$

定理のより詳細な説明と証明のアイデア

Now, let us introduce an auxiliary problem to investigate (SLP). Let $\tilde{V} > 0$ be given, let us consider the problem

$$(AP;\tilde{V}) \begin{cases} \varepsilon^2 W_{xx} + W(W-1)(\tilde{V}+1-W) = 0 \quad \text{in } (0,1), & (2.1) \\ W_x(0) = W_x(1) = 0, & (2.2) \\ W_x(x) > 0 \text{ in } (0,1). & (2.3) \end{cases}$$

We note that (AP;$\tilde{V}$) is equivalent to

$$\begin{cases} \varepsilon^2 W_{xx} + W(W-1)(\tilde{V}+1-W) = 0 \quad \text{in } (0,1), \\ W_x(0) = W_x(1) = 0, \\ 0 < W(0) < \tilde{V}+1, \quad W_x(x) > 0 \text{ in } (0,1) \end{cases}$$

for given $\tilde{V} > 0$, since it is easy to see that a condition $0 < W(0) < \tilde{V}+1$ holds for any solution of (AP;$\tilde{V}$).

The existence and the uniqueness of the solution $W(x)$ of (AP;$\tilde{V}$) is well-known (see, e.g. Smoller-Wasserman [5] and Smoller [6]). However, we need to know more precise information to investigate (SLP). The following theorem gives the representation formula for all solutions of (AP;$\tilde{V}$).

Theorem 2.1. *Let $\tilde{V} > 0$. There exists a solution of (AP;$\tilde{V}$), if and only if $(\tilde{V},\ \varepsilon^2) \in \mathcal{G}$, where*

$$\mathcal{G} := \left\{ (\tilde{V},\ \varepsilon^2) :\ 0 < \varepsilon^2 < \frac{\tilde{V}}{\pi^2} \right\}. \tag{2.4}$$

Moreover, the solution is unique. The solution $W(x;\tilde{V},\varepsilon^2)$ has properties

$$0 < W(x;\tilde{V},\varepsilon^2) < \tilde{V} + 1, \tag{2.5}$$

$$W(x;\tilde{V},\varepsilon^2) = \tilde{V} + 1 - \tilde{V} \cdot W\left(1 - x; \frac{1}{\tilde{V}}, \frac{\varepsilon^2}{\tilde{V}^2}\right). \tag{2.6}$$

The solution $W(x;\tilde{V},\varepsilon^2)$ *is represented by*

$$W(x,\tilde{V},\varepsilon^2) = \frac{\tilde{V}+2}{3}$$
$$+ \frac{1}{\sqrt{3}}\sqrt{\tilde{V}^2+\tilde{V}+1}\cdot\frac{\beta\cdot(1-hs)\mathrm{sn}^2(K(\sqrt{h})x,\sqrt{h})+\alpha\cdot\mathrm{cn}^2(K(\sqrt{h})x,\sqrt{h})}{(1-hs)\mathrm{sn}^2(K(\sqrt{h})x,\sqrt{h})+\mathrm{cn}^2(K(\sqrt{h})x,\sqrt{h})}, \quad (2.7)$$

$$\alpha := \alpha(h,s) = \frac{3hs^2-2(1+h)s+1}{\sqrt{3h^2s^4-4(h^2+h)s^3+(4h^2+2h+4)s^2-4(h+1)s+3}}, \quad (2.8)$$

$$\beta := \beta(h,s) = \frac{-hs^2-2(1-h)s+1}{\sqrt{3h^2s^4-4(h^2+h)s^3+(4h^2+2h+4)s^2-4(h+1)s+3}}, \quad (2.9)$$

where $(h,s) = \Big(h(\tilde{V},\varepsilon^2), s(\tilde{V},\varepsilon^2)\Big)$ *is the unique solution of the following system of transcendental equations*

$$(\mathrm{E})\begin{cases} \mathcal{E}(h,s) = \sqrt{3}\cdot\dfrac{\varepsilon}{\sqrt{\tilde{V}^2+\tilde{V}+1}}, & (2.10)\\ \mathcal{A}(h,s) = \dfrac{1}{3\sqrt{3}}\cdot\dfrac{(1-\tilde{V})(2\tilde{V}+1)(\tilde{V}+2)}{\sqrt{\tilde{V}^2+\tilde{V}+1}^3}, & (2.11)\\ 0<h<1, \quad 0<s<1, & (2.12)\end{cases}$$

where

$$\mathcal{E}(h,s) := \frac{\sqrt{2s(1-s)(1-sh)}/K(\sqrt{h})}{\sqrt{3h^2s^4-4(h^2+h)s^3+(4h^2+2h+4)s^2-4(1+h)s+3}}, \quad (2.13)$$

$$\mathcal{A}(h,s) := \frac{2(hs^2-2sh+1)(hs^2-2s+1)(1-hs^2)}{\sqrt{3h^2s^4-4(h^2+h)s^3+(4h^2+2h+4)s^2-4(h+1)s+3}^3}. \quad (2.14)$$

We show the graph of $\mathcal{A}(h,s)$ and $\mathcal{E}(h,s)$ in Figures 1 and 2.

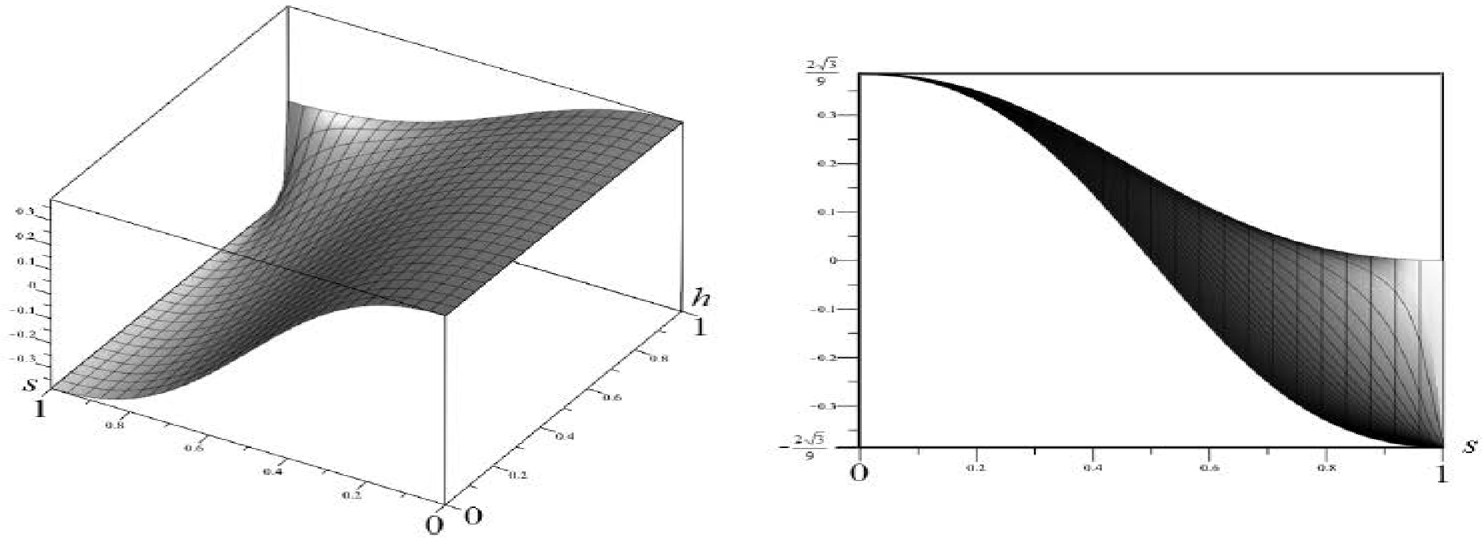


FIGURE 1. Graph of $\mathcal{A}(h,s)$

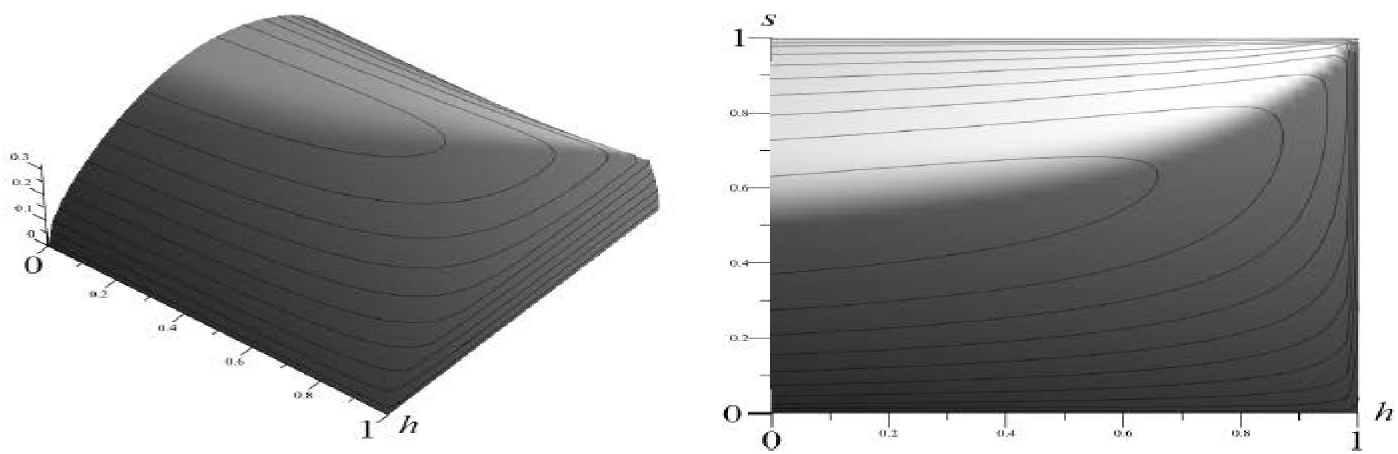


FIGURE 2. Graph of $\mathcal{E}(h,s)$

Theorem 2.2. *Let $W(x;\tilde{V},\varepsilon^2)$ be the unique solution of (AP;$\tilde{V}$), and*

$$m(\tilde{V},\ \varepsilon^2) := \int_0^1 W(x;\tilde{V},\varepsilon^2)dx + \tilde{V}, \tag{2.15}$$

then

$$m(\tilde{V},\varepsilon^2) = 2\tilde{V} + 2 - \tilde{V} m\left(\frac{1}{\tilde{V}}, \frac{\varepsilon^2}{\tilde{V}^2}\right) \quad \text{for any} \quad \tilde{V} > 0, \quad \varepsilon > 0. \tag{2.16}$$

In particular,

$$m(1,\varepsilon^2) = 2 \quad \text{for any} \quad \varepsilon > 0. \tag{2.17}$$

Moreover, it holds that

$$m(\tilde{V},\ \varepsilon^2) = \frac{4\tilde{V}+2}{3} + \frac{1}{\sqrt{3}} \cdot \sqrt{\tilde{V}^2+\tilde{V}+1} \cdot \mathcal{M}(h,s), \tag{2.18}$$

$$\mathcal{M}(h,s) := \frac{-(hs^2 - 2(1+h)s + 3) + 4(1-s)(1-sh)\Pi(-sh,\sqrt{h})/K(\sqrt{h})}{\sqrt{3h^2s^4 - 4(h^2+h)s^3 + (4h^2+2h+4)s^2 - 4(1+h)s + 3}}, \tag{2.19}$$

where $h = h(\tilde{V},\varepsilon^2)$, $s = s(\tilde{V},\varepsilon^2)$ are given in Theorem2.1. Here, $K(\cdot)$ is the complete elliptic integral of the 1st kind, and $\Pi(\cdot,\cdot)$ is the complete elliptic integral of the third kind.

Let us define the global bifurcation sheet S by

$$S := \left\{ \left(\tilde{V}, \varepsilon^2, \int_0^1 W dx + \tilde{V} \right) : (\tilde{V},\ \varepsilon^2) \in \mathcal{G} \right\}.$$

We obtain exact representation the global bifurcation sheet S as

$$S = \left\{ \left(\tilde{V}, \varepsilon^2, m(\tilde{V},\ \varepsilon^2) \right) : (\tilde{V},\ \varepsilon^2) \in \mathcal{G} \right\} \tag{2.20}$$

by Theorem 2.2 . For each m, we can obtain the bifurcation diagram by

$$\left\{ (\tilde{V},\ \varepsilon^2) \in \mathcal{G} : m(\tilde{V},\ \varepsilon^2) = m \right\} \tag{2.21}$$

directly from the global bifurcation sheet S.

We will mathematically investigate precise properties of the global bifurcation sheet and bifurcation diagrams in a forth-coming paper. For instance, we see the following facts:

- For each fixed $\tilde{V} \in (0, \infty)$, $W(x; \tilde{V}, \varepsilon^2) \to 1$ as $\varepsilon^2 \to \tilde{V}/\pi^2$ uniformly on $[0, 1]$.
- For each fixed $\tilde{V} \in (0, \infty)$, $m(\tilde{V}, \varepsilon^2) \to \tilde{V} + 1$ as $\varepsilon^2 \to \tilde{V}/\pi^2$.

- For each fixed $\tilde{V} \in (0, 1)$, $W(x; \tilde{V}, \varepsilon^2) \to \tilde{V} + 1$ as $\varepsilon^2 \to 0$ in $(0, 1]$.
- For each fixed $\tilde{V} \in (1, \infty)$, $W(x; \tilde{V}, \varepsilon^2) \to 0$ as $\varepsilon^2 \to 0$ in $[0, 1)$.

- For each fixed $\tilde{V} \in (0, 1)$, $m(\tilde{V}, \varepsilon^2) \to 2\tilde{V} + 1$ as $\varepsilon^2 \to 0$.
- For each fixed $\tilde{V} \in (1, \infty)$, $m(\tilde{V}, \varepsilon^2) \to \tilde{V}$ as $\varepsilon^2 \to 0$.

- For $m \in (0, 1]$, bifurcation diagrams are the empty set.
- For $m \in (1, \infty)$, bifurcation diagram given by (2.21) are graphs with $\tilde{V}$ axis (smooth single-valued function in $\tilde{V}$) except $m = 2$ with $\tilde{V} = 1$.

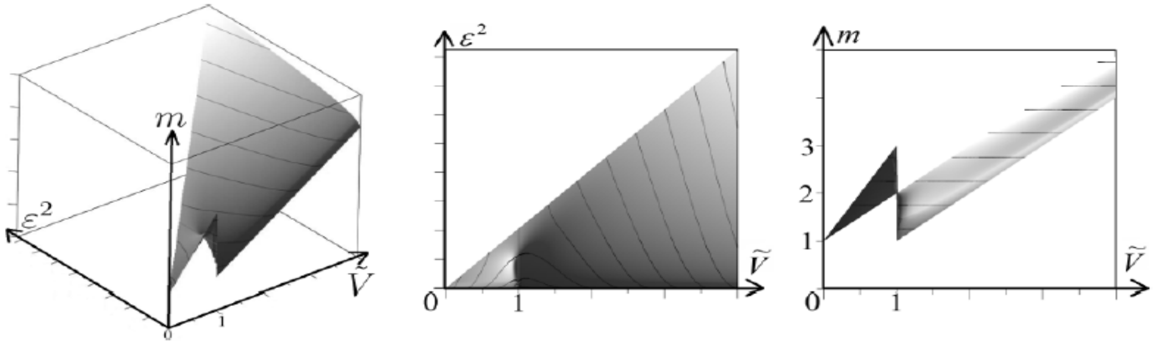


FIGURE 3. Global bifurcation sheet

The Figure 4 show the bifurcation diagrams for various m with the profiles of solutions of (SLP).

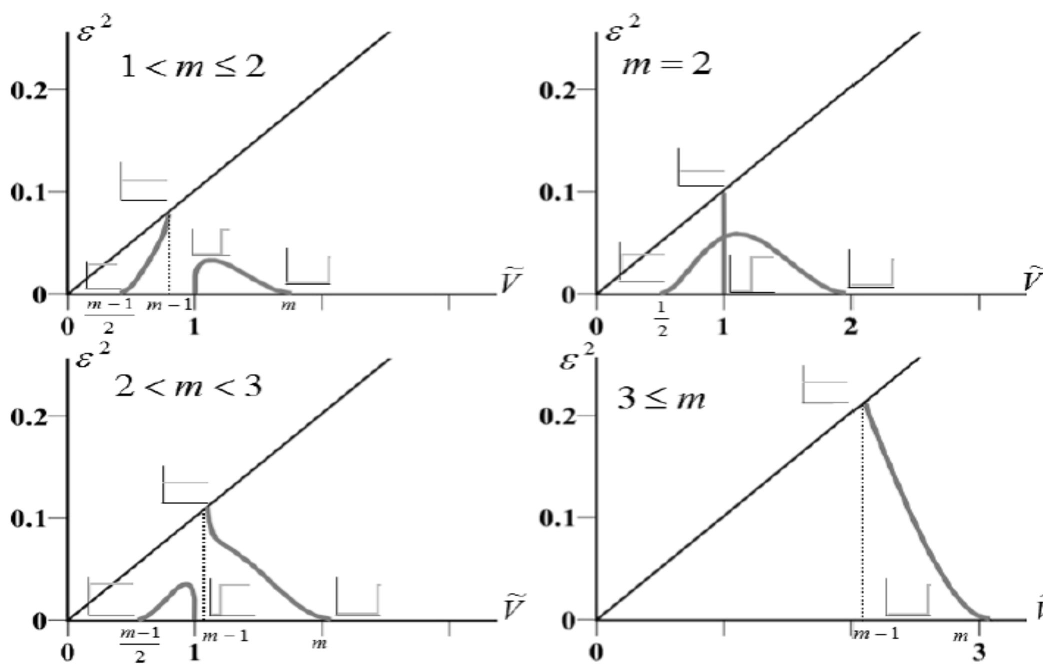


FIGURE 4. Bifurcation diagrams for various m

3. **Proof of Theorems 2.1 and 2.2.** We prepare several propositions to prove Theorem 2.1 and Theorem 2.2.

Proposition 3.1. *Let $W(x)$ be a solution of $(AP;\tilde{V})$, and*

$$u(x) := \frac{\sqrt{3}}{\sqrt{\lambda^2 - \lambda + 1}} \left(\lambda\, W(x) - \left(\frac{1}{3} + \frac{\lambda}{3} \right) \right), \tag{3.1}$$

where

$$\lambda := \frac{1}{\tilde{V} + 1}. \tag{3.2}$$

Then $u(x)$ satisfies

$$\begin{cases} \left(\dfrac{\sqrt{3}\varepsilon}{\sqrt{\tilde{V}^2 + \tilde{V} + 1}} \right)^2 u_{xx} - u^3 + u \\ \qquad + \dfrac{1}{3\sqrt{3}} \cdot \dfrac{(1 - \tilde{V})(2\tilde{V} + 1)(\tilde{V} + 2)}{\sqrt{\tilde{V}^2 + \tilde{V} + 1}^{\,3}} \quad \text{in } (0,1), & (3.3) \\ u_x(0) = u_x(1) = 0, & (3.4) \\ u_x(x) > 0 \quad \text{in } (0,1), & (3.5) \end{cases}$$

and

$$\int_0^1 W(x)dx = \frac{\tilde{V} + 2}{3} + \frac{1}{\sqrt{3}} \cdot \sqrt{\tilde{V}^2 + \tilde{V} + 1} \int_0^1 u(x)dx. \tag{3.6}$$

Proposition 3.1. *Let $W(x)$ be a solution of (AP;$\tilde{V}$), and*

$$u(x) := \frac{\sqrt{3}}{\sqrt{\lambda^2 - \lambda + 1}} \left(\lambda\, W(x) - \left(\frac{1}{3} + \frac{\lambda}{3} \right) \right), \tag{3.1}$$

where

$$\lambda := \frac{1}{\tilde{V} + 1}. \tag{3.2}$$

Then $u(x)$ satisfies

$$\begin{cases} \left(\dfrac{\sqrt{3}\varepsilon}{\sqrt{\tilde{V}^2 + \tilde{V} + 1}} \right)^2 u_{xx} - u^3 + u \\ \qquad + \dfrac{1}{3\sqrt{3}} \cdot \dfrac{(1 - \tilde{V})(2\tilde{V} + 1)(\tilde{V} + 2)}{\sqrt{\tilde{V}^2 + \tilde{V} + 1}^{\,3}} \quad \text{in } (0,1), & (3.3) \\ u_x(0) = u_x(1) = 0, & (3.4) \\ u_x(x) > 0 \quad \text{in } (0,1), & (3.5) \end{cases}$$

Proposition 3.2. *Let $\tilde{V} > 0$. There exists a solution $W(x)$ of $(AP;\tilde{V})$, if and only if (E) has a solution (h, s). For the solution (h, s) of (E), $(AP;\tilde{V})$ has a solution in the form (2.7) with (2.8) and (2.9).*

Proposition 3.3. *Let $\tilde{V} > 0$. There exists a solution $(h, s) = (h(\tilde{V}, \varepsilon^2), s(\tilde{V}, \varepsilon^2))$ of (E), if and only if $(\tilde{V}, \varepsilon^2) \in \mathcal{G}$, where $\mathcal{G}$ is defined by (2.4). Moreover, the solution is unique.*

Proposition 3.4. *Let $\tilde{V} > 0$, $\varepsilon > 0$, $(h, s) = (h(\tilde{V}, \varepsilon^2), s(\tilde{V}, \varepsilon^2))$ be the unique solution of (E), $W(x; \tilde{V}, \varepsilon^2)$ be the unique solution of (E) in the form (2.7) with (2.8) and (2.9), and $u(x)$ be defined by (3.1) and (3.2) with $W(x) = W(x; \tilde{V}, \varepsilon^2)$. Then*

$$\int_0^1 u(x)dx = \mathcal{M}(h(\tilde{V}, \varepsilon^2), s(\tilde{V}, \varepsilon^2)) \tag{3.7}$$

where $\mathcal{M}(h, s)$ is defined by (2.19).

Proof of Theorem 2.1. We see from Proposition 3.2 and Proposition 3.3 that conclusions hold except (2.6).

We see that

$$\tilde{V} + 1 - \tilde{V} \cdot W\left(1 - x; \frac{1}{\tilde{V}}, \frac{\varepsilon^2}{\tilde{V}^2}\right).$$

is a solution of (AP; $\tilde{V}$). Thus, we obtain (2.6) by the uniqueness of solutions of $(AP; \tilde{V})$.

Proof of Theorem 2.2. We obtain conclusions by (2.16), Proposition 3.1, and Proposition 3.4.

Proof of Proposition 3.1. Let us put

$$U(x) := \frac{W(x)}{\tilde{V}+1}.$$

We get

$$\begin{cases} (\varepsilon\lambda)^2 U_{xx} + U(1-U)(U-\lambda) = 0 \quad \text{in } (0,1), \\ U_x(0) = U_x(1) = 0, \\ U_x(x) > 0 \quad \text{in } (0,1), \end{cases}$$

and

$$\int_0^1 W(x)dx = \frac{1}{\lambda}\int_0^1 U(x)dx,$$

where $\lambda = 1/(\tilde{V}+1)$.

We further introduce $u(x)$ by

$$u(x) := \frac{(U(x) - (\frac{1}{3} + \frac{\lambda}{3}))}{c}, \quad c := \frac{\sqrt{\lambda^2 - \lambda + 1}}{\sqrt{3}}.$$

We have

$$U(x) = cu(x) + \frac{1}{3} + \frac{\lambda}{3},$$

and obtain

$$\begin{cases} \left(\frac{\lambda\varepsilon}{c}\right)^2 u_{xx} - u^3 + u + \frac{1}{3\sqrt{3}} \frac{(\lambda-2)(2\lambda-1)(\lambda+1)}{(\lambda^2-\lambda+1)^{3/2}} = 0 \quad \text{in } (0,1), \\ u_x(0) = u_x(1) = 0, \\ u_x(x) > 0 \quad \text{in } (0,1), \end{cases}$$

and

$$\int_0^1 W(x)dx = \frac{1}{\lambda}\left(c\int_0^1 udx + \frac{1+\lambda}{3}\right).$$

Hence, we get

$$\begin{cases} \left(\dfrac{\sqrt{3}\lambda\varepsilon}{\sqrt{\lambda^2-\lambda+1}}\right)^2 u_{xx} - u^3 + u \\ \qquad + \dfrac{1}{3\sqrt{3}} \dfrac{(\lambda-2)(2\lambda-1)(\lambda+1)}{(\lambda^2-\lambda+1)^{3/2}} = 0 \quad \text{in } (0,1), \\ u_x(0) = u_x(1) = 0, \\ u_x(x) > 0 \quad \text{in } (0,1), \end{cases}$$

and

$$\int_0^1 W(x)dx = \frac{1}{\lambda}\left(\frac{\sqrt{\lambda^2-\lambda+1}}{\sqrt{3}}\int_0^1 u(x)dx + \frac{1+\lambda}{3}\right).$$

Therefore, we obtain

$$\begin{cases} \left(\dfrac{\sqrt{3}\varepsilon}{\sqrt{\tilde{V}^2+\tilde{V}+1}}\right)^2 u_{xx} - u^3 + u - \dfrac{1}{3\sqrt{3}} \cdot \dfrac{(1-\tilde{V})(2\tilde{V}+1)(\tilde{V}+2)}{\sqrt{\tilde{V}^2+\tilde{V}+1}^3} \quad \text{in } (0,1), \\ u_x(0) = u_x(1) = 0, \\ u_x(x) > 0 \quad \text{in } (0,1), \end{cases}$$

and

$$\int_0^1 W(x)dx = \frac{\tilde{V}+2}{3} + \frac{1}{3}\cdot\sqrt{\tilde{V}^2+\tilde{V}+1}\int_0^1 u(x)dx.$$

5. **Proof of Proposition 3.2 and 3.4.** In this section we give a proof of Proposition 3.2 and 3.4. We employ representation formulas obtained in Proposition 1.1 and its proof in Kosugi-Morita-Yotsutani[3].

Lemma 5.1. *Let $E > 0$ and A be constants. Then all the solution of*

$$\begin{cases} E^2 u_{xx} - u^3 + u - A = 0 \quad \text{in } (0,1), \\ u_x(0) = u_x(1) = 0, \\ u_x(x) > 0 \quad \text{in } (0,1). \end{cases}$$

are represented by two parameters (h, s) with $0 < h < 1$ and $0 < s < 1$ as follows.

$$u(x; h, s) = \frac{\beta \cdot (1 - hs)\mathrm{sn}^2(K(\sqrt{h})x, \sqrt{h}) + \alpha \cdot \mathrm{cn}^2(K(\sqrt{h})x, \sqrt{h})}{(1 - hs)\mathrm{sn}^2(K(\sqrt{h})x, \sqrt{h}) + \mathrm{cn}^2(K(\sqrt{h})x, \sqrt{h})}, \tag{5.1}$$

$$\alpha := \alpha(h, s), \quad \beta := \beta(h, s), \tag{5.2}$$

where $\alpha(h, s)$ and $\beta(h, s)$ are defined by (2.8) and (2.9), and (h, s) is a solution of the following system of transcendental equations

$$(\mathrm{E}) \begin{cases} \mathcal{E}(h, s) = E & (5.3) \\ \mathcal{A}(h, s) = A & (5.4) \\ 0 < h < 1, \quad 0 < s < 1, & (5.5) \end{cases}$$

where $\mathcal{E}(h, s)$ and $\mathcal{A}(h, s)$ are defined by (2.13) and (2.14) respectively.

Moreover,

$$\int_0^1 u(x)dx = \mathcal{M}(h, s),$$

where $\mathcal{M}(h, s)$ is defined (2.19).

Proof of Proposition 3.2. We obtain conclusions by Proposition3.1 and Lemma5.1.

Proof of Proposition 3.4. We obtain conclusions by Proposition3.2 and Lemma5.1.

6. **Proof of Proposition 3.3.** In this section we give a proof of Proposition 3.3.

First we note that

$$3h^2s^4-4(h^2+h)s^3+(4h^2+2h+4)s^2-4(h+1)s+3$$
$$= s^2(3\,s^2-4\,s+4)h^2-2\,s(2\,s^2-s+2)h+4\,s^2-4\,s+3>0$$

and $\mathcal{A}(h,s)$ and $\mathcal{E}(h,s)$ are well-defined in $(h,s)\in(0,1)\times(0,1)$, since

$$s^2\left(2\,s^2-\;s+2\right)^2-\;s^2\left(3\,s^4-4\,s^3+4\,s^2\right)\left(4\,s^2-4\,s+3\right)$$
$$=-8\,s^2\left(s^2-s+1\right)(s-1)^2<0.$$

there. We prepare several lemmas.

We see from Lemma 3.2 and the proof of Lemma 3.4 of [3] that the following lemma holds.

We show the graph of $\mathcal{A}(h,s)$ and $\mathcal{E}(h,s)$ in Figures 1 and 2.

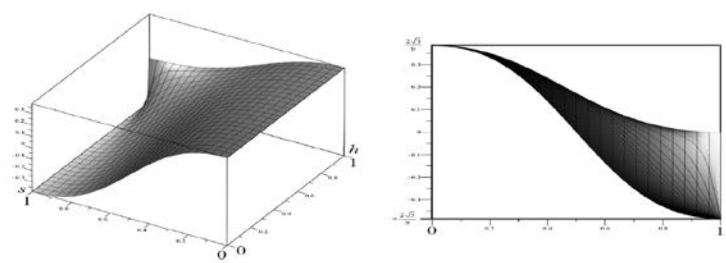


FIGURE 1. Graph of $\mathcal{A}(h,s)$

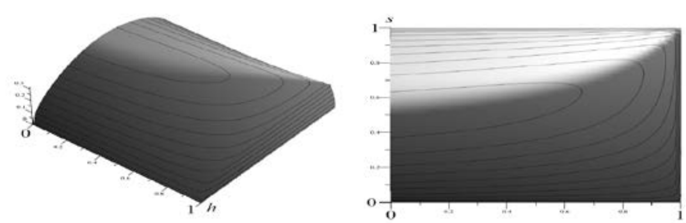


FIGURE 2. Graph of $\mathcal{E}(h,s)$

Lemma 6.1. *Let $\mathcal{E}(h,s)$ be defined by (2.13). The derivative of $\mathcal{E}(h,s)$ with respect to s satisfies*

$$\frac{\partial}{\partial s}\mathcal{E}(h,s)\begin{cases} >0, & s\in(0,\sigma(h)), \quad h\in[0,1),\\ =0, & s=\sigma(h), \quad h\in[0,1),\\ <0, & s\in(\sigma(h),1), \quad h\in[0,1),\end{cases} \tag{6.1}$$

and

$$\mathcal{E}(h,s)<\mathcal{E}(h,\sigma(h)) \quad \text{for all } (h,s)\in[0,1)\times[0,1]\setminus\{(h,s): s=\sigma(h)\}, \tag{6.2}$$

where $\sigma(h):=1/(1+\sqrt{1-h})$.

Moreover,

$$\mathcal{E}(h,\sigma(h))=\frac{1}{\sqrt{2(2-h)}\,K(\sqrt{h})}, \tag{6.3}$$

$$\frac{d}{dh}\mathcal{E}(h,\sigma(h))<0 \quad \text{for } h\in[0,1), \tag{6.4}$$

and

$$\mathcal{E}(0,\sigma(0))=\frac{1}{\pi}, \quad \mathcal{E}(h,\sigma(h))\to 0 \text{ as } h\to 1. \tag{6.5}$$

In addition,

$$\mathcal{E}(0,s)=\frac{2\sqrt{2\,s\,(1-s)}}{\pi\sqrt{4\,s^2-4\,s+3}}. \tag{6.6}$$

Lemma 6.2. *Let*

$$r(v) := \frac{\sqrt{3}}{9} \cdot \frac{(1-v)(2v+1)(v+2)}{\sqrt{v^2+v+1}^3}. \tag{6.7}$$

Then, $r(v)$ is monotone decreasing in $(0, \infty)$ and

$$r(0) = \frac{2\sqrt{3}}{9}, \quad r(v) \to -\frac{2\sqrt{3}}{9} \text{ as } v \to \infty. \tag{6.8}$$

Lemma 6.3. *Let $\mathcal{A}(h, s)$ be defined by (2.14). Then*

$$\mathcal{A}(h, 0) = \frac{2\sqrt{3}}{9}, \quad \mathcal{A}(h, 1) = -\frac{2\sqrt{3}}{9} \quad \text{for all } h \in [0, 1), \tag{6.9}$$

$$\mathcal{A}_s(h, s) < 0 \quad \text{for all } (h, s) \in (0, 1) \times (0, 1). \tag{6.10}$$

Lemma 6.4. *Let $\tilde{V} > 0$ be fixed. There exists a unique curve*

$$s(h;\tilde{V}) \in C^\infty[0,1) \tag{6.15}$$

such that

$$\mathcal{A}(h, s(h;\tilde{V})) = \frac{1}{3\sqrt{3}} \cdot \frac{(1-\tilde{V})(2\tilde{V}+1)(\tilde{V}+2)}{\sqrt{\tilde{V}^2+\tilde{V}+1}^3}, \quad 0 < s(h;\tilde{V}) < 1. \tag{6.16}$$

Moreover

$$s(0;\tilde{V}) = \frac{1}{2} - \frac{1-\tilde{V}}{\sqrt{2}\sqrt{(\tilde{V}+2)(2\tilde{V}+1)}}, \tag{6.17}$$

$$\mathcal{E}(0, s(0;\tilde{V})) = \frac{\sqrt{3}\sqrt{\tilde{V}}}{\pi\sqrt{\tilde{V}^2+\tilde{V}+1}}, \tag{6.18}$$

and

$$\mathcal{E}(h, s(h;\tilde{V})) \to 0 \text{ as } h \to 1. \tag{6.19}$$

Lemma 6.5. *Let $\mathcal{E}(h,s)$ be defined by (2.13), and $s(h;\tilde{V})$ defined in Lemma 6.4, then for each fixed $\tilde{V} > 0$*

$$\frac{d\mathcal{E}(h, s(h;\tilde{V}))}{dh} < 0 \quad \text{in}(0,1). \tag{6.20}$$

Proof of Proposition 3.3 First, we note that

$$0 < \frac{\sqrt{3}\varepsilon}{\sqrt{\tilde{V}^2 + \tilde{V} + 1}} < \mathcal{E}(0, s(0; \tilde{V}))$$

is equivalent to

$$0 < \frac{\sqrt{3}\varepsilon}{\sqrt{\tilde{V}^2 + \tilde{V} + 1}} < \frac{\sqrt{3}\sqrt{\tilde{V}}}{\pi\sqrt{\tilde{V}^2 + \tilde{V} + 1}},$$

that is,

$$0 < \varepsilon < \frac{\sqrt{\tilde{V}}}{\pi}.$$

Thus, we complete the proof by Lemmas 6.4 and 6.5.

We show the graph of $\mathcal{A}(h, s)$ and $\mathcal{E}(h, s)$ in Figures 1 and 2.

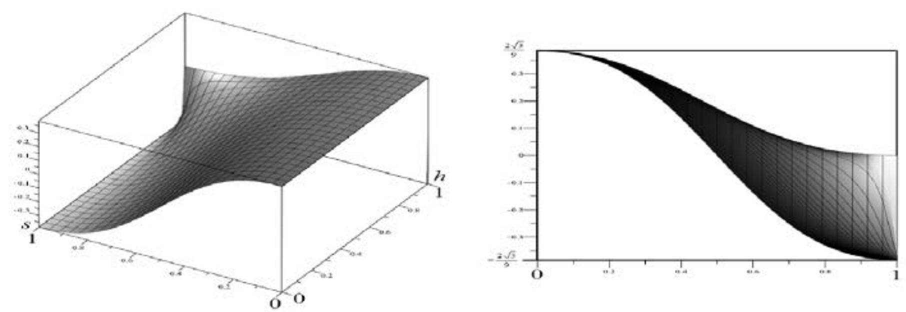


FIGURE 1. Graph of $\mathcal{A}(h, s)$

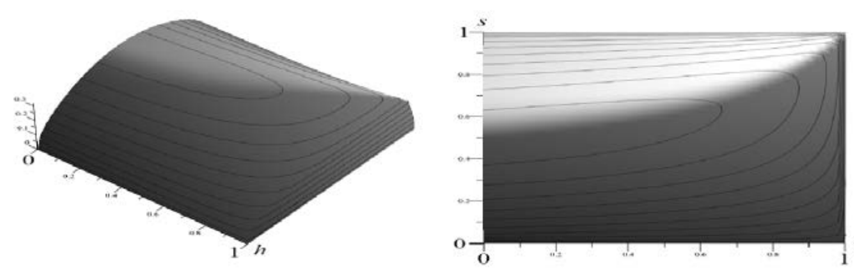


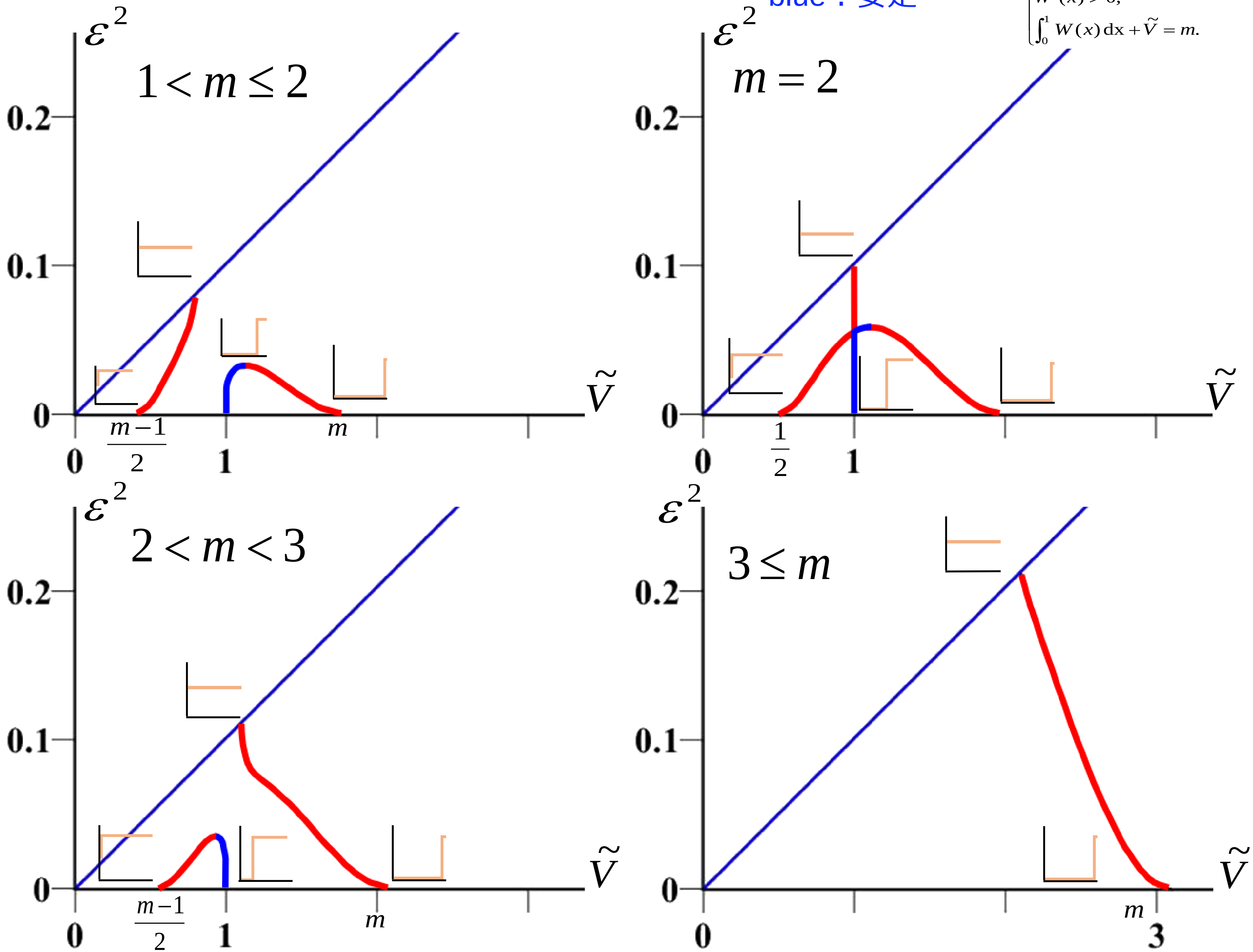
FIGURE 2. Graph of $\mathcal{E}(h, s)$

(SLP)の定常解の安定性

数値計算

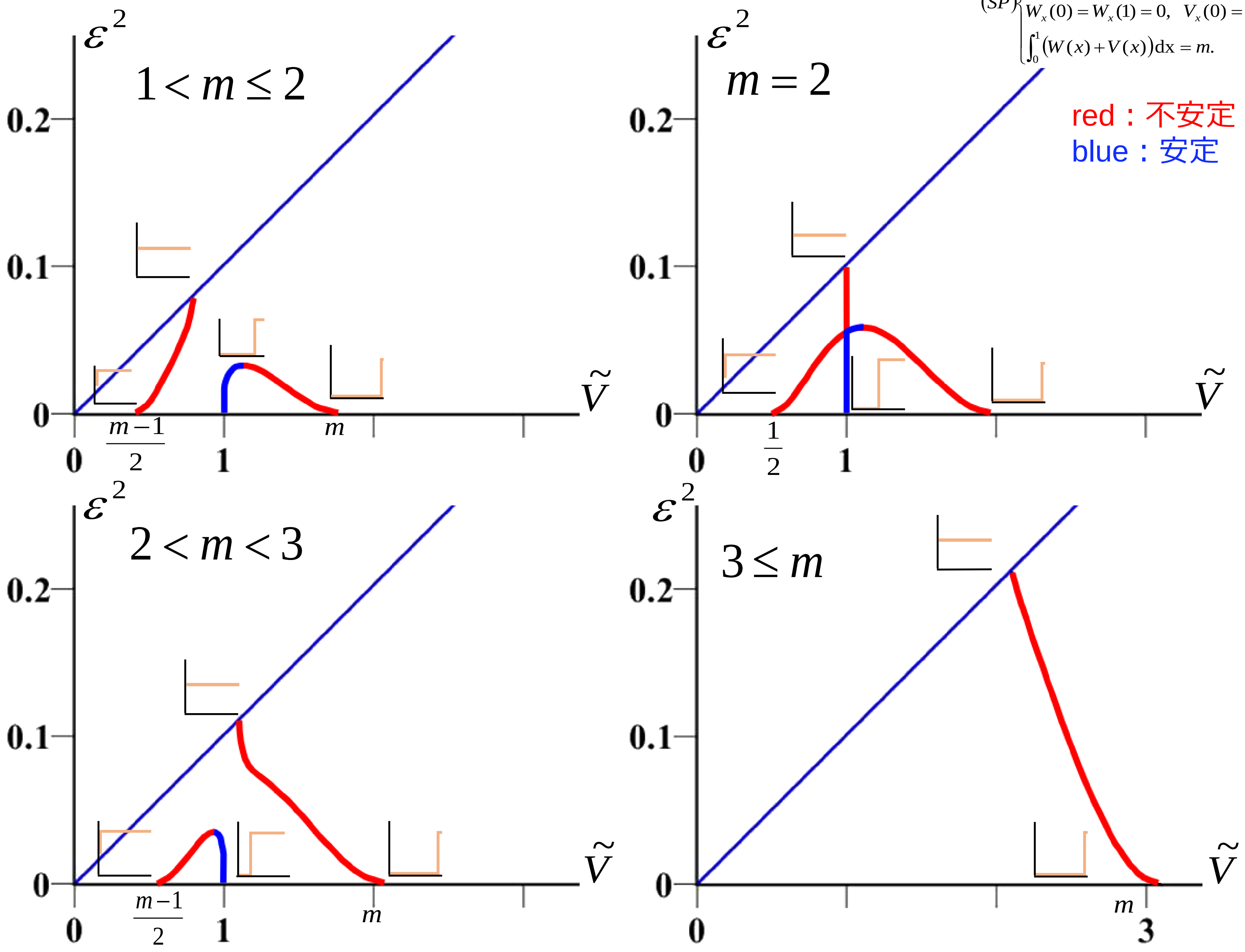
red：不安定
blue：安定

$$
(SLP)\begin{cases}
\varepsilon^2 W_{xx} + W(W-1)(\tilde{V}+1-W) = 0, \\
W_x(0) = W_x(1) = 0, \\
W'(x) > 0, \\
\int_0^1 W(x)\,\mathrm{d}x + \tilde{V} = m.
\end{cases}
$$



(SP)の定常解の安定性 $D=10000$ (SLP)と同様

$$(SP)\begin{cases}\varepsilon^2 W_{xx} + W(W-1)(V+1-W)=0,\\ DV_{xx} - W(W-1)(V+1-W)=0,\\ W_x(0)=W_x(1)=0,\quad V_x(0)=V_x(1)=0,\\ \int_0^1 (W(x)+V(x))\mathrm{d}x = m.\end{cases}$$



Idea of proof of Theorem C

$$\frac{\partial m(\tilde{V},\varepsilon^2)}{\partial \varepsilon} < 0 \textit{ for } \quad 0 < \tilde{V} < 1, \ 0 < \varepsilon^2 < \frac{\tilde{V}}{\pi^2}.$$

$$m(\tilde{V},\ \varepsilon^2) := \int_0^1 W(x;\tilde{V},\varepsilon^2)\,dx + \tilde{V}$$

Idea of proof of Theorem C

$$\frac{\partial m(\tilde{V},\varepsilon^2)}{\partial \varepsilon} < 0 \text{ for } 0 < \tilde{V} < 1,\ 0 < \varepsilon^2 < \frac{\tilde{V}}{\pi^2}.$$

$m(\tilde{V},\varepsilon^2)$ *is represented by*

$$m(\tilde{V},\ \varepsilon^2) := \int_0^1 W(x;\tilde{V},\varepsilon^2)\,dx + \tilde{V}$$

$$m(\tilde{V},\varepsilon^2) := \frac{4\tilde{V}+2}{3} + \frac{1}{\sqrt{3}} \cdot \sqrt{\tilde{V}^2 + \tilde{V} + 1} \cdot M(h,s),$$

$$M(h,\ s) := \frac{-(hs^2 - 2(1+h)s + 3) + 4(1-s)(1-sh)\Pi(-sh,\sqrt{h})/K(\sqrt{h})}{\sqrt{3h^2s^4 - 4(h^2+h)s^3 + (4h^2+2h+4)s^2 - 4(h+1)s + 3}},$$

Idea of proof of Theorem C

$$\frac{\partial m(\tilde{V},\varepsilon^2)}{\partial \varepsilon} < 0$$ *for* $0<\tilde{V}<1,\ 0<\varepsilon^2<\frac{\tilde{V}}{\pi^2}.$

$m(\tilde{V},\varepsilon^2)$ *is represented by*

$$m(\tilde{V},\ \varepsilon^2) := \int_0^1 W(x;\tilde{V},\varepsilon^2)\,dx + \tilde{V}$$

$$m(\tilde{V},\varepsilon^2) := \frac{4\tilde{V}+2}{3} + \frac{1}{\sqrt{3}}\cdot\sqrt{\tilde{V}^2+\tilde{V}+1}\cdot M(h,s),$$

$$M(h,\ s) := \frac{-(hs^2-2(1+h)s+3)+4(1-s)(1-sh)\Pi(-sh,\sqrt{h})\big/K(\sqrt{h})}{\sqrt{3h^2s^4-4(h^2+h)s^3+(4h^2+2h+4)s^2-4(h+1)s+3}},$$

where $(h,\ s)=(h(\tilde{V},\ \varepsilon^2),\ s(\tilde{V},\ \varepsilon^2))$ *is the unique solution of the following system of transcendental equations*

$$\begin{cases} E(h,s) := \dfrac{\sqrt{2s(1-s)(1-sh)}\big/K(\sqrt{h})}{\sqrt{3h^2s^4-4(h^2+h)s^3+(4h^2+2h+4)s^2-4(h+1)s+3}} = \sqrt{3}\cdot\dfrac{\varepsilon}{\sqrt{\tilde{V}^2+\tilde{V}+1}}, \\[2ex] A(h,s) := \dfrac{2(hs^2-sh+1)(hs^2-2s+1)(1-hs^2)}{\sqrt{3h^2s^4-4(h^2+h)s^3+(4h^2+2h+4)s^2-4(h+1)s+3}^{\,3}} = \dfrac{1}{3\sqrt{3}}\cdot\dfrac{(1-\tilde{V})(2\tilde{V}+1)(\tilde{V}+2)}{\sqrt{\tilde{V}^2+\tilde{V}+1}^{\,3}}, \\[2ex] 0<h<1,\quad 0<s<\dfrac{1}{1+\sqrt{1-h}}. \end{cases}$$

Idea of proof of Theorem C

$$\frac{\partial m(\tilde V,\varepsilon^2)}{\partial \varepsilon}<0 \text{ for } 0<\tilde V<1,\ 0<\varepsilon^2<\frac{\tilde V}{\pi^2}.$$

$$m(\tilde V,\ \varepsilon^2):=\int_0^1 W(x;\tilde V,\varepsilon^2)\,dx+\tilde V$$

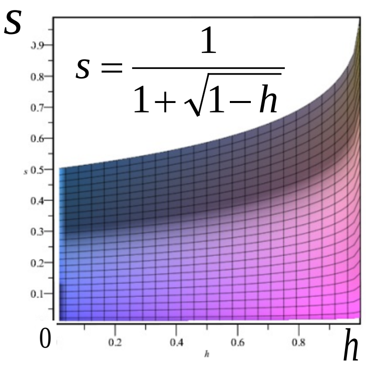
$m(\tilde V,\varepsilon^2)$ is represented by

$$m(\tilde V,\varepsilon^2):=\frac{4\tilde V+2}{3}+\frac{1}{\sqrt{3}}\cdot\sqrt{\tilde V^2+\tilde V+1}\cdot M(h,s),$$

$$M(h,\ s):=\frac{-(hs^2-2(1+h)s+3)+4(1-s)(1-sh)\Pi(-sh,\sqrt{h})/K(\sqrt{h})}{\sqrt{3h^2s^4-4(h^2+h)s^3+(4h^2+2h+4)s^2-4(h+1)s+3}},$$

where $(h,\ s)=(h(\tilde V,\ \varepsilon^2),\ s(\tilde V,\ \varepsilon^2))$ is the unique solution of the following system of transcendental equations

$$\begin{cases} E(h,s):=\dfrac{\sqrt{2s(1-s)(1-sh)}/K(\sqrt{h})}{\sqrt{3h^2s^4-4(h^2+h)s^3+(4h^2+2h+4)s^2-4(h+1)s+3}}=\sqrt{3}\cdot\dfrac{\varepsilon}{\sqrt{\tilde V^2+\tilde V+1}}, \\ A(h,s):=\dfrac{2(hs^2-sh+1)(hs^2-2s+1)(1-hs^2)}{\sqrt{3h^2s^4-4(h^2+h)s^3+(4h^2+2h+4)s^2-4(h+1)s+3}^{\,3}}=\dfrac{1}{3\sqrt{3}}\cdot\dfrac{(1-\tilde V)(2\tilde V+1)(\tilde V+2)}{\sqrt{\tilde V^2+\tilde V+1}^{\,3}}, \\ 0<h<1,\quad 0<s<\dfrac{1}{1+\sqrt{1-h}}. \end{cases}$$



Idea of proof of Theorem C

$\dfrac{\partial m(\tilde{V},\varepsilon^2)}{\partial \varepsilon} < 0$ *for* $0 < \tilde{V} < 1,\ 0 < \varepsilon^2 < \dfrac{\tilde{V}}{\pi^2}.$

$$m(\tilde{V},\ \varepsilon^2) := \int_0^1 W(x;\tilde{V},\varepsilon^2)\,dx + \tilde{V}$$

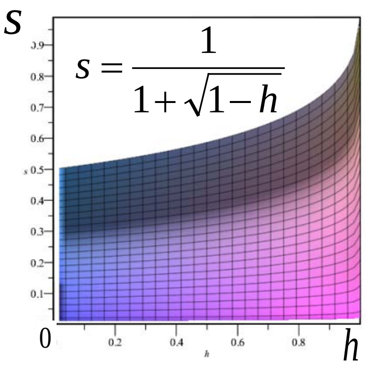
$m(\tilde{V},\varepsilon^2)$ *is represented by*

$$m(\tilde{V},\varepsilon^2) := \frac{4\tilde{V}+2}{3} + \frac{1}{\sqrt{3}}\cdot\sqrt{\tilde{V}^2+\tilde{V}+1}\cdot M(h,s),$$

$$M(h,\ s) := \frac{-(hs^2-2(1+h)s+3)+4(1-s)(1-sh)\Pi(-sh,\sqrt{h})\big/K(\sqrt{h})}{\sqrt{3h^2s^4-4(h^2+h)s^3+(4h^2+2h+4)s^2-4(h+1)s+3}},$$

where $(h,\ s) = (h(\tilde{V},\ \varepsilon^2),\ s(\tilde{V},\ \varepsilon^2))$ *is the unique solution of the following system of transcendental equations*

$$\begin{cases} E(h,s) := \dfrac{\sqrt{2s(1-s)(1-sh)}\big/K(\sqrt{h})}{\sqrt{3h^2s^4-4(h^2+h)s^3+(4h^2+2h+4)s^2-4(h+1)s+3}} = \sqrt{3}\cdot\dfrac{\varepsilon}{\sqrt{\tilde{V}^2+\tilde{V}+1}}, \\[2ex] A(h,s) := \dfrac{2(hs^2-sh+1)(hs^2-2s+1)(1-hs^2)}{\sqrt{3h^2s^4-4(h^2+h)s^3+(4h^2+2h+4)s^2-4(h+1)s+3}^{\,3}} = \dfrac{1}{3\sqrt{3}}\cdot\dfrac{(1-\tilde{V})(2\tilde{V}+1)(\tilde{V}+2)}{\sqrt{\tilde{V}^2+\tilde{V}+1}^{\,3}}, \\[2ex] 0 < h < 1,\quad 0 < s < \dfrac{1}{1+\sqrt{1-h}}. \end{cases}$$



Here, $K(\cdot)$ *is the complete elliptic integral of the first kind,*

$\Pi(\cdot,\cdot)$ *is the complete elliptic integral of the third kind.*

Idea of proof of Theorem C

$$\frac{\partial m(\tilde{V},\varepsilon^2)}{\partial \varepsilon} < 0 \text{ for } 0 < \tilde{V} < 1,\ 0 < \varepsilon^2 < \frac{\tilde{V}}{\pi^2}.$$

$$m(\tilde{V},\ \varepsilon^2) := \int_0^1 W(x;\tilde{V},\varepsilon^2)\,dx + \tilde{V}$$

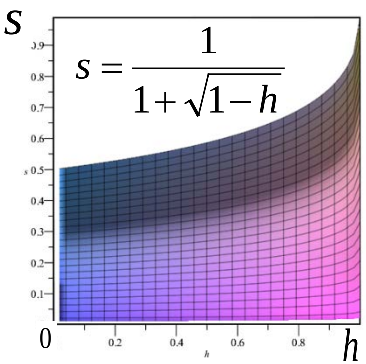
$m(\tilde{V},\varepsilon^2)$ *is represented by*

$$m(\tilde{V},\varepsilon^2) := \frac{4\tilde{V}+2}{3} + \frac{1}{\sqrt{3}} \cdot \sqrt{\tilde{V}^2+\tilde{V}+1} \cdot M(h,s),$$

$$M(h,\ s) := \frac{-(hs^2 - 2(1+h)s + 3) + 4(1-s)(1-sh)\Pi(-sh,\sqrt{h})/K(\sqrt{h})}{\sqrt{3h^2s^4 - 4(h^2+h)s^3 + (4h^2+2h+4)s^2 - 4(h+1)s + 3}},$$

where $(h,\ s) = (h(\tilde{V},\ \varepsilon^2),\ s(\tilde{V},\ \varepsilon^2))$ *is the unique solution of the following system of transcendental equations*

$$\begin{cases} E(h,s) := \dfrac{\sqrt{2s(1-s)(1-sh)}/K(\sqrt{h})}{\sqrt{3h^2s^4 - 4(h^2+h)s^3 + (4h^2+2h+4)s^2 - 4(h+1)s + 3}} = \sqrt{3} \cdot \dfrac{\varepsilon}{\sqrt{\tilde{V}^2+\tilde{V}+1}}, \\[2ex] A(h,s) := \dfrac{2(hs^2 - sh + 1)(hs^2 - 2s + 1)(1 - hs^2)}{\sqrt{3h^2s^4 - 4(h^2+h)s^3 + (4h^2+2h+4)s^2 - 4(h+1)s + 3}^{\,3}} = \dfrac{1}{3\sqrt{3}} \cdot \dfrac{(1-\tilde{V})(2\tilde{V}+1)(\tilde{V}+2)}{\sqrt{\tilde{V}^2+\tilde{V}+1}^{\,3}}, \\[2ex] 0 < h < 1,\quad 0 < s < \dfrac{1}{1+\sqrt{1-h}}. \end{cases}$$

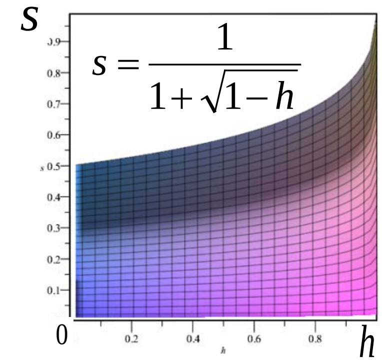


Here, $K(\cdot)$ *is the complete elliptic integral of the first kind,*

$\Pi(\cdot,\cdot)$ *is the complete elliptic integral of the third kind.*

$$K(k) := \int_0^{\frac{\pi}{2}} \frac{d\varphi}{\sqrt{1-k^2\sin^2\varphi}}, \qquad \Pi(\mu,k) := \int_0^{\frac{\pi}{2}} \frac{d\varphi}{(1+\mu\sin^2\varphi)\sqrt{1-k^2\sin^2\varphi}}.$$

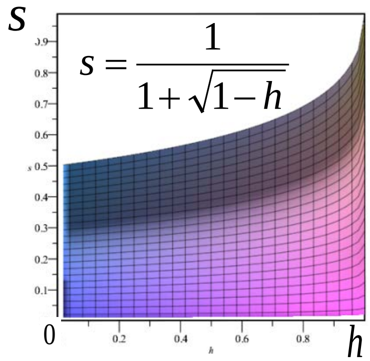
$$\begin{cases} E(h,s) := \dfrac{\sqrt{2s(1-s)(1-sh)}\Big/ K(\sqrt{h})}{\sqrt{3h^2s^4-4(h^2+h)s^3+(4h^2+2h+4)s^2-4(h+1)s+3}} = \sqrt{3}\cdot\dfrac{\varepsilon}{\sqrt{\tilde{V}^2+\tilde{V}+1}}, \\ A(h,s) := \dfrac{2(hs^2-sh+1)(hs^2-2s+1)(1-hs^2)}{\sqrt{3h^2s^4-4(h^2+h)s^3+(4h^2+2h+4)s^2-4(h+1)s+3}^3} = \dfrac{1}{3\sqrt{3}}\cdot\dfrac{(1-\tilde{V})(2\tilde{V}+1)(\tilde{V}+2)}{\sqrt{\tilde{V}^2+\tilde{V}+1}^3}. \end{cases}$$



$$\left(0 < h < 1,\ 0 < s < \frac{1}{1+\sqrt{1-h}}\right)$$

$$
\begin{cases}
E(h,s) := \dfrac{\sqrt{2s(1-s)(1-sh)} \big/ K(\sqrt{h})}{\sqrt{3h^2s^4 - 4(h^2+h)s^3 + (4h^2+2h+4)s^2 - 4(h+1)s+3}} = \sqrt{3} \cdot \dfrac{\varepsilon}{\sqrt{\tilde{V}^2 + \tilde{V} + 1}}, \\[2ex]
A(h,s) := \dfrac{2(hs^2 - sh + 1)(hs^2 - 2s + 1)(1 - hs^2)}{\sqrt{3h^2s^4 - 4(h^2+h)s^3 + (4h^2+2h+4)s^2 - 4(h+1)s+3}^{\,3}} = \dfrac{1}{3\sqrt{3}} \cdot \dfrac{(1-\tilde{V})(2\tilde{V}+1)(\tilde{V}+2)}{\sqrt{\tilde{V}^2 + \tilde{V} + 1}^{\,3}}.
\end{cases}
$$

$$
m(\tilde{V}, \varepsilon^2) := \frac{4\tilde{V} + 2}{3} + \frac{1}{\sqrt{3}} \cdot \sqrt{\tilde{V}^2 + \tilde{V} + 1} \cdot M(h,s),
$$

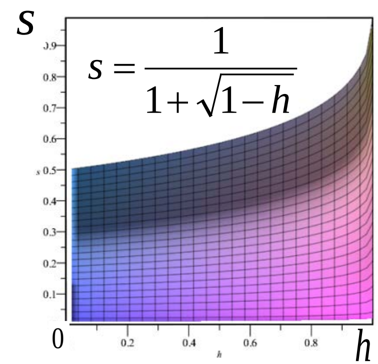


$$\left(0 < h < 1,\ 0 < s < \frac{1}{1+\sqrt{1-h}}\right)$$

$$\begin{cases} E(h,s) := \dfrac{\sqrt{2s(1-s)(1-sh)}\Big/K(\sqrt{h})}{\sqrt{3h^2s^4-4(h^2+h)s^3+(4h^2+2h+4)s^2-4(h+1)s+3}} = \sqrt{3}\cdot\dfrac{\varepsilon}{\sqrt{\tilde{V}^2+\tilde{V}+1}}, \\ A(h,s) := \dfrac{2(hs^2-sh+1)(hs^2-2s+1)(1-hs^2)}{\sqrt{3h^2s^4-4(h^2+h)s^3+(4h^2+2h+4)s^2-4(h+1)s+3}^3} = \dfrac{1}{3\sqrt{3}}\cdot\dfrac{(1-\tilde{V})(2\tilde{V}+1)(\tilde{V}+2)}{\sqrt{\tilde{V}^2+\tilde{V}+1}^3}. \end{cases}$$

$$m(\tilde{V},\varepsilon^2) := \frac{4\tilde{V}+2}{3} + \frac{1}{\sqrt{3}}\cdot\sqrt{\tilde{V}^2+\tilde{V}+1}\cdot M(h,s),$$

$$\frac{\partial m(\tilde{V},\varepsilon^2)}{\partial\varepsilon} = \frac{\partial\left(\dfrac{4\tilde{V}+2}{3}+\sqrt{\dfrac{\tilde{V}^2+\tilde{V}+1}{3}}\cdot M\right)}{\partial\varepsilon} = \sqrt{\frac{\tilde{V}^2+\tilde{V}+1}{3}}\,\frac{\partial M}{\partial\varepsilon}$$



$$\left(0 < h < 1,\ 0 < s < \frac{1}{1+\sqrt{1-h}}\right)$$

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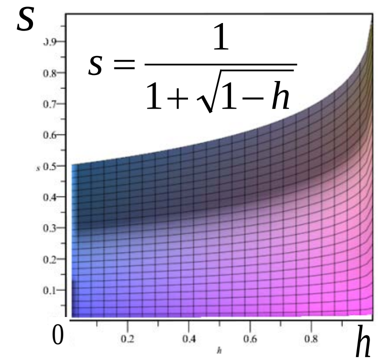
$$= \frac{\dfrac{dM(h,s(h;\tilde{V}))}{dh}}{\dfrac{dE(h,s(h;\tilde{V}))}{dh}}$$



$$\left(0<h<1,\ 0<s<\frac{1}{1+\sqrt{1-h}}\right)$$

$$\begin{cases} E(h,s) := \dfrac{\sqrt{2s(1-s)(1-sh)}\Big/K(\sqrt{h})}{\sqrt{3h^2s^4-4(h^2+h)s^3+(4h^2+2h+4)s^2-4(h+1)s+3}} = \sqrt{3}\cdot\dfrac{\varepsilon}{\sqrt{\tilde{V}^2+\tilde{V}+1}}, \\ A(h,s) := \dfrac{2(hs^2-sh+1)(hs^2-2s+1)(1-hs^2)}{\sqrt{3h^2s^4-4(h^2+h)s^3+(4h^2+2h+4)s^2-4(h+1)s+3}^3} = \dfrac{1}{3\sqrt{3}}\cdot\dfrac{(1-\tilde{V})(2\tilde{V}+1)(\tilde{V}+2)}{\sqrt{\tilde{V}^2+\tilde{V}+1}^3}. \end{cases}$$

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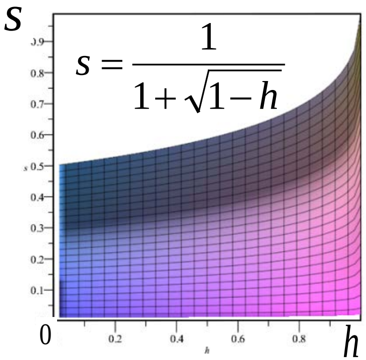
$$\frac{\partial m(\tilde{V},\varepsilon^2)}{\partial\varepsilon} = \frac{\partial\left(\dfrac{4\tilde{V}+2}{3}+\sqrt{\dfrac{\tilde{V}^2+\tilde{V}+1}{3}}\cdot M\right)}{\partial\varepsilon} = \sqrt{\frac{\tilde{V}^2+\tilde{V}+1}{3}}\,\frac{\partial M}{\partial\varepsilon}$$

$$= \frac{\dfrac{dM(h,s(h;\tilde{V}))}{dh}}{\dfrac{dE(h,s(h;\tilde{V}))}{dh}}$$

$$= \frac{M_h + M_s\cdot\dfrac{ds(h;\tilde{V})}{dh}}{E_h + E_s\cdot\dfrac{ds(h;\tilde{V})}{dh}}$$



$$\left(0<h<1,\ 0<s<\frac{1}{1+\sqrt{1-h}}\right)$$

$$\begin{cases} E(h,s) := \dfrac{\sqrt{2s(1-s)(1-sh)}\,\big/\,K(\sqrt{h})}{\sqrt{3h^2s^4-4(h^2+h)s^3+(4h^2+2h+4)s^2-4(h+1)s+3}} = \sqrt{3}\cdot\dfrac{\varepsilon}{\sqrt{\tilde{V}^2+\tilde{V}+1}}, \\ A(h,s) := \dfrac{2(hs^2-sh+1)(hs^2-2s+1)(1-hs^2)}{\sqrt{3h^2s^4-4(h^2+h)s^3+(4h^2+2h+4)s^2-4(h+1)s+3}^{\,3}} = \dfrac{1}{3\sqrt{3}}\cdot\dfrac{(1-\tilde{V})(2\tilde{V}+1)(\tilde{V}+2)}{\sqrt{\tilde{V}^2+\tilde{V}+1}^{\,3}}. \end{cases}$$

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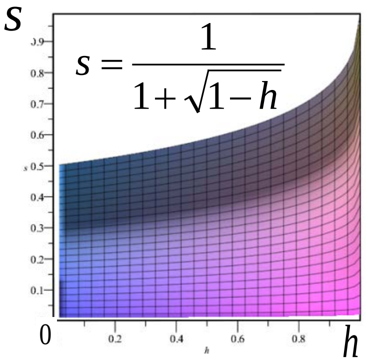


$\left(0 < h < 1,\ 0 < s < \dfrac{1}{1+\sqrt{1-h}}\right)$

$$\frac{\partial m(\tilde{V},\varepsilon^2)}{\partial\varepsilon} = \frac{\partial\left(\dfrac{4\tilde{V}+2}{3} + \sqrt{\dfrac{\tilde{V}^2+\tilde{V}+1}{3}}\cdot M\right)}{\partial\varepsilon} = \sqrt{\frac{\tilde{V}^2+\tilde{V}+1}{3}}\,\frac{\partial M}{\partial\varepsilon}$$

$$= \frac{\dfrac{dM(h,s(h;\tilde{V}))}{dh}}{\dfrac{dE(h,s(h;\tilde{V}))}{dh}}$$

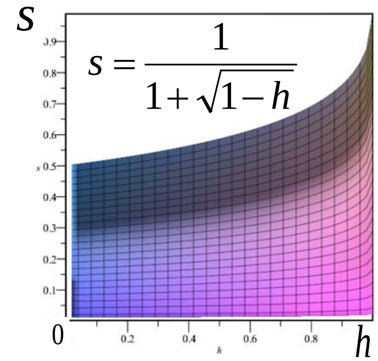
$$= \frac{M_h + M_s\cdot\dfrac{ds(h;\tilde{V})}{dh}}{E_h + E_s\cdot\dfrac{ds(h;\tilde{V})}{dh}}$$

By $\quad A(h,s) = \dfrac{1}{3\sqrt{3}}\cdot\dfrac{(1-\tilde{V})(2\tilde{V}+1)(\tilde{V}+2)}{\sqrt{\tilde{V}^2+\tilde{V}+1}^{\,3}},$

we have $A_h + A_s\cdot\dfrac{ds(h;\tilde{V})}{dh} = 0. \qquad \therefore \dfrac{ds(h;\tilde{V})}{dh} = -\dfrac{A_h}{A_s}.$

$$\begin{cases} E(h,s) := \dfrac{\sqrt{2s(1-s)(1-sh)} \big/ K(\sqrt{h})}{\sqrt{3h^2s^4 - 4(h^2+h)s^3 + (4h^2+2h+4)s^2 - 4(h+1)s + 3}} = \sqrt{3} \cdot \dfrac{\varepsilon}{\sqrt{\tilde{V}^2 + \tilde{V} + 1}}, \\ A(h,s) := \dfrac{2(hs^2 - sh + 1)(hs^2 - 2s + 1)(1 - hs^2)}{\sqrt{3h^2s^4 - 4(h^2+h)s^3 + (4h^2+2h+4)s^2 - 4(h+1)s + 3}^3} = \dfrac{1}{3\sqrt{3}} \cdot \dfrac{(1-\tilde{V})(2\tilde{V}+1)(\tilde{V}+2)}{\sqrt{\tilde{V}^2 + \tilde{V} + 1}^3}. \end{cases}$$

$$m(\tilde{V}, \varepsilon^2) := \frac{4\tilde{V}+2}{3} + \frac{1}{\sqrt{3}} \cdot \sqrt{\tilde{V}^2 + \tilde{V} + 1} \cdot M(h,s),$$



$\left(0 < h < 1,\ 0 < s < \dfrac{1}{1+\sqrt{1-h}}\right)$

$$\frac{\partial m(\tilde{V}, \varepsilon^2)}{\partial \varepsilon} = \frac{\partial\left(\dfrac{4\tilde{V}+2}{3} + \sqrt{\dfrac{\tilde{V}^2+\tilde{V}+1}{3}} \cdot M\right)}{\partial \varepsilon} = \sqrt{\frac{\tilde{V}^2+\tilde{V}+1}{3}} \frac{\partial M}{\partial \varepsilon}$$

$$= \frac{\dfrac{dM(h, s(h;\tilde{V}))}{dh}}{\dfrac{dE(h, s(h;\tilde{V}))}{dh}}$$

$$= \frac{M_h + M_s \cdot \dfrac{ds(h;\tilde{V})}{dh}}{E_h + E_s \cdot \dfrac{ds(h;\tilde{V})}{dh}}$$

$$= \frac{M_h + M_s \cdot \left(-\dfrac{A_h}{A_s}\right)}{E_h + E_s \cdot \left(-\dfrac{A_h}{A_s}\right)}$$

By $\quad A(h,s) = \dfrac{1}{3\sqrt{3}} \cdot \dfrac{(1-\tilde{V})(2\tilde{V}+1)(\tilde{V}+2)}{\sqrt{\tilde{V}^2+\tilde{V}+1}^3},$

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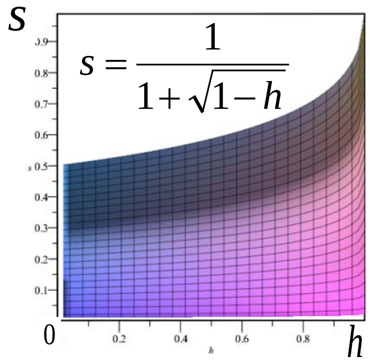

$$\left(0<h<1,\ 0<s<\frac{1}{1+\sqrt{1-h}}\right)$$

$$\frac{\partial m(\tilde{V},\varepsilon^2)}{\partial\varepsilon} = \frac{\partial\left(\dfrac{4\tilde{V}+2}{3}+\sqrt{\dfrac{\tilde{V}^2+\tilde{V}+1}{3}}\cdot M\right)}{\partial\varepsilon} = \sqrt{\frac{\tilde{V}^2+\tilde{V}+1}{3}}\,\frac{\partial M}{\partial\varepsilon}$$

$$= \frac{\dfrac{dM(h,s(h;\tilde{V}))}{dh}}{\dfrac{dE(h,s(h;\tilde{V}))}{dh}}$$

$$= \frac{M_h+M_s\cdot\dfrac{ds(h;\tilde{V})}{dh}}{E_h+E_s\cdot\dfrac{ds(h;\tilde{V})}{dh}}$$

$$= \frac{M_h+M_s\cdot\left(-\dfrac{A_h}{A_s}\right)}{E_h+E_s\cdot\left(-\dfrac{A_h}{A_s}\right)}$$

$$= \frac{M_h\cdot A_s-M_s\cdot A_h}{E_h\cdot A_s-E_s\cdot A_h}.$$

By $\quad A(h,s) = \dfrac{1}{3\sqrt{3}}\cdot\dfrac{(1-\tilde{V})(2\tilde{V}+1)(\tilde{V}+2)}{\sqrt{\tilde{V}^2+\tilde{V}+1}^{\,3}},$

we have $\ A_h + A_s\cdot\dfrac{ds(h;\tilde{V})}{dh} = 0. \qquad \therefore\ \dfrac{ds(h;\tilde{V})}{dh} = -\dfrac{A_h}{A_s}.$

$$
\begin{cases}
E(h,s) := \dfrac{\sqrt{2s(1-s)(1-sh)}\,/\,K(\sqrt{h})}{\sqrt{3h^2s^4-4(h^2+h)s^3+(4h^2+2h+4)s^2-4(h+1)s+3}} = \sqrt{3}\cdot\dfrac{\varepsilon}{\sqrt{\tilde{V}^2+\tilde{V}+1}}, \\[2ex]
A(h,s) := \dfrac{2(hs^2-sh+1)(hs^2-2s+1)(1-hs^2)}{\sqrt{3h^2s^4-4(h^2+h)s^3+(4h^2+2h+4)s^2-4(h+1)s+3}^{\,3}} = \dfrac{1}{3\sqrt{3}}\cdot\dfrac{(1-\tilde{V})(2\tilde{V}+1)(\tilde{V}+2)}{\sqrt{\tilde{V}^2+\tilde{V}+1}^{\,3}}.
\end{cases}
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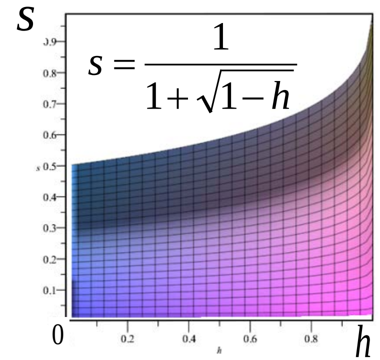
$$
m(\tilde{V},\varepsilon^2) := \frac{4\tilde{V}+2}{3} + \frac{1}{\sqrt{3}}\cdot\sqrt{\tilde{V}^2+\tilde{V}+1}\cdot M(h,s),
$$



$\left(0<h<1,\ 0<s<\frac{1}{1+\sqrt{1-h}}\right)$

$$
\frac{\partial m(\tilde{V},\varepsilon^2)}{\partial\varepsilon} = \frac{\partial\left(\dfrac{4\tilde{V}+2}{3} + \sqrt{\dfrac{\tilde{V}^2+\tilde{V}+1}{3}}\cdot M\right)}{\partial\varepsilon} = \sqrt{\frac{\tilde{V}^2+\tilde{V}+1}{3}}\,\frac{\partial M}{\partial\varepsilon}
$$

$$
= \frac{\dfrac{dM(h,s(h;\tilde{V}))}{dh}}{\dfrac{dE(h,s(h;\tilde{V}))}{dh}}
= \frac{M_h + M_s\cdot\dfrac{ds(h;\tilde{V})}{dh}}{E_h + E_s\cdot\dfrac{ds(h;\tilde{V})}{dh}}
= \frac{M_h + M_s\cdot\left(-\dfrac{A_h}{A_s}\right)}{E_h + E_s\cdot\left(-\dfrac{A_h}{A_s}\right)}
= \frac{M_h\cdot A_s - M_s\cdot A_h}{E_h\cdot A_s - E_s\cdot A_h}.
$$

By $\quad A(h,s) = \dfrac{1}{3\sqrt{3}}\cdot\dfrac{(1-\tilde{V})(2\tilde{V}+1)(\tilde{V}+2)}{\sqrt{\tilde{V}^2+\tilde{V}+1}^{\,3}},$

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$$
A(h,s) := \frac{2(hs^2-sh+1)(hs^2-2s+1)(1-hs^2)}{\sqrt{3h^2s^4-4(h^2+h)s^3+(4h^2+2h+4)s^2-4(h+1)s+3}^{\,3}}
$$

$$
E(h,s) := \frac{\sqrt{2s(1-s)(1-sh)}\,/\,K(\sqrt{h})}{\sqrt{3h^2s^4-4(h^2+h)s^3+(4h^2+2h+4)s^2-4(h+1)s+3}}
$$

$$
M(h,\ s) := \frac{-(hs^2-2(1+h)s+3)+4(1-s)(1-sh)\Pi(-sh,\sqrt{h})\,/\,K(\sqrt{h})}{\sqrt{3h^2s^4-4(h^2+h)s^3+(4h^2+2h+4)s^2-4(h+1)s+3}},
$$

Plan of proof(continued)

$$\frac{\partial m(\tilde{V},\varepsilon^2)}{\partial \varepsilon} = \sqrt{\frac{\tilde{V}^2+\tilde{V}+1}{3}} \cdot \frac{M_h \cdot A_s - M_s \cdot A_h}{E_h \cdot A_s - E_s \cdot A_h}.$$

Plan of proof(continued)

$$\frac{\partial m(\tilde{V},\varepsilon^2)}{\partial \varepsilon} = \sqrt{\frac{\tilde{V}^2+\tilde{V}+1}{3}} \cdot \frac{M_h \cdot A_s - M_s \cdot A_h}{E_h \cdot A_s - E_s \cdot A_h}.$$

Here,

$$M(h,\ s) := \frac{-(hs^2-2(1+h)s+3)+4(1-s)(1-sh)\Pi(-sh,\sqrt{h})\big/K(\sqrt{h})}{\sqrt{3h^2s^4-4(h^2+h)s^3+(4h^2+2h+4)s^2-4(h+1)s+3}}$$

$$A(h,s) := \frac{2(hs^2-sh+1)(hs^2-2s+1)(1-hs^2)}{\sqrt{3h^2s^4-4(h^2+h)s^3+(4h^2+2h+4)s^2-4(h+1)s+3}^{\,3}}$$

$$E(h,s) := \frac{\sqrt{2s(1-s)(1-sh)}\big/K(\sqrt{h})}{\sqrt{3h^2s^4-4(h^2+h)s^3+(4h^2+2h+4)s^2-4(h+1)s+3}}$$

Plan of proof(continued)

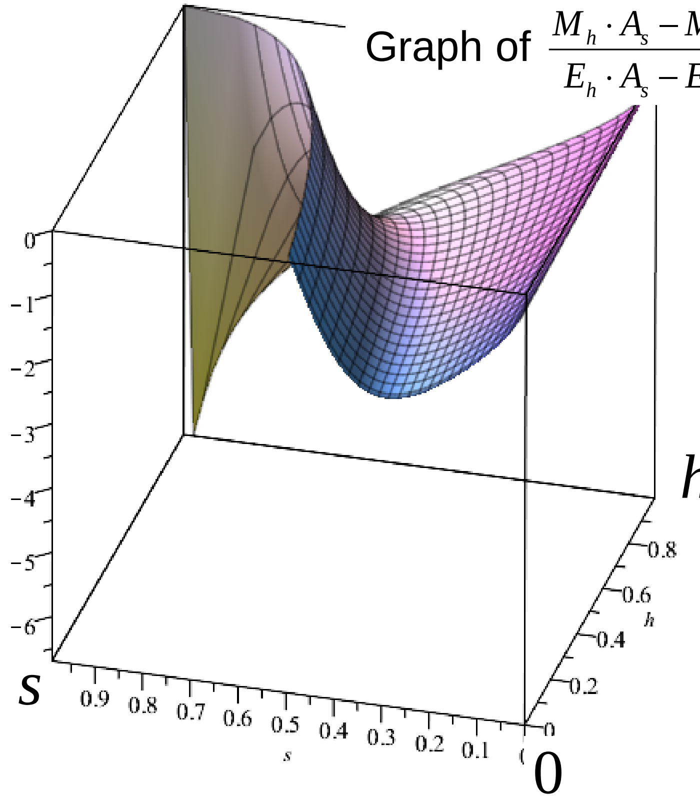
$$\frac{\partial m(\tilde{V},\varepsilon^2)}{\partial \varepsilon} = \sqrt{\frac{\tilde{V}^2+\tilde{V}+1}{3}} \cdot \frac{M_h \cdot A_s - M_s \cdot A_h}{E_h \cdot A_s - E_s \cdot A_h}.$$

Here,

$$M(h,\ s) := \frac{-(hs^2 - 2(1+h)s+3) + 4(1-s)(1-sh)\Pi(-sh,\sqrt{h})/K(\sqrt{h})}{\sqrt{3h^2s^4 - 4(h^2+h)s^3 + (4h^2+2h+4)s^2 - 4(h+1)s+3}}$$

$$A(h,s) := \frac{2(hs^2 - sh + 1)(hs^2 - 2s + 1)(1 - hs^2)}{\sqrt{3h^2s^4 - 4(h^2+h)s^3 + (4h^2+2h+4)s^2 - 4(h+1)s+3}^{\,3}}$$

$$E(h,s) := \frac{\sqrt{2s(1-s)(1-sh)}/K(\sqrt{h})}{\sqrt{3h^2s^4 - 4(h^2+h)s^3 + (4h^2+2h+4)s^2 - 4(h+1)s+3}}$$



Graph of $\frac{M_h \cdot A_s - M_s \cdot A_h}{E_h \cdot A_s - E_s \cdot A_h}$

Plan of proof(continued)

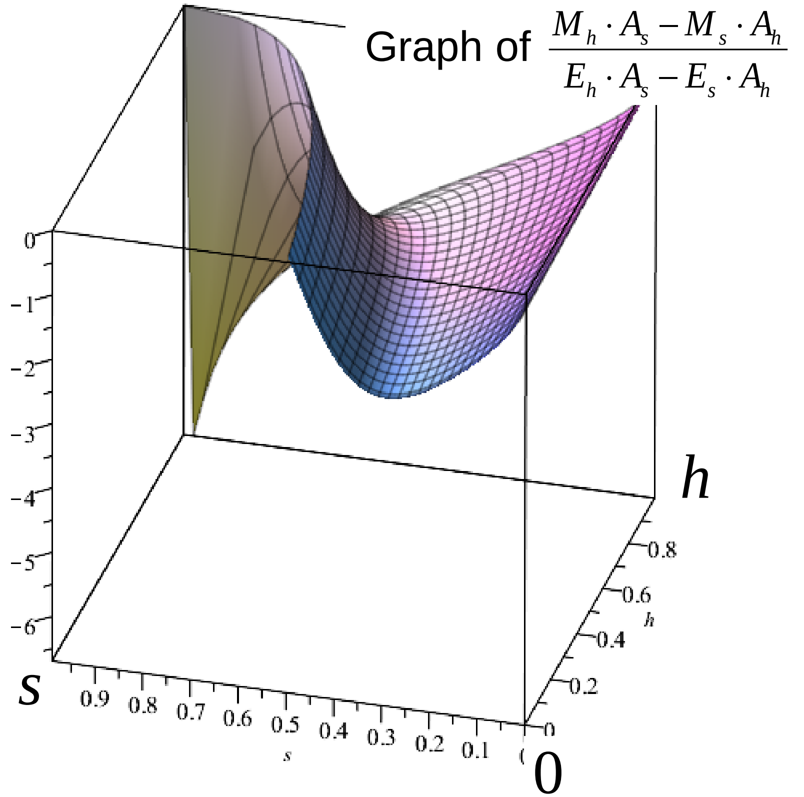
$$\frac{\partial m(\tilde{V},\varepsilon^2)}{\partial \varepsilon} = \sqrt{\frac{\tilde{V}^2+\tilde{V}+1}{3}} \cdot \frac{M_h \cdot A_s - M_s \cdot A_h}{E_h \cdot A_s - E_s \cdot A_h}.$$

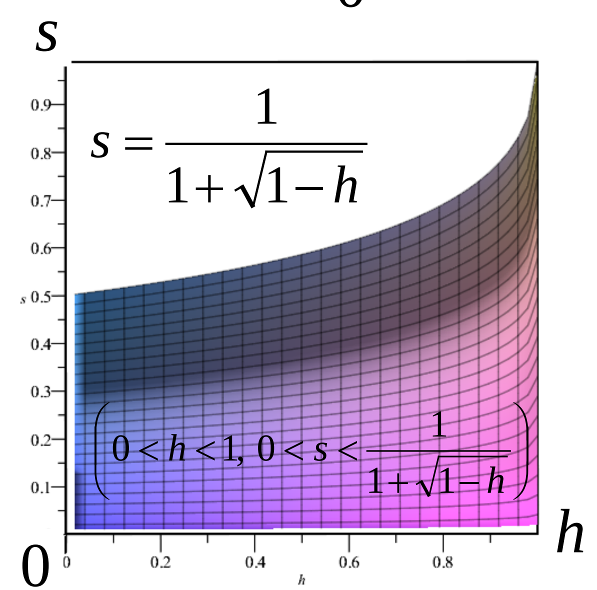
Here,

$$M(h,s) := \frac{-(hs^2-2(1+h)s+3)+4(1-s)(1-sh)\Pi(-sh,\sqrt{h})/K(\sqrt{h})}{\sqrt{3h^2s^4-4(h^2+h)s^3+(4h^2+2h+4)s^2-4(h+1)s+3}}$$

$$A(h,s) := \frac{2(hs^2-sh+1)(hs^2-2s+1)(1-hs^2)}{\sqrt{3h^2s^4-4(h^2+h)s^3+(4h^2+2h+4)s^2-4(h+1)s+3}^{\,3}}$$

$$E(h,s) := \frac{\sqrt{2s(1-s)(1-sh)}/K(\sqrt{h})}{\sqrt{3h^2s^4-4(h^2+h)s^3+(4h^2+2h+4)s^2-4(h+1)s+3}}$$





Plan of proof(continued)

$$\frac{\partial m(\tilde{V},\varepsilon^2)}{\partial \varepsilon}=\sqrt{\frac{\tilde{V}^2+\tilde{V}+1}{3}}\cdot\frac{M_h\cdot A_s-M_s\cdot A_h}{E_h\cdot A_s-E_s\cdot A_h}.$$

Here,

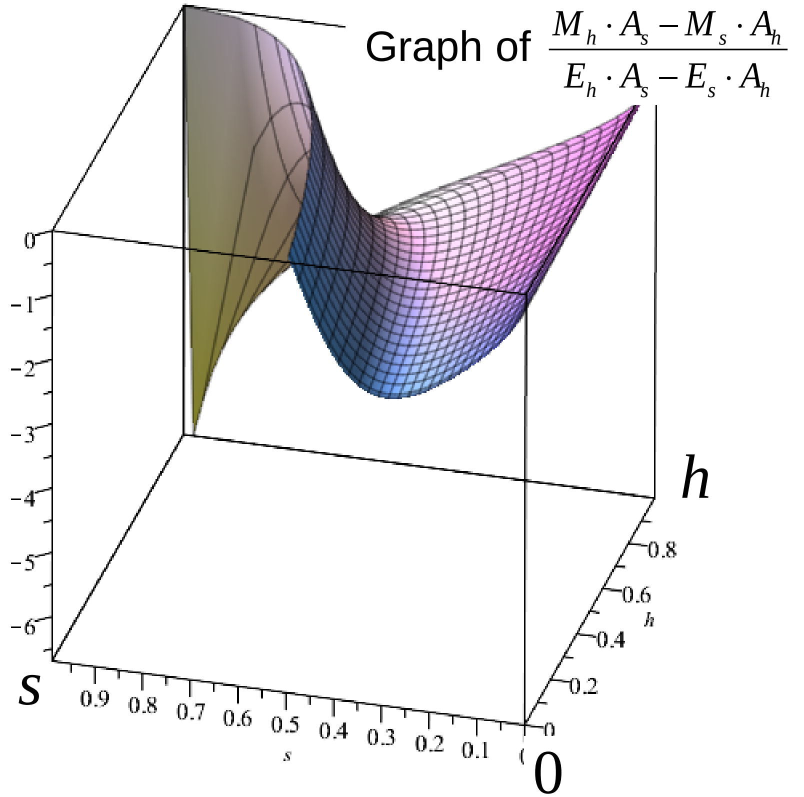
$$M(h,\ s):=\frac{-(hs^2-2(1+h)s+3)+4(1-s)(1-sh)\Pi(-sh,\sqrt{h})/K(\sqrt{h})}{\sqrt{3h^2s^4-4(h^2+h)s^3+(4h^2+2h+4)s^2-4(h+1)s+3}}$$

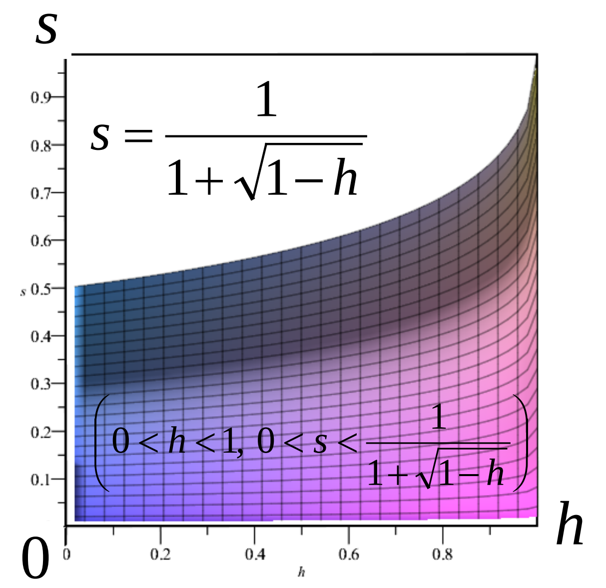
$$A(h,s):=\frac{2(hs^2-sh+1)(hs^2-2s+1)(1-hs^2)}{\sqrt{3h^2s^4-4(h^2+h)s^3+(4h^2+2h+4)s^2-4(h+1)s+3}^3}$$

$$E(h,s):=\frac{\sqrt{2s(1-s)(1-sh)}/K(\sqrt{h})}{\sqrt{3h^2s^4-4(h^2+h)s^3+(4h^2+2h+4)s^2-4(h+1)s+3}}$$

We may show the following inequality equations.

$$M_h\cdot A_s-M_s\cdot A_h<0,\quad E_h\cdot A_s-E_s\cdot A_h>0.$$





Plan of proof(continued)

$$\frac{\partial m(\tilde{V},\varepsilon^2)}{\partial \varepsilon}=\sqrt{\frac{\tilde{V}^2+\tilde{V}+1}{3}}\cdot\frac{M_h\cdot A_s-M_s\cdot A_h}{E_h\cdot A_s-E_s\cdot A_h}.$$

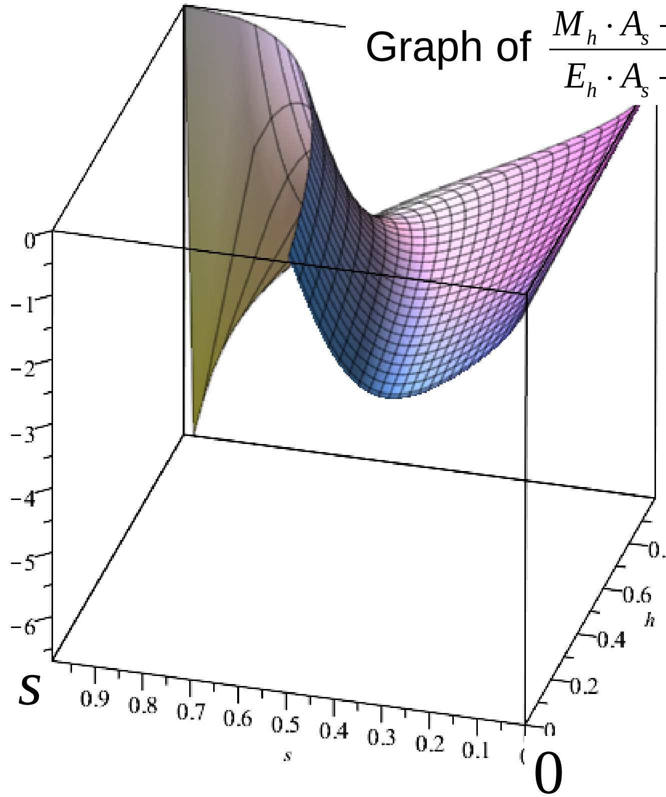
Here,

$$M(h,\, s):=\frac{-(hs^2-2(1+h)s+3)+4(1-s)(1-sh)\Pi(-sh,\sqrt{h})/K(\sqrt{h})}{\sqrt{3h^2s^4-4(h^2+h)s^3+(4h^2+2h+4)s^2-4(h+1)s+3}}$$

$$A(h,s):=\frac{2(hs^2-sh+1)(hs^2-2s+1)(1-hs^2)}{\sqrt{3h^2s^4-4(h^2+h)s^3+(4h^2+2h+4)s^2-4(h+1)s+3}^{\,3}}$$

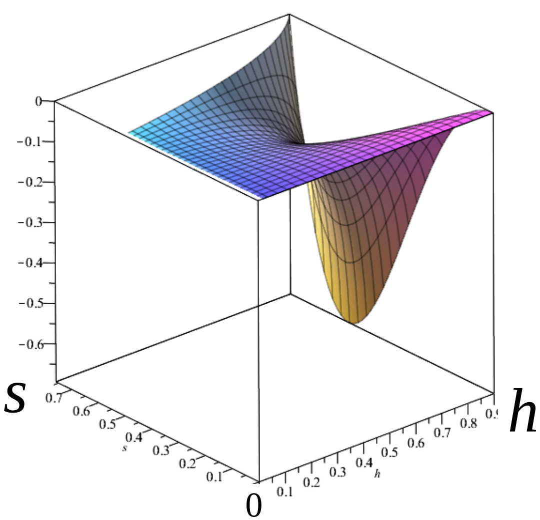
$$E(h,s):=\frac{\sqrt{2s(1-s)(1-sh)}/K(\sqrt{h})}{\sqrt{3h^2s^4-4(h^2+h)s^3+(4h^2+2h+4)s^2-4(h+1)s+3}}$$

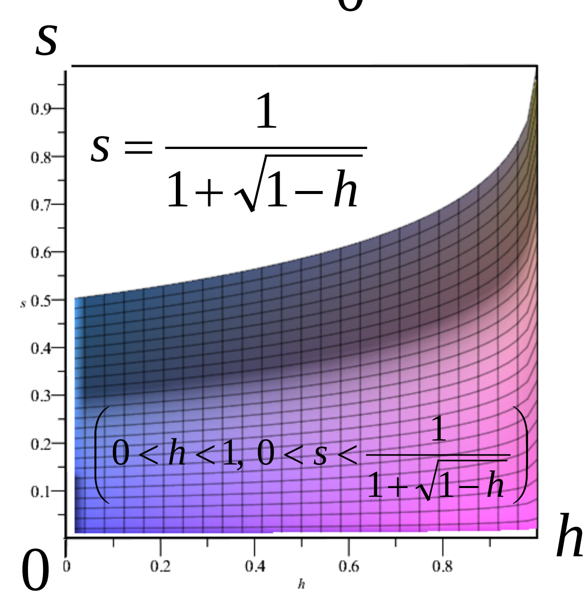
Graph of $\frac{M_h\cdot A_s-M_s\cdot A_h}{E_h\cdot A_s-E_s\cdot A_h}$



We may show the following inequality equations.

$$M_h\cdot A_s-M_s\cdot A_h<0,\quad E_h\cdot A_s-E_s\cdot A_h>0.$$





Plan of proof(continued)

$$\frac{\partial m(\tilde{V},\varepsilon^2)}{\partial \varepsilon}=\sqrt{\frac{\tilde{V}^2+\tilde{V}+1}{3}}\cdot\frac{M_h\cdot A_s-M_s\cdot A_h}{E_h\cdot A_s-E_s\cdot A_h}.$$

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$$M(h,\,s):=\frac{-(hs^2-2(1+h)s+3)+4(1-s)(1-sh)\Pi(-sh,\sqrt{h})/K(\sqrt{h})}{\sqrt{3h^2s^4-4(h^2+h)s^3+(4h^2+2h+4)s^2-4(h+1)s+3}}$$

$$A(h,s):=\frac{2(hs^2-sh+1)(hs^2-2s+1)(1-hs^2)}{\sqrt{3h^2s^4-4(h^2+h)s^3+(4h^2+2h+4)s^2-4(h+1)s+3}^{\,3}}$$

$$E(h,s):=\frac{\sqrt{2s(1-s)(1-sh)}/K(\sqrt{h})}{\sqrt{3h^2s^4-4(h^2+h)s^3+(4h^2+2h+4)s^2-4(h+1)s+3}}$$

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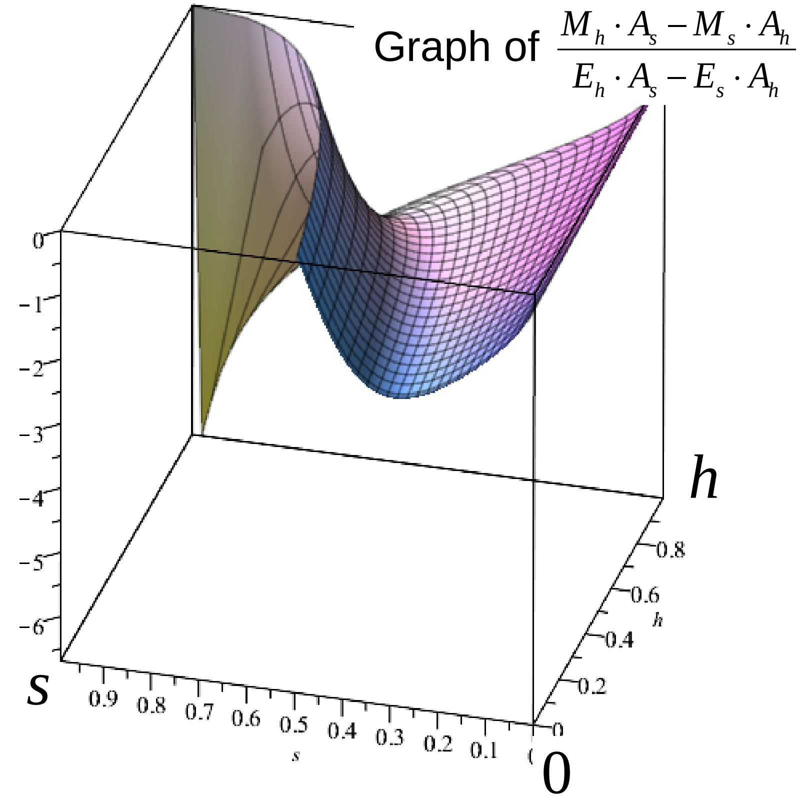

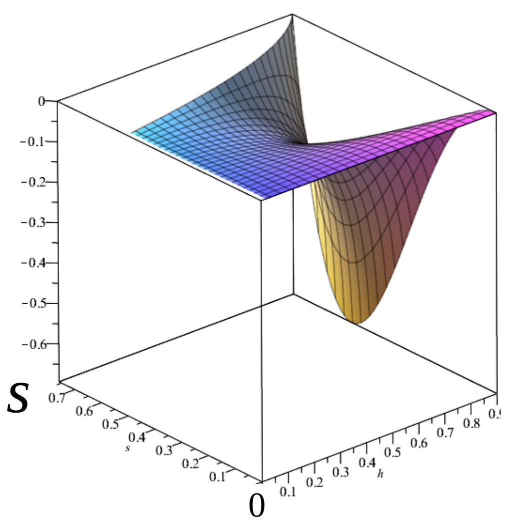

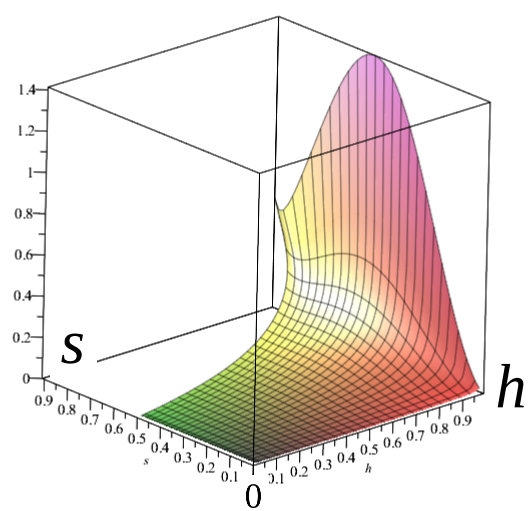

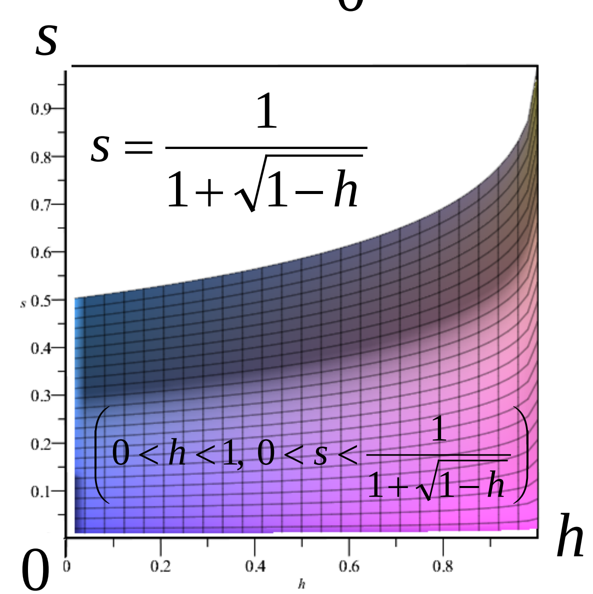

Plan of proof(continued)

$$\frac{\partial m(\tilde{V},\varepsilon^2)}{\partial \varepsilon} = \sqrt{\frac{\tilde{V}^2+\tilde{V}+1}{3}} \cdot \frac{M_h \cdot A_s - M_s \cdot A_h}{E_h \cdot A_s - E_s \cdot A_h}.$$

Here,

$$M(h,\ s) := \frac{-(hs^2-2(1+h)s+3)+4(1-s)(1-sh)\Pi(-sh,\sqrt{h})/K(\sqrt{h})}{\sqrt{3h^2s^4-4(h^2+h)s^3+(4h^2+2h+4)s^2-4(h+1)s+3}}$$

$$A(h,s) := \frac{2(hs^2-sh+1)(hs^2-2s+1)(1-hs^2)}{\sqrt{3h^2s^4-4(h^2+h)s^3+(4h^2+2h+4)s^2-4(h+1)s+3}^{\,3}}$$

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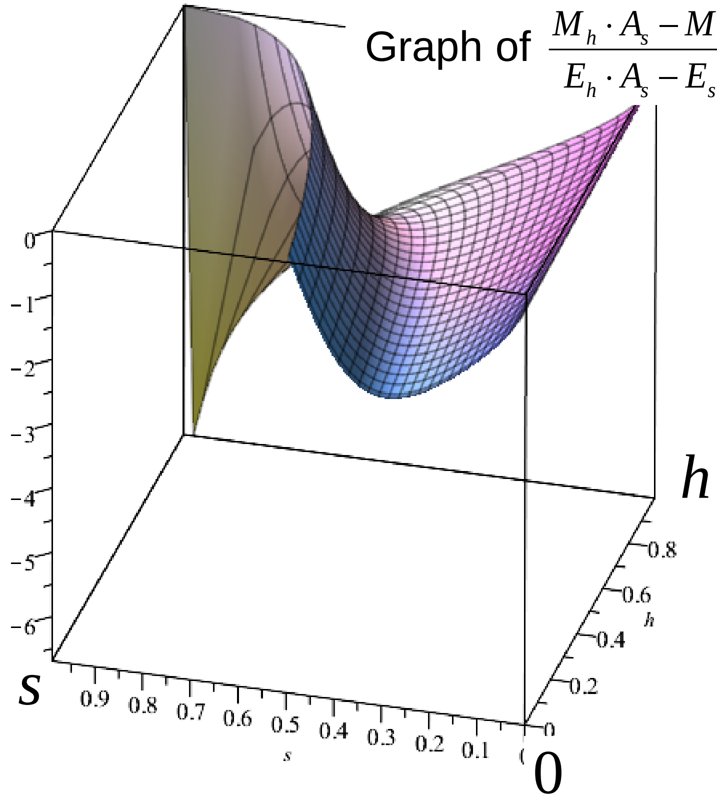
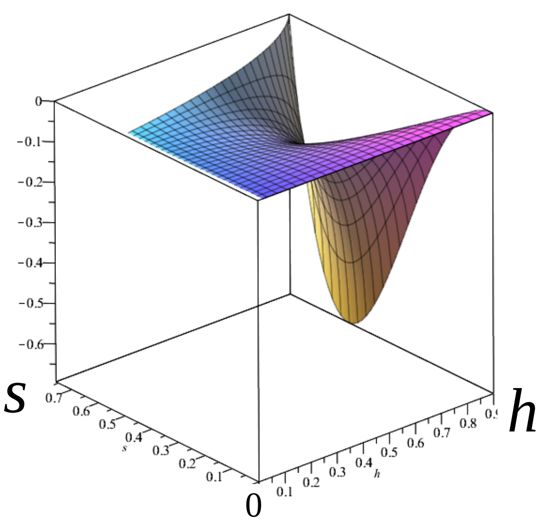
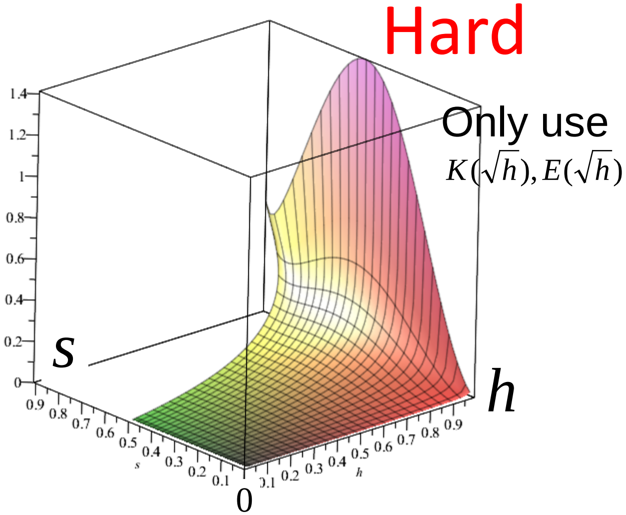
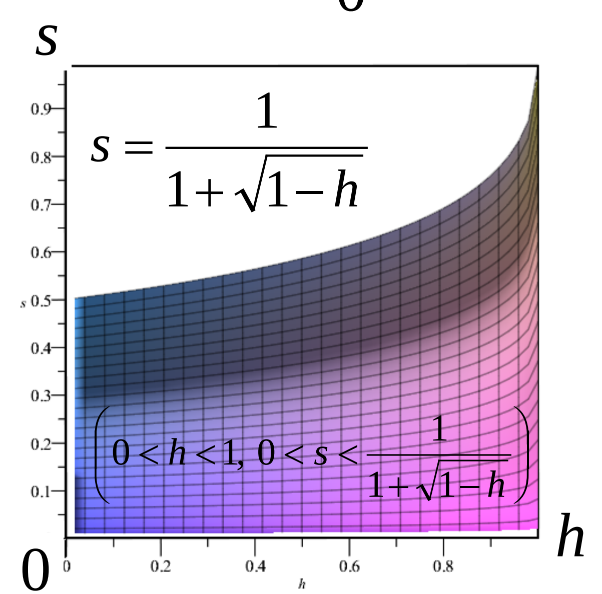


We may show the following inequality equations.

$$M_h \cdot A_s - M_s \cdot A_h < 0, \quad E_h \cdot A_s - E_s \cdot A_h > 0.$$







Plan of proof(continued)

$$\frac{\partial m(\tilde{V},\varepsilon^2)}{\partial \varepsilon}=\sqrt{\frac{\tilde{V}^2+\tilde{V}+1}{3}}\cdot\frac{M_h\cdot A_s-M_s\cdot A_h}{E_h\cdot A_s-E_s\cdot A_h}.$$

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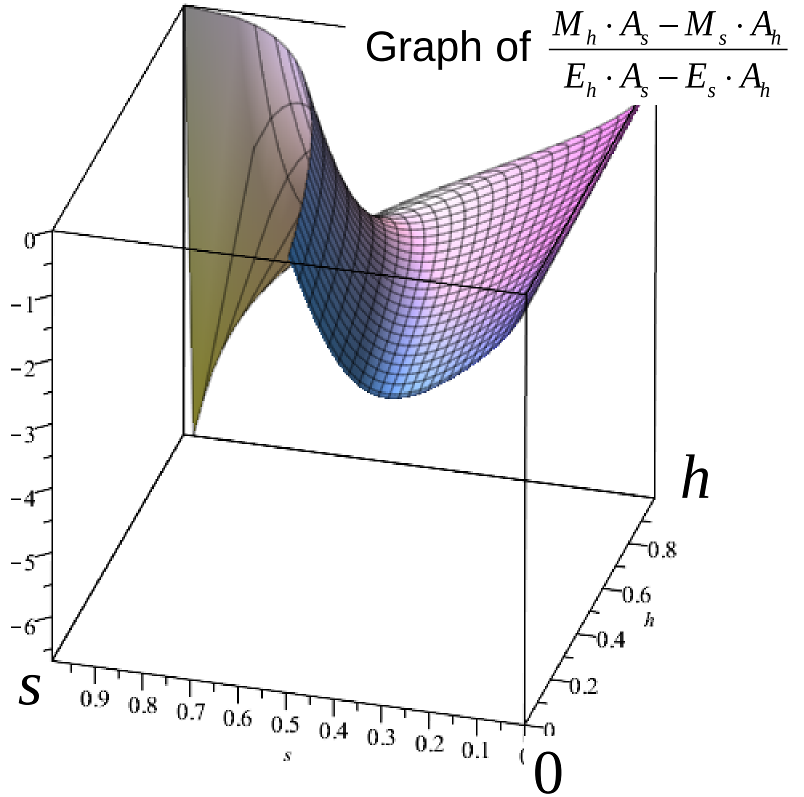
$$A(h,s):=\frac{2(hs^2-sh+1)(hs^2-2s+1)(1-hs^2)}{\sqrt{3h^2s^4-4(h^2+h)s^3+(4h^2+2h+4)s^2-4(h+1)s+3}^{\,3}}$$

$$E(h,s):=\frac{\sqrt{2s(1-s)(1-sh)}/K(\sqrt{h})}{\sqrt{3h^2s^4-4(h^2+h)s^3+(4h^2+2h+4)s^2-4(h+1)s+3}}$$



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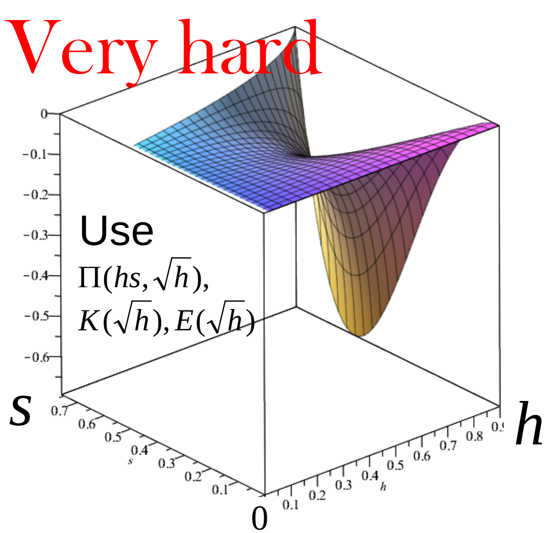

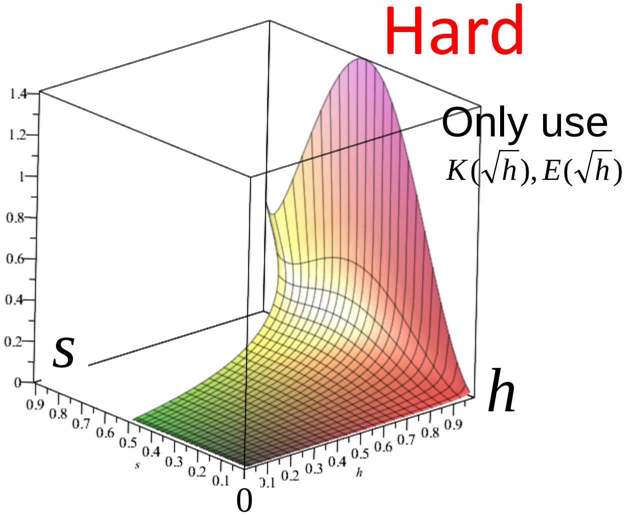

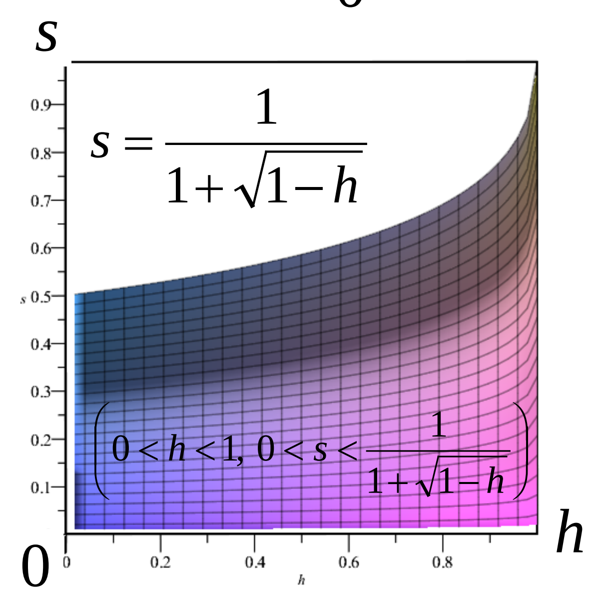

Plan of proof(continued)

$$\frac{\partial m(\tilde{V},\varepsilon^2)}{\partial \varepsilon} = \sqrt{\frac{\tilde{V}^2+\tilde{V}+1}{3}} \cdot \frac{M_h \cdot A_s - M_s \cdot A_h}{E_h \cdot A_s - E_s \cdot A_h}.$$

Here,

$$M(h,\ s) := \frac{-(hs^2-2(1+h)s+3)+4(1-s)(1-sh)\Pi(-sh,\sqrt{h})/K(\sqrt{h})}{\sqrt{3h^2s^4-4(h^2+h)s^3+(4h^2+2h+4)s^2-4(h+1)s+3}}$$

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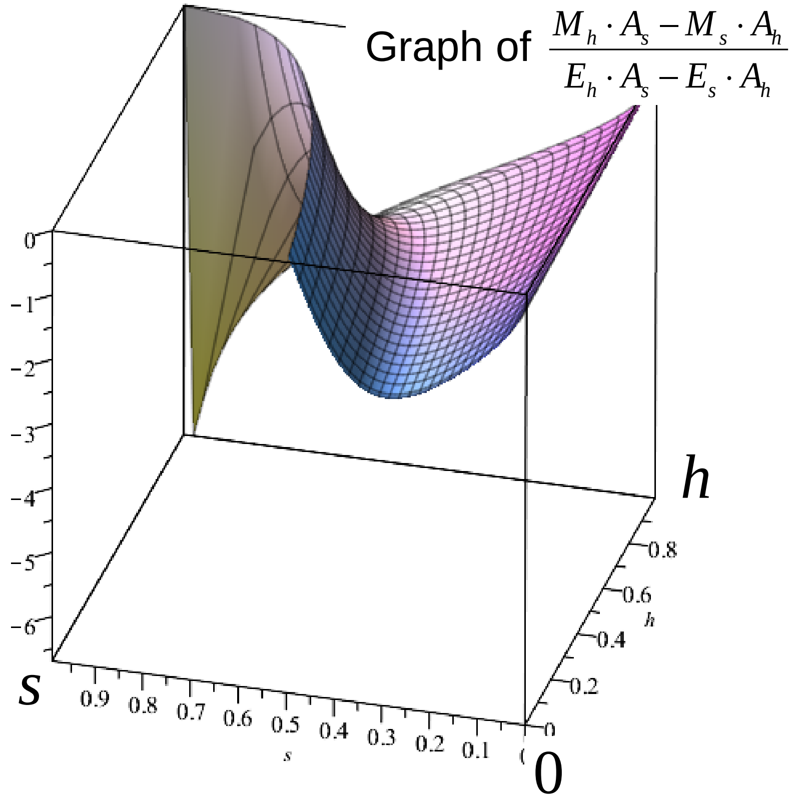

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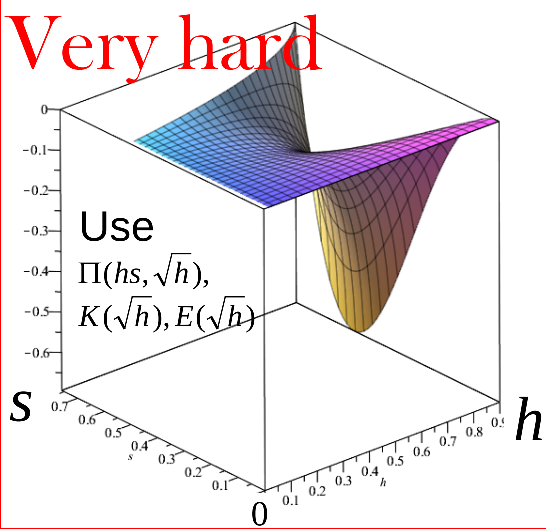
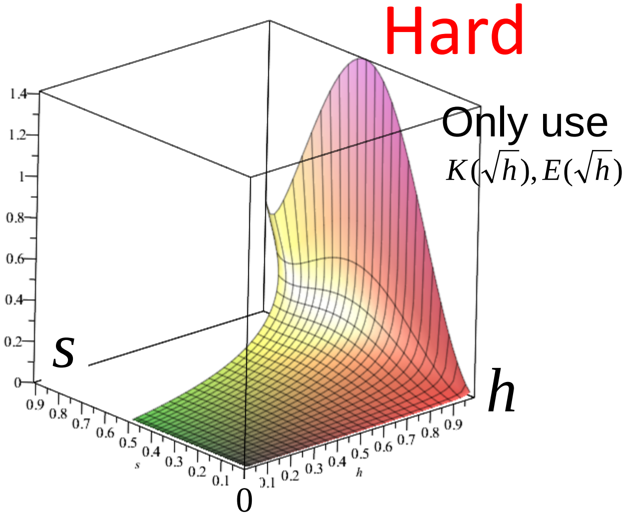
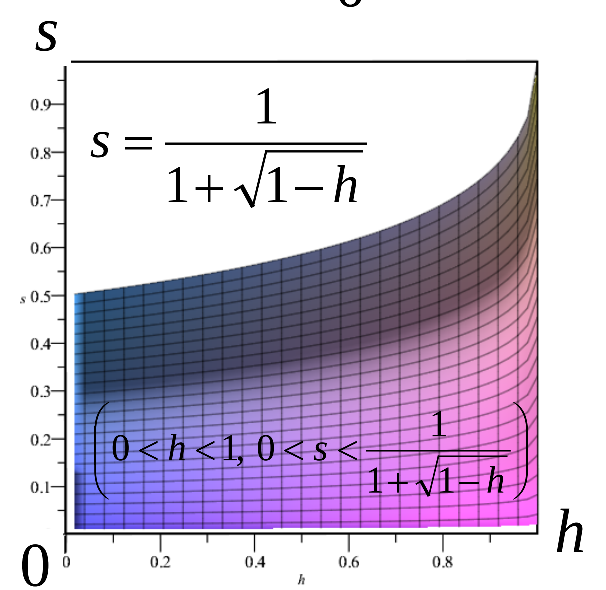
$$M_h \cdot A_s - M_s \cdot A_h < 0, \quad E_h \cdot A_s - E_s \cdot A_h > 0.$$







We show $M_h \cdot A_s - M_s \cdot A_h < 0$

$$\mathcal{M}_h \cdot \mathcal{A}_s - \mathcal{M}_s \cdot \mathcal{A}_h =$$

$$M(h,\ s) := \frac{-(hs^2 - 2(1+h)s + 3) + 4(1-s)(1-sh)\Pi(-sh, \sqrt{h}) / K(\sqrt{h})}{\sqrt{3h^2 s^4 - 4(h^2+h)s^3 + (4h^2+2h+4)s^2 - 4(h+1)s + 3}}$$

$$A(h,s) := \frac{2(hs^2 - sh + 1)(hs^2 - 2s + 1)(1 - hs^2)}{\sqrt{3h^2 s^4 - 4(h^2+h)s^3 + (4h^2+2h+4)s^2 - 4(h+1)s + 3}^{\,3}}$$

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$$A(h,s) := \frac{2(hs^2-sh+1)(hs^2-2s+1)(1-hs^2)}{\sqrt{3h^2s^4-4(h^2+h)s^3+(4h^2+2h+4)s^2-4(h+1)s+3}^3}$$

$$\mathcal{M}_h \cdot \mathcal{A}_s - \mathcal{M}_s \cdot \mathcal{A}_h =$$

$$32s(1-hs)^2(1-s)^2\Bigg(\Big(\big(-2h^4s^4+2h^3s^4+4h^3s^3-2h^2s^4+4h^2s^3-12h^2s^2$$
$$+4h^2s-2h^2+4hs+2h-2\big)\,E(\sqrt{h})+\big(h^4s^4-3h^3s^4+4h^3s^3+2h^2s^4-6h^3s^2$$
$$-4h^2s^3+6h^2s^2+4h^2s+h^2-4hs-3h+2\big)\,K(\sqrt{h})\Big)\Pi(hs,\sqrt{h})$$
$$+\big(-h^3s^3-h^2s^3+6h^2s^2-3h^2s+2h^2-3hs-2h+2\big)\,K(\sqrt{h})E(\sqrt{h})$$
$$+\big(-h^3s^3+3h^3s^2+h^2s^3-3h^2s^2-3h^2s-h^2+3hs+3h-2\big)\,K(\sqrt{h})^2\Bigg)$$
$$\Big/ K(\sqrt{h})^2\,(-1+h)\,h\,\big(3h^2s^4-4h^2s^3+4h^2s^2-4hs^3+2hs^2-4hs+4s^2-4s+3\big)^3$$

We show $M_h \cdot A_s - M_s \cdot A_h < 0$

$$M(h,s) := \frac{-(hs^2 - 2(1+h)s + 3) + 4(1-s)(1-sh)\Pi(-sh,\sqrt{h})/K(\sqrt{h})}{\sqrt{3h^2s^4 - 4(h^2+h)s^3 + (4h^2+2h+4)s^2 - 4(h+1)s + 3}}$$

$$A(h,s) := \frac{2(hs^2 - sh + 1)(hs^2 - 2s + 1)(1 - hs^2)}{\sqrt{3h^2s^4 - 4(h^2+h)s^3 + (4h^2+2h+4)s^2 - 4(h+1)s + 3}^3}$$

$$\mathcal{M}_h \cdot \mathcal{A}_s - \mathcal{M}_s \cdot \mathcal{A}_h =$$

$$32s(1-hs)^2(1-s)^2\Bigg(\bigg(\big(-2h^4s^4 + 2h^3s^4 + 4h^3s^3 - 2h^2s^4 + 4h^2s^3 - 12h^2s^2$$
$$+4h^2s - 2h^2 + 4hs + 2h - 2\big)\,E(\sqrt{h}) + \big(h^4s^4 - 3h^3s^4 + 4h^3s^3 + 2h^2s^4 - 6h^3s^2$$
$$-4h^2s^3 + 6h^2s^2 + 4h^2s + h^2 - 4hs - 3h + 2\big)\,K(\sqrt{h})\bigg)\Pi(-hs,\sqrt{h})$$
$$+\big(-h^3s^3 - h^2s^3 + 6h^2s^2 - 3h^2s + 2h^2 - 3hs - 2h + 2\big)\,K(\sqrt{h})E(\sqrt{h})$$
$$+\big(-h^3s^3 + 3h^3s^2 + h^2s^3 - 3h^2s^2 - 3h^2s - h^2 + 3hs + 3h - 2\big)\,K(\sqrt{h})^2\Bigg)$$
$$\Big/ K(\sqrt{h})^2\,(-1+h)\,h\,\big(3h^2s^4 - 4h^2s^3 + 4h^2s^2 - 4hs^3 + 2hs^2 - 4hs + 4s^2 - 4s + 3\big)^3$$

We show $M_h \cdot A_s - M_s \cdot A_h < 0$

$$M(h,\, s) := \frac{-(hs^2 - 2(1+h)s + 3) + 4(1-s)(1-sh)\Pi(-sh, \sqrt{h})/K(\sqrt{h})}{\sqrt{3h^2s^4 - 4(h^2+h)s^3 + (4h^2+2h+4)s^2 - 4(h+1)s + 3}}$$

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$$\mathcal{M}_h \cdot \mathcal{A}_s - \mathcal{M}_s \cdot \mathcal{A}_h =$$

$$32s(1-hs)^2(1-s)^2 \Big(\Big((-2h^4s^4 + 2h^3s^4 + 4h^3s^3 - 2h^2s^4 + 4h^2s^3 - 12h^2s^2$$
$$+4h^2s - 2h^2 + 4hs + 2h - 2)\, E(\sqrt{h}) + (h^4s^4 - 3h^3s^4 + 4h^3s^3 + 2h^2s^4 - 6h^3s^2$$
$$-4h^2s^3 + 6h^2s^2 + 4h^2s + h^2 - 4hs - 3h + 2)\, K(\sqrt{h}) \Big) \Pi(-hs, \sqrt{h})$$
$$+ (-h^3s^3 - h^2s^3 + 6h^2s^2 - 3h^2s + 2h^2 - 3hs - 2h + 2)\, K(\sqrt{h}) E(\sqrt{h})$$
$$+ (-h^3s^3 + 3h^3s^2 + h^2s^3 - 3h^2s^2 - 3h^2s - h^2 + 3hs + 3h - 2)\, K(\sqrt{h})^2 \Big)$$
$$\Big/ K(\sqrt{h})^2 (-1+h)\, h \left(3h^2s^4 - 4h^2s^3 + 4h^2s^2 - 4hs^3 + 2hs^2 - 4hs + 4s^2 - 4s + 3\right)^3$$

negative positive

We show $M_h \cdot A_s - M_s \cdot A_h < 0$

$$M(h,s) := \frac{-(hs^2 - 2(1+h)s + 3) + 4(1-s)(1-sh)\Pi(-sh,\sqrt{h})/K(\sqrt{h})}{\sqrt{3h^2s^4 - 4(h^2+h)s^3 + (4h^2+2h+4)s^2 - 4(h+1)s + 3}}$$

$$A(h,s) := \frac{2(hs^2 - sh + 1)(hs^2 - 2s + 1)(1 - hs^2)}{\sqrt{3h^2s^4 - 4(h^2+h)s^3 + (4h^2+2h+4)s^2 - 4(h+1)s + 3}^{\,3}}$$

$$\mathcal{M}_h \cdot \mathcal{A}_s - \mathcal{M}_s \cdot \mathcal{A}_h =$$

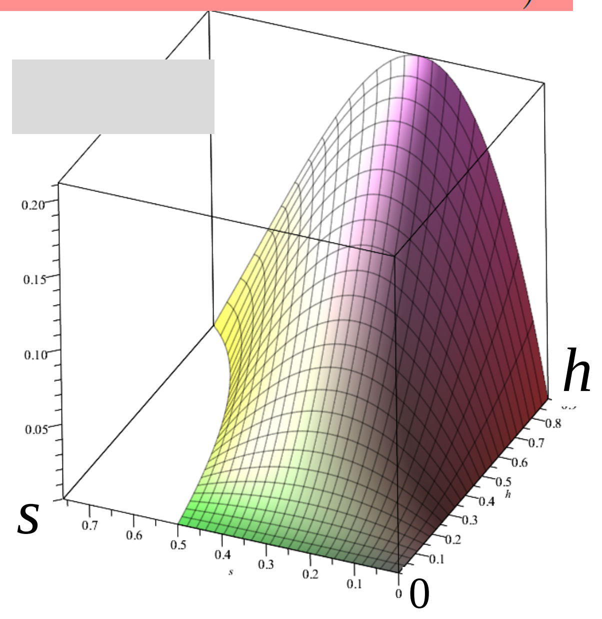
$$32s(1-hs)^2(1-s)^2\Bigg(\Big(\big(-2h^4s^4 + 2h^3s^4 + 4h^3s^3 - 2h^2s^4 + 4h^2s^3 - 12h^2s^2$$
$$+4h^2s - 2h^2 + 4hs + 2h - 2\big)\,E(\sqrt{h}) + \big(h^4s^4 - 3h^3s^4 + 4h^3s^3 + 2h^2s^4 - 6h^3s^2$$
$$-4h^2s^3 + 6h^2s^2 + 4h^2s + h^2 - 4hs - 3h + 2\big)\,K(\sqrt{h})\Big)\Pi(-hs,\sqrt{h})$$
$$+\big(-h^3s^3 - h^2s^3 + 6h^2s^2 - 3h^2s + 2h^2 - 3hs - 2h + 2\big)\,K(\sqrt{h})E(\sqrt{h})$$
$$+\big(-h^3s^3 + 3h^3s^2 + h^2s^3 - 3h^2s^2 - 3h^2s - h^2 + 3hs + 3h - 2\big)\,K(\sqrt{h})^2\Bigg)$$

$$\Big/\; K(\sqrt{h})^2\,(-1+h)\,h\,\big(3h^2s^4 - 4h^2s^3 + 4h^2s^2 - 4hs^3 + 2hs^2 - 4hs + 4s^2 - 4s + 3\big)^3$$

negative positive

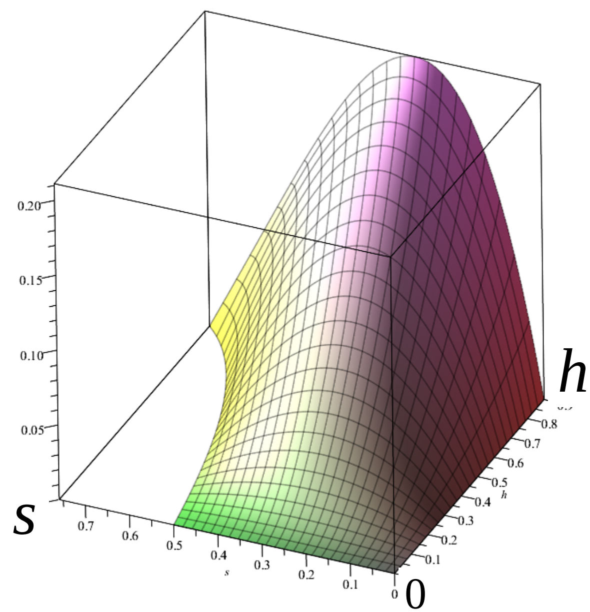
We may show that part of [ ] is positive.

Graph of [ ]



$$
\begin{aligned}
&\Big(\Big(\big(-2h^4s^4+2h^3s^4+4h^3s^3-2h^2s^4+4h^2s^3-12h^2s^2\\
&+4h^2s-2h^2+4hs+2h-2\big)\,E(\sqrt{h})+\big(h^4s^4-3h^3s^4+4h^3s^3+2h^2s^4-6h^3s^2\\
&-4h^2s^3+6h^2s^2+4h^2s+h^2-4hs-3h+2\big)\,K(\sqrt{h})\Big)\Pi(hs,\sqrt{h})\\
&+\big(-h^3s^3-h^2s^3+6h^2s^2-3h^2s+2h^2-3hs-2h+2\big)\,K(\sqrt{h})E(\sqrt{h})\\
&+\big(-h^3s^3+3h^3s^2+h^2s^3-3h^2s^2-3h^2s-h^2+3hs+3h-2\big)\,K(\sqrt{h})^2\Big)
\end{aligned}
$$

We may show that part of is positive.



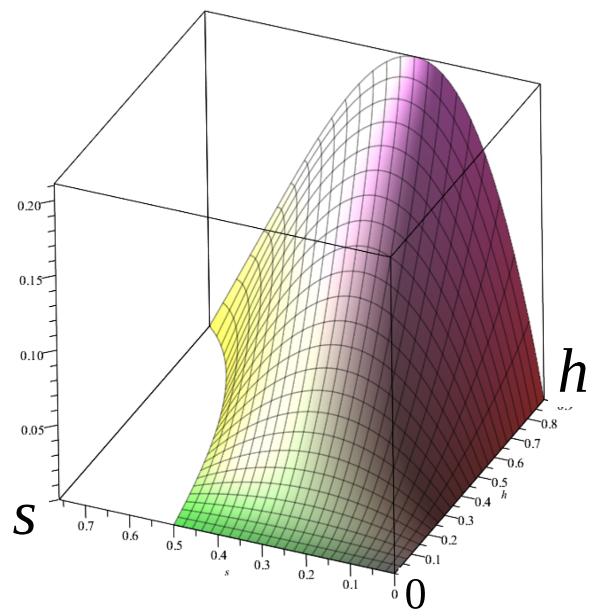
$$
\Big(\Big(\big(-2h^4s^4+2h^3s^4+4h^3s^3-2h^2s^4+4h^2s^3-12h^2s^2
+4h^2s-2h^2+4hs+2h-2\big)\,E(\sqrt{h})+\big(h^4s^4-3h^3s^4+4h^3s^3+2h^2s^4-6h^3s^2
-4h^2s^3+6h^2s^2+4h^2s+h^2-4hs-3h+2\big)\,K(\sqrt{h})\Big)\Pi(hs,\sqrt{h})
+\big(-h^3s^3-h^2s^3+6h^2s^2-3h^2s+2h^2-3hs-2h+2\big)\,K(\sqrt{h})E(\sqrt{h})
+\big(-h^3s^3+3h^3s^2+h^2s^3-3h^2s^2-3h^2s-h^2+3hs+3h-2\big)\,K(\sqrt{h})^2\Big)
$$

We may show that part of ▭ is positive.

- We can prove to use only $K(\sqrt{h}), E(\sqrt{h})$.

(∵ Take the derivative of $K(\sqrt{h}), E(\sqrt{h})$ with respect to h

Only $K(\sqrt{h}), E(\sqrt{h})$)



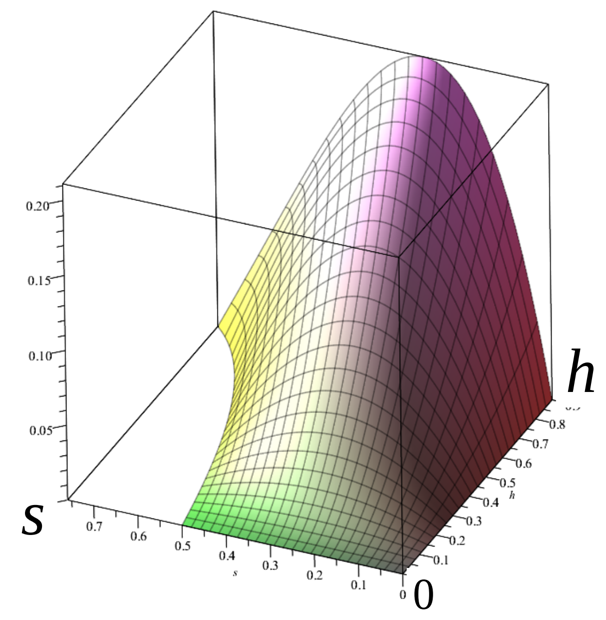
$$\Bigg(\bigg(\Big(-2h^4s^4+2h^3s^4+4h^3s^3-2h^2s^4+4h^2s^3-12h^2s^2$$
$$+4h^2s-2h^2+4hs+2h-2\Big)E(\sqrt{h})+\Big(h^4s^4-3h^3s^4+4h^3s^3+2h^2s^4-6h^3s^2$$
$$-4h^2s^3+6h^2s^2+4h^2s+h^2-4hs-3h+2\Big)K(\sqrt{h})\bigg)\Pi(-hs,\sqrt{h})$$
$$+\Big(-h^3s^3-h^2s^3+6h^2s^2-3h^2s+2h^2-3hs-2h+2\Big)K(\sqrt{h})E(\sqrt{h})$$
$$+\Big(-h^3s^3+3h^3s^2+h^2s^3-3h^2s^2-3h^2s-h^2+3hs+3h-2\Big)K(\sqrt{h})^2\Bigg)$$

We may show that part of [grey box] is positive.

- We can prove to use only $K(\sqrt{h}), E(\sqrt{h})$.

(∵ Take the derivative of $K(\sqrt{h}), E(\sqrt{h})$ with respect to h

Only $K(\sqrt{h}), E(\sqrt{h})$



$$
\Bigg(\bigg(\Big(-2h^4s^4+2h^3s^4+4h^3s^3-2h^2s^4+4h^2s^3-12h^2s^2
+4h^2s-2h^2+4hs+2h-2\Big)E(\sqrt{h})+\Big(h^4s^4-3h^3s^4+4h^3s^3+2h^2s^4-6h^3s^2
-4h^2s^3+6h^2s^2+4h^2s+h^2-4hs-3h+2\Big)K(\sqrt{h})\bigg)\Pi(-hs,\sqrt{h})
+\Big(-h^3s^3-h^2s^3+6h^2s^2-3h^2s+2h^2-3hs-2h+2\Big)K(\sqrt{h})E(\sqrt{h})
+\Big(-h^3s^3+3h^3s^2+h^2s^3-3h^2s^2-3h^2s-h^2+3hs+3h-2\Big)K(\sqrt{h})^2\Bigg)
$$

We may show that part of [grey box] is positive.

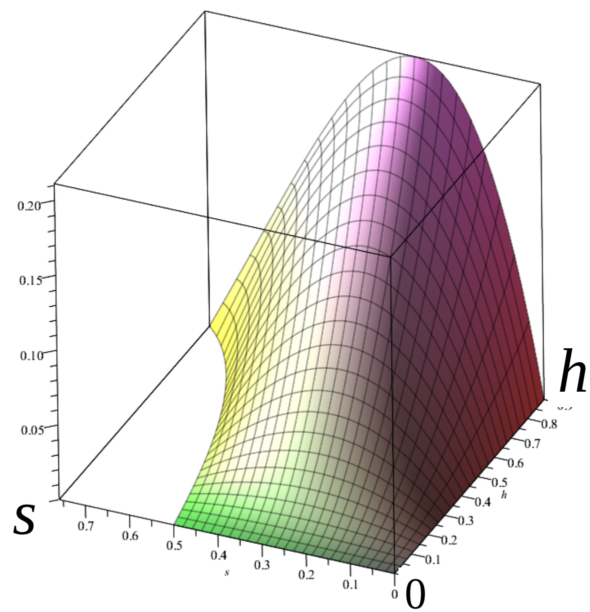
- We can prove to use only $K(\sqrt{h}), E(\sqrt{h})$.

(∵ Take the derivative of $K(\sqrt{h}), E(\sqrt{h})$ with respect to h

Only $K(\sqrt{h}), E(\sqrt{h})$)

- Using $\Pi(hs, \sqrt{h})$ case

We can prove but hard calculation.



Differential of a $\Pi(-hs,\sqrt{h})$

$\Pi(-hs,\sqrt{h})$ は微分すると $\Pi(-hs,\sqrt{h}), K(\sqrt{h}), E(\sqrt{h})$ がでる.

$$\frac{\partial}{\partial s}\Pi(-hs,\sqrt{h}) = \frac{1}{2}\frac{(1-hs^2)\Pi(-hs,\sqrt{h}) - sE(\sqrt{h}) - (1-s)K(\sqrt{h})}{s(1-s)(1-sh)}$$

$$\frac{\partial}{\partial h}\Pi(-hs,\sqrt{h}) = \frac{1}{2}\frac{sh(1-h)\Pi(-hs,\sqrt{h}) + E(\sqrt{h}) - (1-h)K(\sqrt{h})}{h(1-h)(1-sh)}$$

Idea of except $\Pi(-hs,\sqrt{h})$

K.Kosugi, Y. Morita and Y. Yotsutani, DCDS 19(2007),

Take the derivative of $\dfrac{2\sqrt{(1-s)(1-sh)}}{\sqrt{s}}\cdot\Pi(-hs,\sqrt{h})$ with respect to h

Only $K(\sqrt{h}), E(\sqrt{h})$

$$\frac{\partial}{\partial s}\left(\frac{2\sqrt{(1-s)(1-sh)}}{\sqrt{s}}\cdot\Pi(-hs,\sqrt{h})\right) = -\frac{sE(\sqrt{h}) + (1-s)K(\sqrt{h})}{s\sqrt{s(1-s)(1-sh)}}$$

$$\frac{\partial}{\partial h}\left(\frac{2\sqrt{(1-s)(1-sh)}}{\sqrt{s}}\cdot\Pi(-hs,\sqrt{h})\right) = \frac{(1-s)\left(E(\sqrt{h}) - (1-h)K(\sqrt{h})\right)}{h(1-h)\sqrt{s(1-s)(1-sh)}}$$

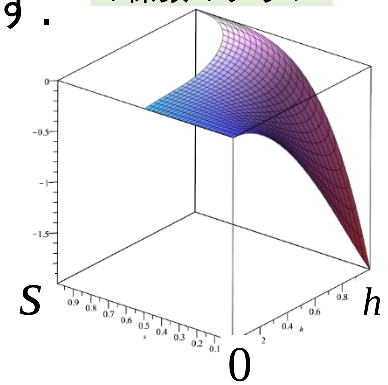
$$\Big(\big(-2h^4s^4+2h^3s^4+4h^3s^3-2h^2s^4+4h^2s^3-12h^2s^2+4h^2s-2h^2+4hs+2h-2\big)\,E(\sqrt{h})$$
$$+\big(h^4s^4-3h^3s^4+4h^3s^3+2h^2s^4-6h^3s^2\;-4h^2s^3+6h^2s^2+4h^2s+h^2-4hs-3h+2\big)\,K(\sqrt{h})\Big)\Pi(hs,\sqrt{h})$$
$$+\big(-h^3s^3-h^2s^3+6h^2s^2-3h^2s+2h^2-3hs-2h+2\big)\,K(\sqrt{h})E(\sqrt{h})$$
$$+\big(-h^3s^3+3h^3s^2+h^2s^3-3h^2s^2-3h^2s-h^2+3hs+3h-2\big)\,K(\sqrt{h})^2>0$$

を示す.

$$\Big(\big(-2h^4s^4+2h^3s^4+4h^3s^3-2h^2s^4+4h^2s^3-12h^2s^2+4h^2s-2h^2+4hs+2h-2\big)E(\sqrt{h})$$
$$+\big(h^4s^4-3h^3s^4+4h^3s^3+2h^2s^4-6h^3s^2-4h^2s^3+6h^2s^2+4h^2s+h^2-4hs-3h+2\big)K(\sqrt{h})\Big)\Pi(hs,\sqrt{h})$$
$$+\big(-h^3s^3-h^2s^3+6h^2s^2-3h^2s+2h^2-3hs-2h+2\big)K(\sqrt{h})E(\sqrt{h})$$
$$+\big(-h^3s^3+3h^3s^2+h^2s^3-3h^2s^2-3h^2s-h^2+3hs+3h-2\big)K(\sqrt{h})^2>0$$

を示す.

$\Pi(hs,\sqrt{h})$ の係数のグラフ

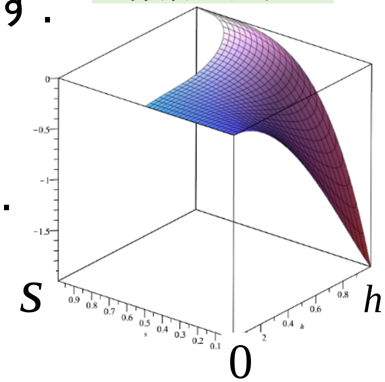


$\Pi(hs,\sqrt{h})$ の係数は負であることを確認する.

$$
\begin{aligned}
&\Big(\left(-2h^4s^4 + 2h^3s^4 + 4h^3s^3 - 2h^2s^4 + 4h^2s^3 - 12h^2s^2 + 4h^2s - 2h^2 + 4hs + 2h - 2\right) E(\sqrt{h}) \\
&+ \left(h^4s^4 - 3h^3s^4 + 4h^3s^3 + 2h^2s^4 - 6h^3s^2 - 4h^2s^3 + 6h^2s^2 + 4h^2s + h^2 - 4hs - 3h + 2\right) K(\sqrt{h})\Big)\Pi(hs, \sqrt{h}) \\
&+ \left(-h^3s^3 - h^2s^3 + 6h^2s^2 - 3h^2s + 2h^2 - 3hs - 2h + 2\right) K(\sqrt{h})E(\sqrt{h}) \\
&+ \left(-h^3s^3 + 3h^3s^2 + h^2s^3 - 3h^2s^2 - 3h^2s - h^2 + 3hs + 3h - 2\right) K(\sqrt{h})^2 > 0
\end{aligned}
$$

を示す.

$\Pi(hs, \sqrt{h})$ の係数のグラフ

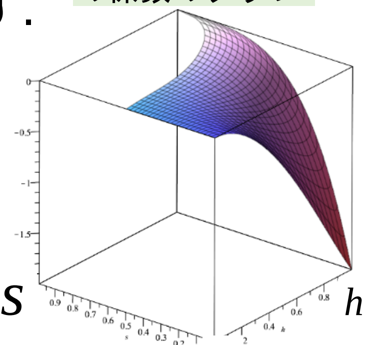


$\Pi(hs, \sqrt{h})$ の係数は負であることを確認する.

両辺を $\Pi(hs, \sqrt{h})$ の係数で割り,さらに $-\dfrac{2\sqrt{2(1-s)(1-sh)}}{\sqrt{s}}$ をかける.

$$\Big(\big(-2h^4s^4+2h^3s^4+4h^3s^3-2h^2s^4+4h^2s^3-12h^2s^2+4h^2s-2h^2+4hs+2h-2\big)E(\sqrt{h})$$
$$+\big(h^4s^4-3h^3s^4+4h^3s^3+2h^2s^4-6h^3s^2-4h^2s^3+6h^2s^2+4h^2s+h^2-4hs-3h+2\big)K(\sqrt{h})\Big)\Pi(hs,\sqrt{h})$$
$$+\big(-h^3s^3-h^2s^3+6h^2s^2-3h^2s+2h^2-3hs-2h+2\big)K(\sqrt{h})E(\sqrt{h})$$
$$+\big(-h^3s^3+3h^3s^2+h^2s^3-3h^2s^2-3h^2s-h^2+3hs+3h-2\big)K(\sqrt{h})^2>0$$

を示す.



$\Pi(hs,\sqrt{h})$ の係数は負であることを確認する.

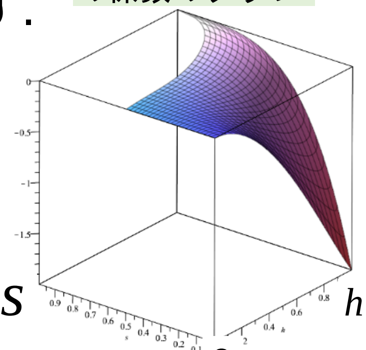
両辺を $\Pi(hs,\sqrt{h})$ の係数で割り,さらに $-\dfrac{2\sqrt{2(1-s)(1-sh)}}{\sqrt{s}}$ をかける.

$$-\frac{2\sqrt{(1-s)(1-sh)}}{\sqrt{s}}\cdot\Pi(hs,\sqrt{h})$$
$$+\frac{2\sqrt{(1-s)(1-sh)}}{\sqrt{s}}\cdot\Big\{\big(h^3s^3+h^2s^3-6h^2s^2+3h^2s-2h^2+3hs+2h-2\big)K(\sqrt{h})E(\sqrt{h})$$
$$+\big(h^3s^3-3h^3s^2-h^2s^3+3h^2s^2+3h^2s+h^2-3hs-3h+2\big)K(\sqrt{h})^2\Big\}$$
$$\Big/\Big\{\big(-2h^4s^4+2h^3s^4+4h^3s^3-2h^2s^4+4h^2s^3-12h^2s^2+4h^2s-2h^2+4hs+2h-2\big)E(\sqrt{h})$$
$$+\big(h^4s^4-3h^3s^4+4h^3s^3+2h^2s^4-6h^3s^2-4h^2s^3+6h^2s^2+4h^2s+h^2-4hs-3h+2\big)K(\sqrt{h})\Big\}$$
$$>0$$

$$
\begin{aligned}
&\Big(\big(-2h^4s^4+2h^3s^4+4h^3s^3-2h^2s^4+4h^2s^3-12h^2s^2+4h^2s-2h^2+4hs+2h-2\big)E(\sqrt{h})\\
&+\big(h^4s^4-3h^3s^4+4h^3s^3+2h^2s^4-6h^3s^2-4h^2s^3+6h^2s^2+4h^2s+h^2-4hs-3h+2\big)K(\sqrt{h})\Big)\Pi(hs,\sqrt{h})\\
&+\big(-h^3s^3-h^2s^3+6h^2s^2-3h^2s+2h^2-3hs-2h+2\big)K(\sqrt{h})E(\sqrt{h})\\
&+\big(-h^3s^3+3h^3s^2+h^2s^3-3h^2s^2-3h^2s-h^2+3hs+3h-2\big)K(\sqrt{h})^2>0
\end{aligned}
$$

を示す.

$\Pi(hs,\sqrt{h})$ の係数のグラフ



$\Pi(hs,\sqrt{h})$ の係数は負であることを確認する.

両辺を $\Pi(hs,\sqrt{h})$ の係数で割り,さらに $-\dfrac{2\sqrt{2(1-s)(1-sh)}}{\sqrt{s}}$ をかける.

$$
\begin{aligned}
&-\frac{2\sqrt{(1-s)(1-sh)}}{\sqrt{s}}\cdot\Pi(hs,\sqrt{h})\\
&+\frac{2\sqrt{(1-s)(1-sh)}}{\sqrt{s}}\cdot\Big\{\big(h^3s^3+h^2s^3-6h^2s^2+3h^2s-2h^2+3hs+2h-2\big)K(\sqrt{h})E(\sqrt{h})\\
&+\big(h^3s^3-3h^3s^2-h^2s^3+3h^2s^2+3h^2s+h^2-3hs-3h+2\big)K(\sqrt{h})^2\Big\}\\
&\Big/\Big\{\big(-2h^4s^4+2h^3s^4+4h^3s^3-2h^2s^4+4h^2s^3-12h^2s^2+4h^2s-2h^2+4hs+2h-2\big)E(\sqrt{h})\\
&+\big(h^4s^4-3h^3s^4+4h^3s^3+2h^2s^4-6h^3s^2-4h^2s^3+6h^2s^2+4h^2s+h^2-4hs-3h+2\big)K(\sqrt{h})\Big\}\\
&>0
\end{aligned}
$$

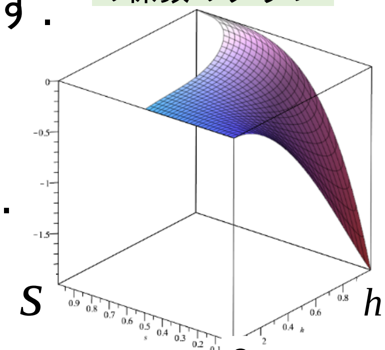
s で微分したものが

$K(\sqrt{h}),E(\sqrt{h})$ のみで書けるように変形した.

$$\Big(\big(-2h^4s^4+2h^3s^4+4h^3s^3-2h^2s^4+4h^2s^3-12h^2s^2+4h^2s-2h^2+4hs+2h-2\big)E(\sqrt{h})$$
$$+\big(h^4s^4-3h^3s^4+4h^3s^3+2h^2s^4-6h^3s^2-4h^2s^3+6h^2s^2+4h^2s+h^2-4hs-3h+2\big)K(\sqrt{h})\Big)\Pi(hs,\sqrt{h})$$
$$+\big(-h^3s^3-h^2s^3+6h^2s^2-3h^2s+2h^2-3hs-2h+2\big)K(\sqrt{h})E(\sqrt{h})$$
$$+\big(-h^3s^3+3h^3s^2+h^2s^3-3h^2s^2-3h^2s-h^2+3hs+3h-2\big)K(\sqrt{h})^2>0$$

を示す.

$\Pi(hs,\sqrt{h})$ の係数のグラフ



$\Pi(hs,\sqrt{h})$ の係数は負であることを確認する.

両辺を $\Pi(hs,\sqrt{h})$ の係数で割り,さらに $-\dfrac{2\sqrt{2(1-s)(1-sh)}}{\sqrt{s}}$ をかける.

$$f(h,s):=-\frac{2\sqrt{(1-s)(1-sh)}}{\sqrt{s}}\cdot\Pi(hs,\sqrt{h})$$
$$+\frac{2\sqrt{(1-s)(1-sh)}}{\sqrt{s}}\cdot\Big\{\big(h^3s^3+h^2s^3-6h^2s^2+3h^2s-2h^2+3hs+2h-2\big)K(\sqrt{h})E(\sqrt{h})$$
$$+\big(h^3s^3-3h^3s^2-h^2s^3+3h^2s^2+3h^2s+h^2-3hs-3h+2\big)K(\sqrt{h})^2\Big\}$$
$$\Big/\Big\{\big(-2h^4s^4+2h^3s^4+4h^3s^3-2h^2s^4+4h^2s^3-12h^2s^2+4h^2s-2h^2+4hs+2h-2\big)E(\sqrt{h})$$
$$+\big(h^4s^4-3h^3s^4+4h^3s^3+2h^2s^4-6h^3s^2-4h^2s^3+6h^2s^2+4h^2s+h^2-4hs-3h+2\big)K(\sqrt{h})\Big\}$$
$$>0$$

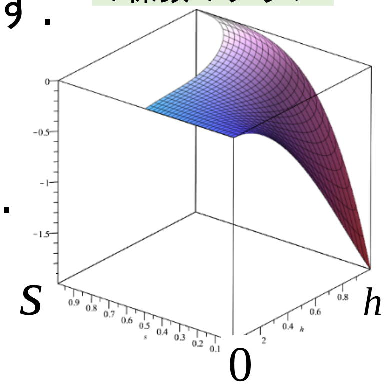
s で微分したものが

$K(\sqrt{h}),E(\sqrt{h})$ のみで書けるように変形した.

$$\Big(\big(-2h^4s^4+2h^3s^4+4h^3s^3-2h^2s^4+4h^2s^3-12h^2s^2+4h^2s-2h^2+4hs+2h-2\big)E(\sqrt{h})$$
$$+\big(h^4s^4-3h^3s^4+4h^3s^3+2h^2s^4-6h^3s^2-4h^2s^3+6h^2s^2+4h^2s+h^2-4hs-3h+2\big)K(\sqrt{h})\Big)\Pi(hs,\sqrt{h})$$
$$+\big(-h^3s^3-h^2s^3+6h^2s^2-3h^2s+2h^2-3hs-2h+2\big)K(\sqrt{h})E(\sqrt{h})$$
$$+\big(-h^3s^3+3h^3s^2+h^2s^3-3h^2s^2-3h^2s-h^2+3hs+3h-2\big)K(\sqrt{h})^2>0$$

を示す.

$\Pi(hs,\sqrt{h})$ の係数のグラフ



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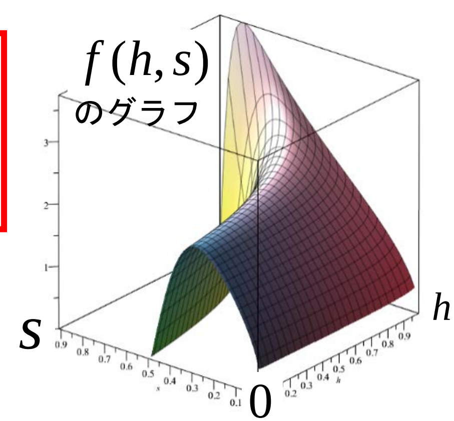
$$f(h,s):=-\frac{2\sqrt{(1-s)(1-sh)}}{\sqrt{s}}\cdot\Pi(hs,\sqrt{h})$$
$$+\frac{2\sqrt{(1-s)(1-sh)}}{\sqrt{s}}\cdot\Big\{\big(h^3s^3+h^2s^3-6h^2s^2+3h^2s-2h^2+3hs+2h-2\big)K(\sqrt{h})E(\sqrt{h})$$
$$+\big(h^3s^3-3h^3s^2-h^2s^3+3h^2s^2+3h^2s+h^2-3hs-3h+2\big)K(\sqrt{h})^2\Big\}$$
$$\Big/\Big\{\big(-2h^4s^4+2h^3s^4+4h^3s^3-2h^2s^4+4h^2s^3-12h^2s^2+4h^2s-2h^2+4hs+2h-2\big)E(\sqrt{h})$$
$$+\big(h^4s^4-3h^3s^4+4h^3s^3+2h^2s^4-6h^3s^2-4h^2s^3+6h^2s^2+4h^2s+h^2-4hs-3h+2\big)K(\sqrt{h})\Big\}$$
$$>0$$

s で微分したものが
$K(\sqrt{h}),E(\sqrt{h})$ のみで書けるように変形した.

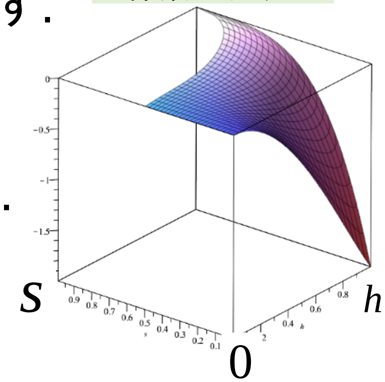


$$\left(\begin{array}{l}0<h<1,\\ 0<s<\dfrac{1}{1+\sqrt{1-h}}\end{array}\right)$$

$$\Big(\big(-2h^4s^4+2h^3s^4+4h^3s^3-2h^2s^4+4h^2s^3-12h^2s^2+4h^2s-2h^2+4hs+2h-2\big)E(\sqrt{h})$$
$$+\big(h^4s^4-3h^3s^4+4h^3s^3+2h^2s^4-6h^3s^2-4h^2s^3+6h^2s^2+4h^2s+h^2-4hs-3h+2\big)K(\sqrt{h})\Big)\Pi(hs,\sqrt{h})$$
$$+\big(-h^3s^3-h^2s^3+6h^2s^2-3h^2s+2h^2-3hs-2h+2\big)K(\sqrt{h})E(\sqrt{h})$$
$$+\big(-h^3s^3+3h^3s^2+h^2s^3-3h^2s^2-3h^2s-h^2+3hs+3h-2\big)K(\sqrt{h})^2>0$$

を示す.

$\Pi(hs,\sqrt{h})$ の係数のグラフ



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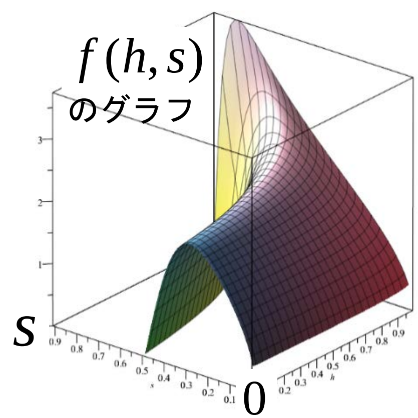
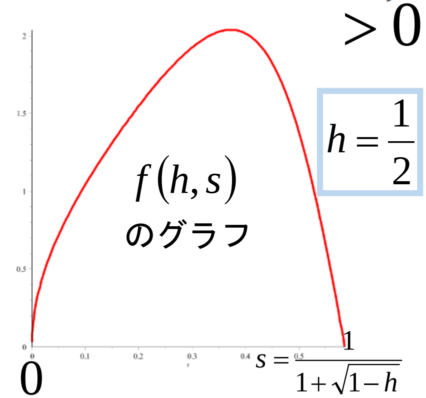
$$f(h,s):=-\frac{2\sqrt{(1-s)(1-sh)}}{\sqrt{s}}\cdot\Pi(hs,\sqrt{h})$$
$$+\frac{2\sqrt{(1-s)(1-sh)}}{\sqrt{s}}\cdot\Big\{\big(h^3s^3+h^2s^3-6h^2s^2+3h^2s-2h^2+3hs+2h-2\big)K(\sqrt{h})E(\sqrt{h})$$
$$+\big(h^3s^3-3h^3s^2-h^2s^3+3h^2s^2+3h^2s+h^2-3hs-3h+2\big)K(\sqrt{h})^2\Big\}$$
$$\Big/\Big\{\big(-2h^4s^4+2h^3s^4+4h^3s^3-2h^2s^4+4h^2s^3-12h^2s^2+4h^2s-2h^2+4hs+2h-2\big)E(\sqrt{h})$$
$$+\big(h^4s^4-3h^3s^4+4h^3s^3+2h^2s^4-6h^3s^2-4h^2s^3+6h^2s^2+4h^2s+h^2-4hs-3h+2\big)K(\sqrt{h})\Big\}$$
$$>0$$

以下, h を固定して s の関数と思う.

$$\lim_{s\to 0} f(h,0)>0,\quad f\left(h,\frac{1}{1+\sqrt{1-h}}\right)=0$$





hを固定して, s で偏微分する.

$f_s(h, s)$

hを固定して, s で偏微分する.

$$f_s(h,s) = 2\sqrt{2}\cdot\Big\{2\Big(h^2\left(h^2-h+1\right)s^4-2h^2\left(h+1\right)s^3+6h^2s^2-2h\left(h+1\right)s+h^2-h+1\Big)^2 E(\sqrt{h})^3$$

$$+\left(1-h\right)\Big(-3h^4\left(2-h\right)\left(h^2-h+1\right)s^8+6h^4\left(3h^2-3h+4\right)s^7-2h^4\left(23h^2-6h+25\right)s^6$$

$$+2h^3\left(16h^3+31h^2+41h-4\right)s^5-2h^2\left(8h^4+17h^3+87h^2-5h-2\right)s^4$$

$$+2h^2\left(16h^3+31h^2+41h-4\right)s^3-2h^2\left(23h^2-6h+25\right)s^2+6h\left(3h^2-3h+4\right)s$$

$$-3\left(2-h\right)\left(h^2-h+1\right)\Big)K(\sqrt{h})E(\sqrt{h})^2$$

$$+\left(1-h\right)^2\Big(3h^4\left(h^2-2h+2\right)s^8-12h^4\left(h^2-2h+2\right)s^7+4h^4\left(3h^2-4h+4\right)s^6$$

$$-4h^3\left(h^2+6h-10\right)s^5-2h^2\left(8h^3-33h^2+30h+10\right)s^4-4h^2\left(h^2+6h-10\right)s^3$$

$$+4h^2\left(3h^2-4h+4\right)s^2-12h\left(h^2-2h+2\right)s+3h^2-6h+6\Big)K(\sqrt{h})^2E(\sqrt{h})$$

$$+\left(1-h\right)^3\Big(h^4\left(h-2\right)s^8-2h^4\left(3h-4\right)s^7+2h^4\left(3h+1\right)s^6-2h^3\left(5h+12\right)s^5$$

$$-2h^2\left(2h^2-21h-6\right)s^4-2h^2\left(5h+12\right)s^3+2h^2\left(3h+1\right)s^2$$

$$-2h\left(3h-4\right)s+h-2\Big)K(\sqrt{h})^3\Big\}$$

$$\Big/\sqrt{s}\sqrt{\left(s-1\right)\left(hs-1\right)}\Big(\left(2h^4s^4-2h^3s^4-4h^3s^3+2h^2s^4-4h^2s^3+12h^2s^2-4h^2s\right.$$

$$\left.+2h^2-4hs-2h+2\right)E(\sqrt{h})+\left(-h^4s^4+3h^3s^4-4h^3s^3-2h^2s^4+6h^3s^2\right.$$

$$\left.+4h^2s^3-6h^2s^2-4h^2s-h^2+4hs+3h-2\right)K(\sqrt{h})\Big)^2$$

hを固定して, s で偏微分する.

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$$+ (1-h)\left(-3h^4(2-h)\left(h^2-h+1\right)s^8 + 6h^4\left(3h^2-3h+4\right)s^7 - 2h^4\left(23h^2-6h+25\right)s^6\right.$$
$$+ 2h^3\left(16h^3+31h^2+41h-4\right)s^5 - 2h^2\left(8h^4+17h^3+87h^2-5h-2\right)s^4$$
$$+ 2h^2\left(16h^3+31h^2+41h-4\right)s^3 - 2h^2\left(23h^2-6h+25\right)s^2 + 6h\left(3h^2-3h+4\right)s$$
$$\left.-3(2-h)\left(h^2-h+1\right)\right)K(\sqrt{h})E(\sqrt{h})^2$$
$$+ (1-h)^2\left(3h^4\left(h^2-2h+2\right)s^8 - 12h^4\left(h^2-2h+2\right)s^7 + 4h^4\left(3h^2-4h+4\right)s^6\right.$$
$$- 4h^3\left(h^2+6h-10\right)s^5 - 2h^2\left(8h^3-33h^2+30h+10\right)s^4 - 4h^2\left(h^2+6h-10\right)s^3$$
$$\left.+ 4h^2\left(3h^2-4h+4\right)s^2 - 12h\left(h^2-2h+2\right)s + 3h^2 - 6h + 6\right)K(\sqrt{h})^2E(\sqrt{h})$$
$$+ (1-h)^3\left(h^4(h-2)s^8 - 2h^4(3h-4)s^7 + 2h^4(3h+1)s^6 - 2h^3(5h+12)s^5\right.$$
$$- 2h^2\left(2h^2-21h-6\right)s^4 - 2h^2(5h+12)s^3 + 2h^2(3h+1)s^2$$
$$\left.-2h(3h-4)s + h - 2\right)K(\sqrt{h})^3\Big\}$$
$$\Big/ \sqrt{s}\sqrt{(s-1)(hs-1)}\Big(\left(2h^4s^4 - 2h^3s^4 - 4h^3s^3 + 2h^2s^4 - 4h^2s^3 + 12h^2s^2 - 4h^2s\right.$$
$$\left.+2h^2 - 4hs - 2h + 2\right)E(\sqrt{h}) + \left(-h^4s^4 + 3h^3s^4 - 4h^3s^3 - 2h^2s^4 + 6h^3s^2\right.$$
$$\left.+4h^2s^3 - 6h^2s^2 - 4h^2s - h^2 + 4hs + 3h - 2\right)K(\sqrt{h})\Big)^2$$

正

h を固定して, s で偏微分する.

$$f_s(h,s) = 2\sqrt{2} \cdot \Big\{ 2\left(h^2(h^2-h+1)s^4 - 2h^2(h+1)s^3 + 6h^2s^2 - 2h(h+1)s + h^2 - h + 1\right)^2 E(\sqrt{h})^3$$
$$+ (1-h)\left(-3h^4(2-h)(h^2-h+1)s^8 + 6h^4(3h^2-3h+4)s^7 - 2h^4(23h^2-6h+25)s^6\right.$$
$$+ 2h^3(16h^3+31h^2+41h-4)s^5 - 2h^2(8h^4+17h^3+87h^2-5h-2)s^4$$
$$+ 2h^2(16h^3+31h^2+41h-4)s^3 - 2h^2(23h^2-6h+25)s^2 + 6h(3h^2-3h+4)s$$
$$\left.-3(2-h)(h^2-h+1)\right)K(\sqrt{h})E(\sqrt{h})^2$$
$$+ (1-h)^2\left(3h^4(h^2-2h+2)s^8 - 12h^4(h^2-2h+2)s^7 + 4h^4(3h^2-4h+4)s^6\right.$$
$$- 4h^3(h^2+6h-10)s^5 - 2h^2(8h^3-33h^2+30h+10)s^4 - 4h^2(h^2+6h-10)s^3$$
$$\left.+ 4h^2(3h^2-4h+4)s^2 - 12h(h^2-2h+2)s + 3h^2 - 6h + 6\right)K(\sqrt{h})^2E(\sqrt{h})$$
$$+ (1-h)^3\left(h^4(h-2)s^8 - 2h^4(3h-4)s^7 + 2h^4(3h+1)s^6 - 2h^3(5h+12)s^5\right.$$
$$- 2h^2(2h^2-21h-6)s^4 - 2h^2(5h+12)s^3 + 2h^2(3h+1)s^2$$
$$\left.-2h(3h-4)s + h - 2\right)K(\sqrt{h})^3\Big\}$$
$$\Big/ \sqrt{s}\sqrt{(s-1)(hs-1)}\Big(\big(2h^4s^4 - 2h^3s^4 - 4h^3s^3 + 2h^2s^4 - 4h^2s^3 + 12h^2s^2 - 4h^2s$$
$$+2h^2 - 4hs - 2h + 2\big)E(\sqrt{h}) + \big(-h^4s^4 + 3h^3s^4 - 4h^3s^3 - 2h^2s^4 + 6h^3s^2$$
$$+4h^2s^3 - 6h^2s^2 - 4h^2s - h^2 + 4hs + 3h - 2\big)K(\sqrt{h})\Big)^2$$

この分子が正から負に1回だけ
符号変化することを示せばよい

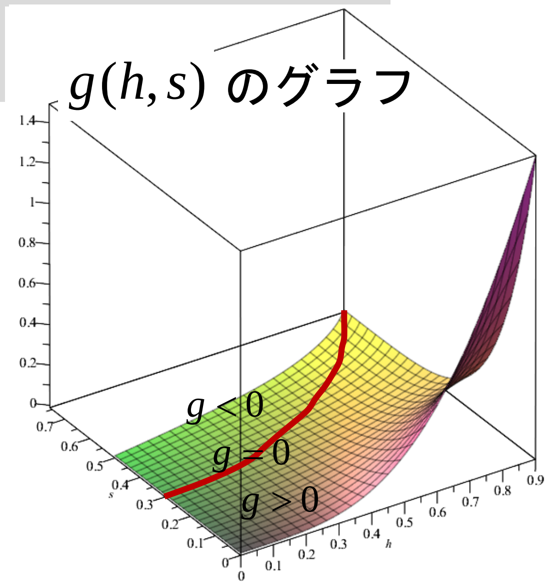
正

$$
\begin{aligned}
g(h,s) :=\ &\Big\{2\Big(h^2\left(h^2-h+1\right)s^4-2h^2\left(h+1\right)s^3+6h^2s^2-2h\left(h+1\right)s+h^2-h+1\Big)^2E(\sqrt{h})^3\\
&+(1-h)\Big(-3h^4(2-h)\left(h^2-h+1\right)s^8+6h^4\left(3h^2-3h+4\right)s^7-2h^4\left(23h^2-6h+25\right)s^6\\
&+2h^3\left(16h^3+31h^2+41h-4\right)s^5-2h^2\left(8h^4+17h^3+87h^2-5h-2\right)s^4\\
&+2h^2\left(16h^3+31h^2+41h-4\right)s^3-2h^2\left(23h^2-6h+25\right)s^2+6h\left(3h^2-3h+4\right)s\\
&-3(2-h)\left(h^2-h+1\right)\Big)K(\sqrt{h})E(\sqrt{h})^2\\
&+(1-h)^2\Big(3h^4\left(h^2-2h+2\right)s^8-12h^4\left(h^2-2h+2\right)s^7+4h^4\left(3h^2-4h+4\right)s^6\\
&-4h^3\left(h^2+6h-10\right)s^5-2h^2\left(8h^3-33h^2+30h+10\right)s^4-4h^2\left(h^2+6h-10\right)s^3\\
&+4h^2\left(3h^2-4h+4\right)s^2-12h\left(h^2-2h+2\right)s+3h^2-6h+6\Big)K(\sqrt{h})^2E(\sqrt{h})\\
&+(1-h)^3\Big(h^4(h-2)s^8-2h^4(3h-4)s^7+2h^4(3h+1)s^6-2h^3(5h+12)s^5\\
&-2h^2\left(2h^2-21h-6\right)s^4-2h^2(5h+12)s^3+2h^2(3h+1)s^2\\
&-2h(3h-4)s+h-2\Big)K(\sqrt{h})^3\Big\}
\end{aligned}
$$

これが正から負に1回だけ符号変化
することを示せばよい

$$
\begin{aligned}
g(h,s) := \Big\{ & 2\Big(h^2\left(h^2-h+1\right)s^4-2h^2\left(h+1\right)s^3+6h^2s^2-2h\left(h+1\right)s+h^2-h+1\Big)^2 E(\sqrt{h})^3 \\
& +\left(1-h\right)\Big(-3h^4\left(2-h\right)\left(h^2-h+1\right)s^8+6h^4\left(3h^2-3h+4\right)s^7-2h^4\left(23h^2-6h+25\right)s^6 \\
& +2h^3\left(16h^3+31h^2+41h-4\right)s^5-2h^2\left(8h^4+17h^3+87h^2-5h-2\right)s^4 \\
& +2h^2\left(16h^3+31h^2+41h-4\right)s^3-2h^2\left(23h^2-6h+25\right)s^2+6h\left(3h^2-3h+4\right)s \\
& -3\left(2-h\right)\left(h^2-h+1\right)\Big)K(\sqrt{h})E(\sqrt{h})^2 \\
& +\left(1-h\right)^2\Big(3h^4\left(h^2-2h+2\right)s^8-12h^4\left(h^2-2h+2\right)s^7+4h^4\left(3h^2-4h+4\right)s^6 \\
& -4h^3\left(h^2+6h-10\right)s^5-2h^2\left(8h^3-33h^2+30h+10\right)s^4-4h^2\left(h^2+6h-10\right)s^3 \\
& +4h^2\left(3h^2-4h+4\right)s^2-12h\left(h^2-2h+2\right)s+3h^2-6h+6\Big)K(\sqrt{h})^2E(\sqrt{h}) \\
& +\left(1-h\right)^3\Big(h^4\left(h-2\right)s^8-2h^4\left(3h-4\right)s^7+2h^4\left(3h+1\right)s^6-2h^3\left(5h+12\right)s^5 \\
& -2h^2\left(2h^2-21h-6\right)s^4-2h^2\left(5h+12\right)s^3+2h^2\left(3h+1\right)s^2 \\
& -2h\left(3h-4\right)s+h-2\Big)K(\sqrt{h})^3\Big\}
\end{aligned}
$$

これが正から負に1回だけ符号変化することを示せばよい



$$
\begin{aligned}
g(h,s) := \Bigl\{ & 2\left(h^2\left(h^2-h+1\right)s^4-2h^2(h+1)s^3+6h^2s^2-2h(h+1)s+h^2-h+1\right)^2 E(\sqrt{h})^3 \\
& +(1-h)\Bigl(-3h^4(2-h)\left(h^2-h+1\right)s^8+6h^4\left(3h^2-3h+4\right)s^7-2h^4\left(23h^2-6h+25\right)s^6 \\
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& -3(2-h)\left(h^2-h+1\right)\Bigr)K(\sqrt{h})E(\sqrt{h})^2 \\
& +(1-h)^2\Bigl(3h^4\left(h^2-2h+2\right)s^8-12h^4\left(h^2-2h+2\right)s^7+4h^4\left(3h^2-4h+4\right)s^6 \\
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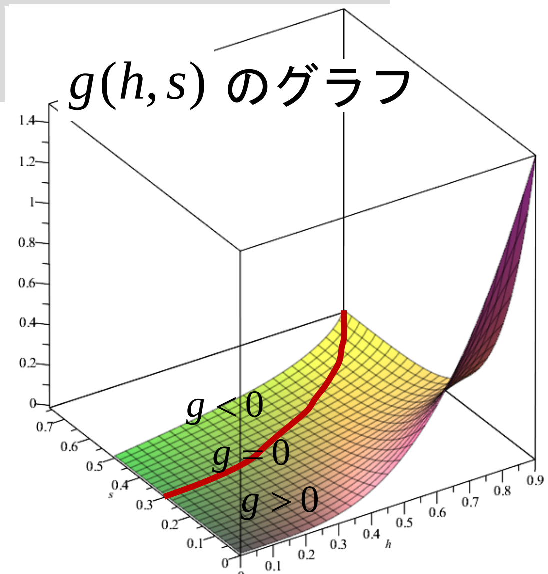
& +(1-h)^3\Bigl(h^4(h-2)s^8-2h^4(3h-4)s^7+2h^4(3h+1)s^6-2h^3(5h+12)s^5 \\
& -2h^2\left(2h^2-21h-6\right)s^4-2h^2(5h+12)s^3+2h^2(3h+1)s^2 \\
& -2h(3h-4)s+h-2\Bigr)K(\sqrt{h})^3\Bigr\}
\end{aligned}
$$

これが正から負に1回だけ符号変化
することを示せばよい

$h\in(0,1)$ をとめるごとに

$$g(h,s)=0 \quad \left(0<s<\frac{1}{1+\sqrt{1-h}}\right)$$

となる s がただ一つ存在することを示す.



フーリエの定理　$p(s)$ を実係数の n 次の多項式とする. $p(s)$ の導関数の列

$$p(s),\ p^{(1)}(s),\ p^{(2)}(s), \cdots, p^{(n)}(s)$$

の s における符号変化数を $v(s)$ とおく. $p(a) \neq 0,\ p(b) \neq 0$ ならば,
区間 (a,b) における $p(s)=0$ の実根の個数は $v(a)-v(b)$ に等しいか,
またそれより偶数だけ少ない.

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系　$v(a)-v(b)=1$ のとき,

区間 (a,b) における $p(s)=0$ の実根の個数は1である.

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<u>系</u>　$v(a)-v(b)=1$ のとき,

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各 $h \in (0,1)$ を固定するごとに, s についての 8次多項式 $g(h,s)$ に対して,

区間 $\left(0, \dfrac{1}{1+\sqrt{1-h}}\right)$ において上記の系を適用する. したがって,

<u>フーリエの定理</u>　$p(s)$ を実係数の n 次の多項式とする. $p(s)$ の導関数の列

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$v(0):=$ $g(h,0),\ g_s^{(1)}(h,0),\cdots,g_s^{(8)}(h,0)$の符号変化数

$v\left(\dfrac{1}{1+\sqrt{1-h}}\right):=$ $g\left(h,\dfrac{1}{1+\sqrt{1-h}}\right),\ g_s^{(1)}\left(h,\dfrac{1}{1+\sqrt{1-h}}\right),\cdots,\ g_s^{(8)}\left(h,\dfrac{1}{1+\sqrt{1-h}}\right)$の符号変化数

とするとき

$$v(0)-v\left(\frac{1}{1+\sqrt{1-h}}\right)=1$$

を示せばよい.

各 $h \in (0,1)$ を固定するごとに, s についての 8次多項式 $g(h,s)$ に対する,

符号変化数 $v(0)$, $v\left(\frac{1}{1+\sqrt{1-h}}\right)$ を調べ, $v(0)-v\left(\frac{1}{1+\sqrt{1-h}}\right)=1$ を確認する.

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	$s=0$
	$0<h<1$
g	$+$
$g_s^{(1)}$	$-$
$g_s^{(2)}$	$+$
$g_s^{(3)}$	$-$
$g_s^{(4)}$	$+$
$g_s^{(5)}$	$-$
$g_s^{(6)}$	$+$
$g_s^{(7)}$	$-$
$g_s^{(8)}$	$+$
符号変化数	8

$s=\frac{1}{1+\sqrt{1-h}}$								
0	$\cdots$	h_1	$\cdots$	h_2	$\cdots$	h_3	$\cdots$	1
	$-$	$-$	$-$	$-$	$-$	$-$	$-$	
	$+$	$+$	$+$	$+$	$+$	$+$	$+$	
	$+$	0	$-$	$-$	$-$	$-$	$-$	
	$-$	$-$	$-$	0	$+$	$+$	$+$	
	$+$	$+$	$+$	$+$	$+$	0	$-$	
	$-$	$-$	$-$	$-$	$-$	$-$	$-$	
	$+$	$+$	$+$	$+$	$+$	$+$	$+$	
	$-$	$-$	$-$	$-$	$-$	$-$	$-$	
	$+$	$+$	$+$	$+$	$+$	$+$	$+$	
	7	7	7	7	7	7	7	

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符号変化数 $v(0)$, $v\left(\frac{1}{1+\sqrt{1-h}}\right)$ を調べ, $v(0)-v\left(\frac{1}{1+\sqrt{1-h}}\right)=1$ を確認する.

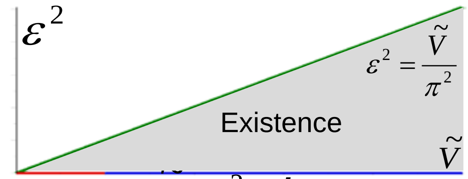
	$s=0$
	$0<h<1$
g	$+$
$g_s^{(1)}$	$-$
$g_s^{(2)}$	$+$
$g_s^{(3)}$	$-$
$g_s^{(4)}$	$+$
$g_s^{(5)}$	$-$
$g_s^{(6)}$	$+$
$g_s^{(7)}$	$-$
$g_s^{(8)}$	$+$
符号変化数	8

$s=\frac{1}{1+\sqrt{1-h}}$								
0	$\cdots$	h_1	$\cdots$	h_2	$\cdots$	h_3	$\cdots$	1
	$-$	$-$	$-$	$-$	$-$	$-$	$-$	
	$+$	$+$	$+$	$+$	$+$	$+$	$+$	
	$+$	0	$-$	$-$	$-$	$-$	$-$	
	$-$	$-$	$-$	0	$+$	$+$	$+$	
	$+$	$+$	$+$	$+$	$+$	0	$-$	
	$-$	$-$	$-$	$-$	$-$	$-$	$-$	
	$+$	$+$	$+$	$+$	$+$	$+$	$+$	
	$-$	$-$	$-$	$-$	$-$	$-$	$-$	
	$+$	$+$	$+$	$+$	$+$	$+$	$+$	
	7	7	7	7	7	7	7	

$$v(0)-v\left(\frac{1}{1+\sqrt{1-h}}\right)=8-7=1$$

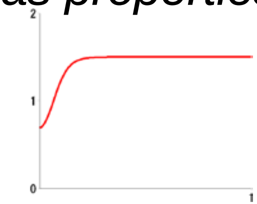
Theorem 1 *Let $\tilde{V} > 0$. There exists a solution of $(AP;\tilde{V})$, if and only if $(\tilde{V},\, \varepsilon^2) \in G$, where*

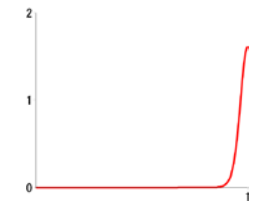
$$G := \left\{ (\tilde{V},\, \varepsilon^2) : 0 < \varepsilon^2 < \frac{\tilde{V}}{\pi^2} \right\}.$$



Moreover, the solution is unique. The solution $W(x;\tilde{V},\varepsilon^2)$ has properties

$$0 < W(x;\tilde{V},\varepsilon^2) < \tilde{V}+1, \qquad W(x;\tilde{V},\varepsilon^2) = \tilde{V}+1-\tilde{V}\cdot W\left(1-x\,;\, \frac{1}{\tilde{V}},\, \frac{\varepsilon^2}{\tilde{V}^2}\right).$$



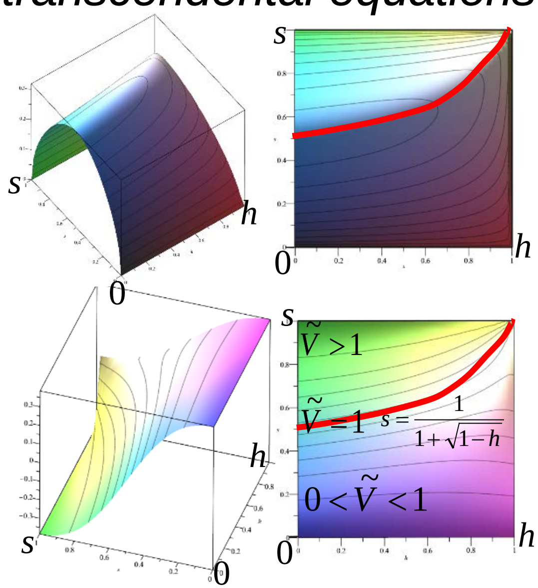


The solution $W(x;\tilde{V},\varepsilon^2)$ is represented by

$$W(x;\tilde{V},\varepsilon^2) = \frac{\tilde{V}+2}{3} + \frac{1}{\sqrt{3}} \cdot \sqrt{\tilde{V}^2+\tilde{V}+1} \cdot \frac{\beta\cdot(1-hs)\mathrm{sn}^2(K(\sqrt{h})x,\sqrt{h}) + \alpha\cdot \mathrm{cn}^2(K(\sqrt{h})x,\sqrt{h})}{(1-hs)\mathrm{sn}^2(K(\sqrt{h})x,\sqrt{h}) + \mathrm{cn}^2(K(\sqrt{h})x,\sqrt{h})},$$

$$\alpha := \frac{3hs^2-2(1+h)s+1}{\sqrt{3h^2s^4-4(h^2+h)s^3+(4h^2+2h+4)s^2-4(h+1)s+3}}, \quad \beta := \frac{-hs^2-2(1-h)s+1}{\sqrt{3h^2s^4-4(h^2+h)s^3+(4h^2+2h+4)s^2-4(h+1)s+3}},$$

where $(h,\, s) = (h(\tilde{V},\, \varepsilon^2),\, s(\tilde{V},\, \varepsilon^2))$ is the unique solution of the following system of transcendental equations



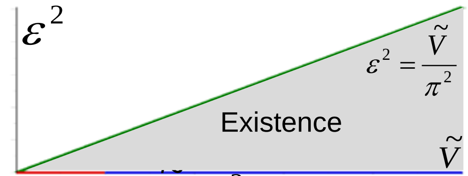
$$\begin{cases} \dfrac{\sqrt{2s(1-s)(1-sh)}\big/K(\sqrt{h})}{\sqrt{3h^2s^4-4(h^2+h)s^3+(4h^2+2h+4)s^2-4(h+1)s+3}} = \sqrt{3}\cdot\dfrac{\varepsilon}{\sqrt{\tilde{V}^2+\tilde{V}+1}}, \\[2ex] \dfrac{2(hs^2-sh+1)(hs^2-2s+1)(1-hs^2)}{\sqrt{3h^2s^4-4(h^2+h)s^3+(4h^2+2h+4)s^2-4(h+1)s+3}^{\,3}} = \dfrac{1}{3\sqrt{3}}\cdot\dfrac{(1-\tilde{V})(2\tilde{V}+1)(\tilde{V}+2)}{\sqrt{\tilde{V}^2+\tilde{V}+1}^{\,3}}, \\[2ex] 0 < h < 1, \quad 0 < s < 1. \end{cases}$$

Here, $sn(\cdot,\cdot), cn(\cdot,\cdot)$ are Jacobi's elliptic function,

$K(\cdot)$ is complete elliptic integral of the 1st kind.

Theorem 1 *Let $\tilde{V} > 0$. There exists a solution of* $(AP;\tilde{V})$, *if and only if* $(\tilde{V},\, \varepsilon^2) \in G$, *where*

$$G := \left\{ (\tilde{V},\, \varepsilon^2) : 0 < \varepsilon^2 < \frac{\tilde{V}}{\pi^2} \right\}.$$



Moreover, the solution is unique. The solution $W(x;\tilde{V},\varepsilon^2)$ *has properties*

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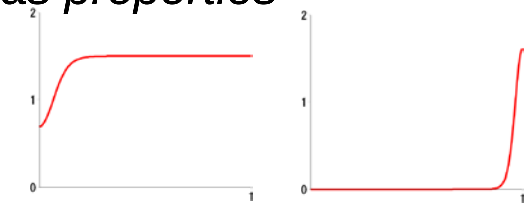

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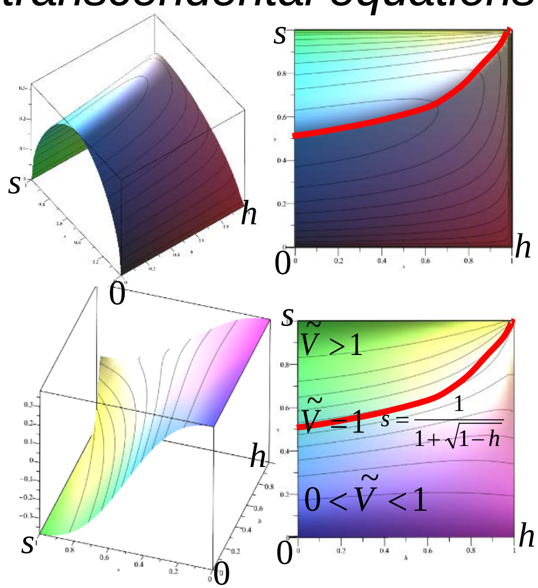
where $(h,\, s) = (h(\tilde{V},\, \varepsilon^2),\, s(\tilde{V},\, \varepsilon^2))$ *is the unique solution of the following system of transcendental equations*



$$\begin{cases} \dfrac{\sqrt{2s(1-s)(1-sh)}\,/\,K(\sqrt{h})}{\sqrt{3h^2s^4-4(h^2+h)s^3+(4h^2+2h+4)s^2-4(h+1)s+3}} = \sqrt{3}\cdot\dfrac{\varepsilon}{\sqrt{\tilde{V}^2+\tilde{V}+1}}, \\[2ex] \dfrac{2(hs^2-sh+1)(hs^2-2s+1)(1-hs^2)}{\sqrt{3h^2s^4-4(h^2+h)s^3+(4h^2+2h+4)s^2-4(h+1)s+3}^{\,3}} = \dfrac{1}{3\sqrt{3}}\cdot\dfrac{(1-\tilde{V})(2\tilde{V}+1)(\tilde{V}+2)}{\sqrt{\tilde{V}^2+\tilde{V}+1}^{\,3}}, \\[2ex] 0 < h < 1, \quad 0 < s < 1. \end{cases}$$

POINT : Parameterization for all W in $(h,s) \in (0,1)\times(0,1)$

Here, $sn(\cdot,\cdot), cn(\cdot,\cdot)$ *are Jacobi's elliptic function,*

$K(\cdot)$ *is complete elliptic integral of the 1st kind.*

Theorem 2 *Let* $W(x;\tilde{V},\varepsilon^2)$ *be the unique solution of* $(AP;\tilde{V})$*, and*

$$m(\tilde{V},\varepsilon^2) := \int_0^1 W(x;\tilde{V},\varepsilon^2)\,dx + \tilde{V},$$

then

$$m(\tilde{V},\varepsilon^2) = 2\tilde{V} + 2 - \tilde{V}\cdot m\left(\frac{1}{\tilde{V}}, \frac{\varepsilon^2}{\tilde{V}^2}\right) \quad \text{for any} \quad \tilde{V} > 0,\ \ \varepsilon > 0.$$

In particular, $m(1,\varepsilon^2) = 2$ *for any* $\varepsilon > 0.$

Theorem 2 *Let* $W(x;\tilde{V},\varepsilon^2)$ *be the unique solution of* $(AP;\tilde{V})$, *and*

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In particular, $m(1,\varepsilon^2) = 2$ *for any* $\varepsilon > 0.$

Moreover, it holds that

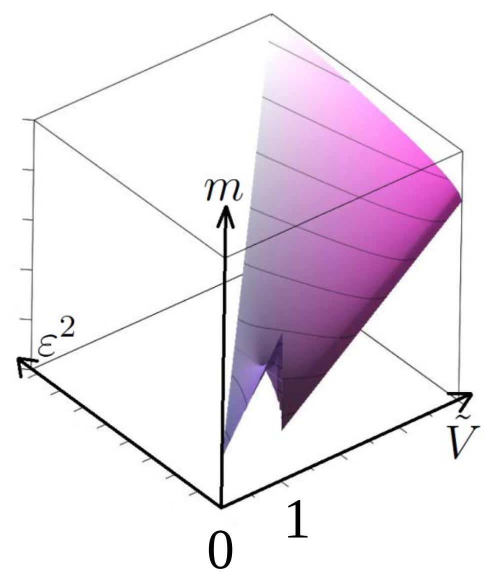
$$m(\tilde{V},\varepsilon^2) := \frac{4\tilde{V}+2}{3} + \frac{1}{\sqrt{3}}\cdot\sqrt{\tilde{V}^2+\tilde{V}+1}\cdot M(h,s)$$

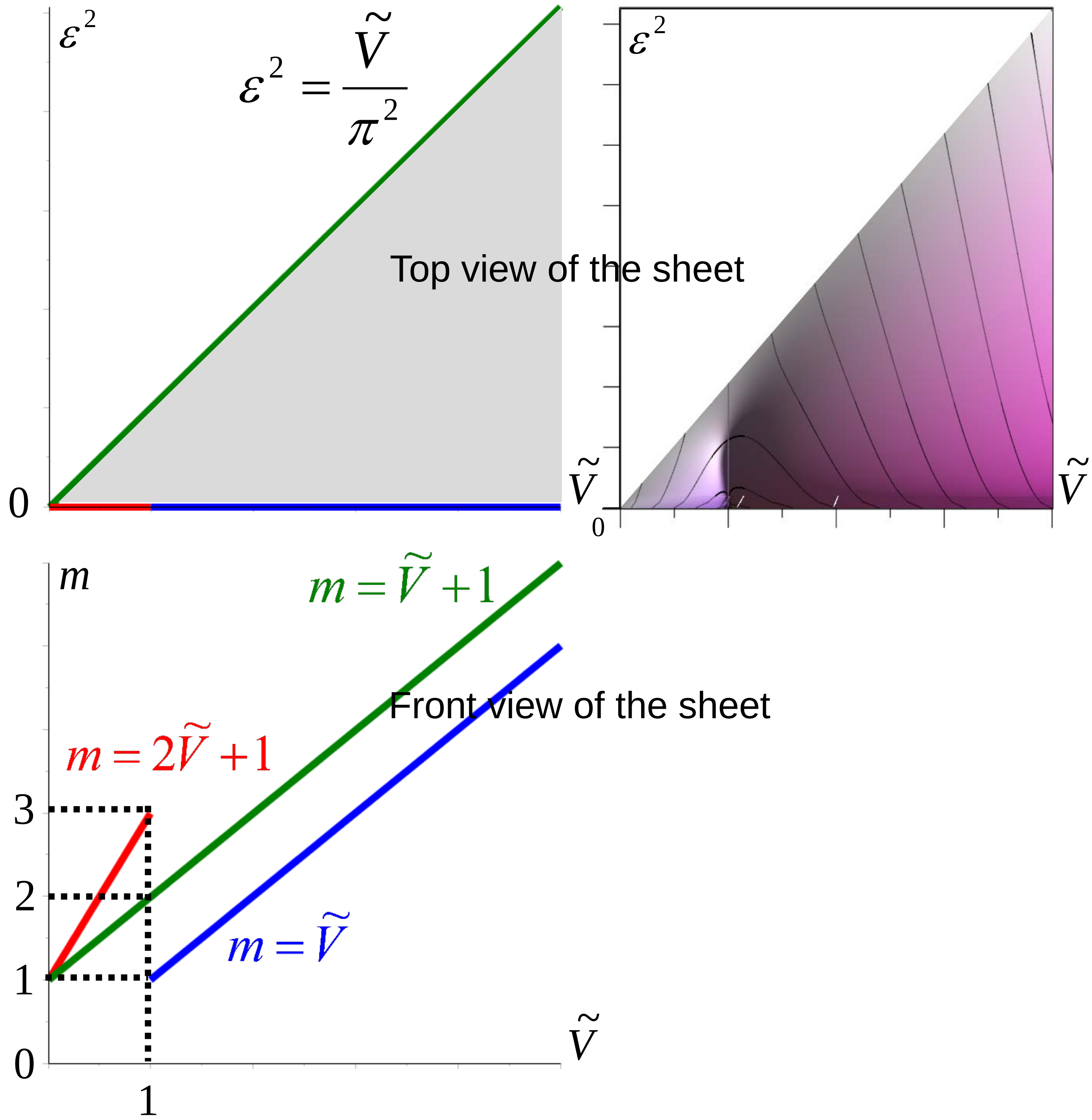
$$M(h,\, s) := \frac{-(hs^2 - 2(1+h)s + 3) + 4(1-s)(1-sh)\Pi(-sh,\sqrt{h})\big/K(\sqrt{h})}{\sqrt{3h^2s^4 - 4(h^2+h)s^3 + (4h^2+2h+4)s^2 - 4(h+1)s + 3}}.$$

where $h = h(\tilde{V},\, \varepsilon^2),\ \ s = s(\tilde{V},\, \varepsilon^2)$ *are given in Theorem 1.*

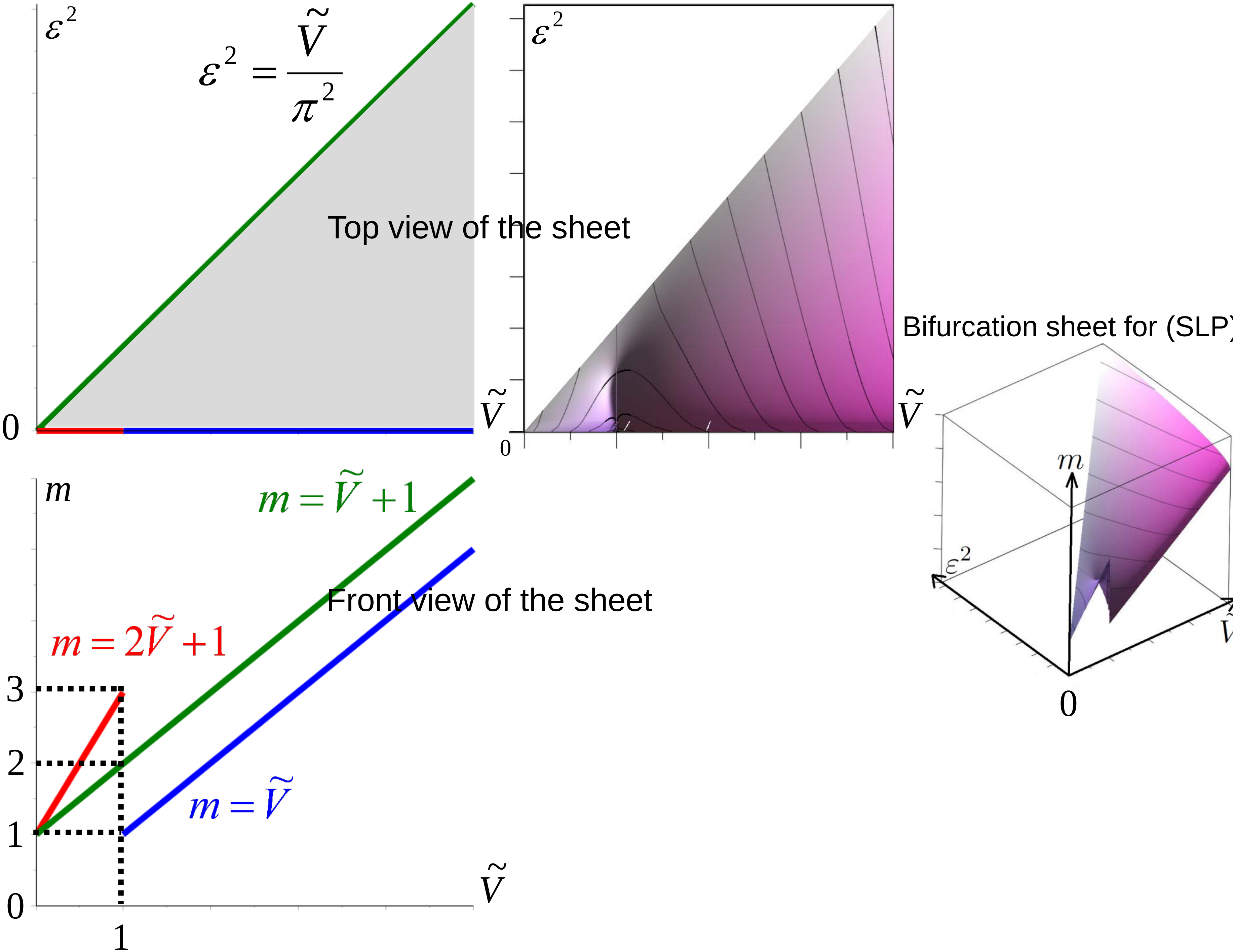
Here, $K(\cdot)$ *is complete elliptic integral of the 1st kind,*

$\Pi(\cdot,\cdot)$ *is complete elliptic integral of the 3rd kind.*

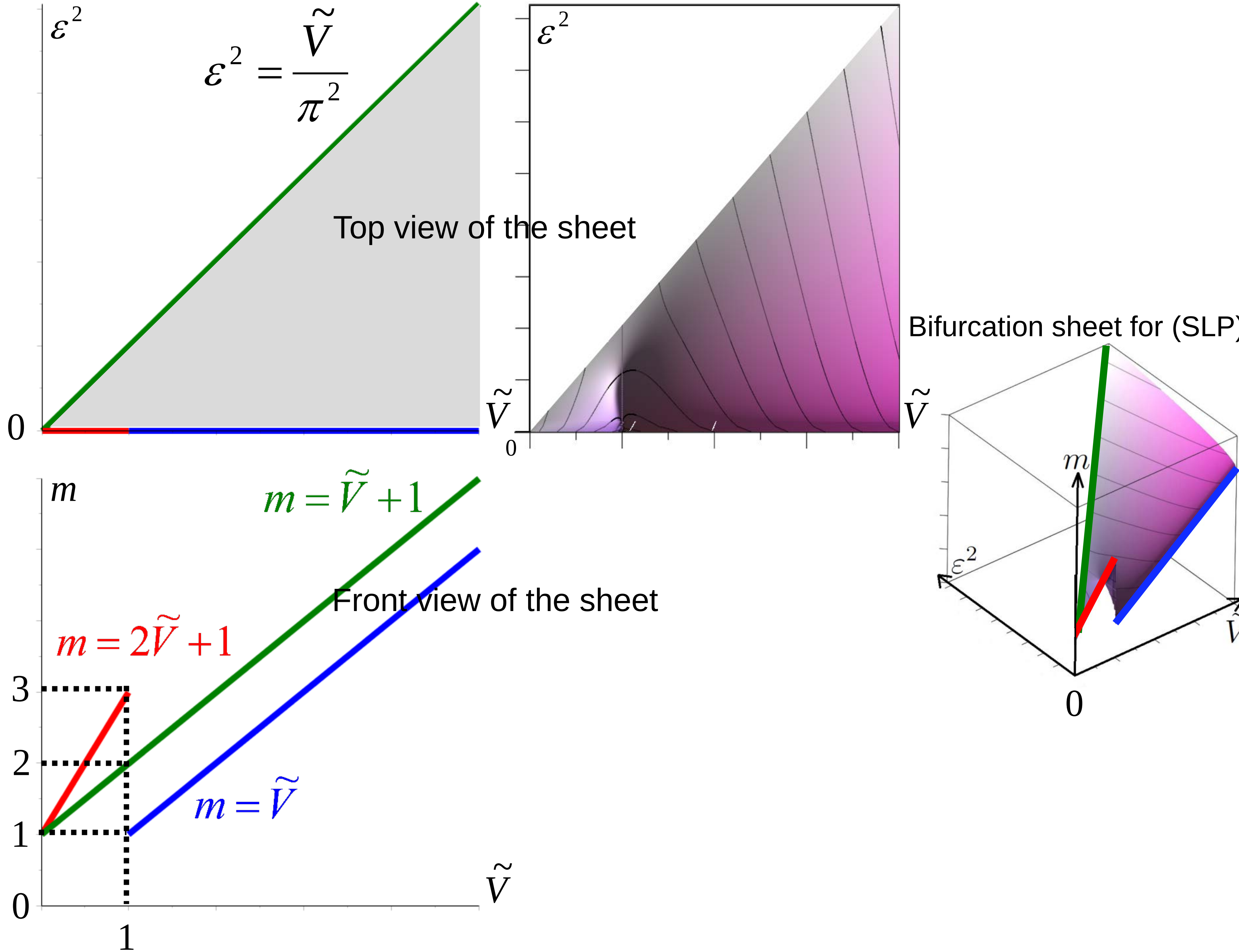




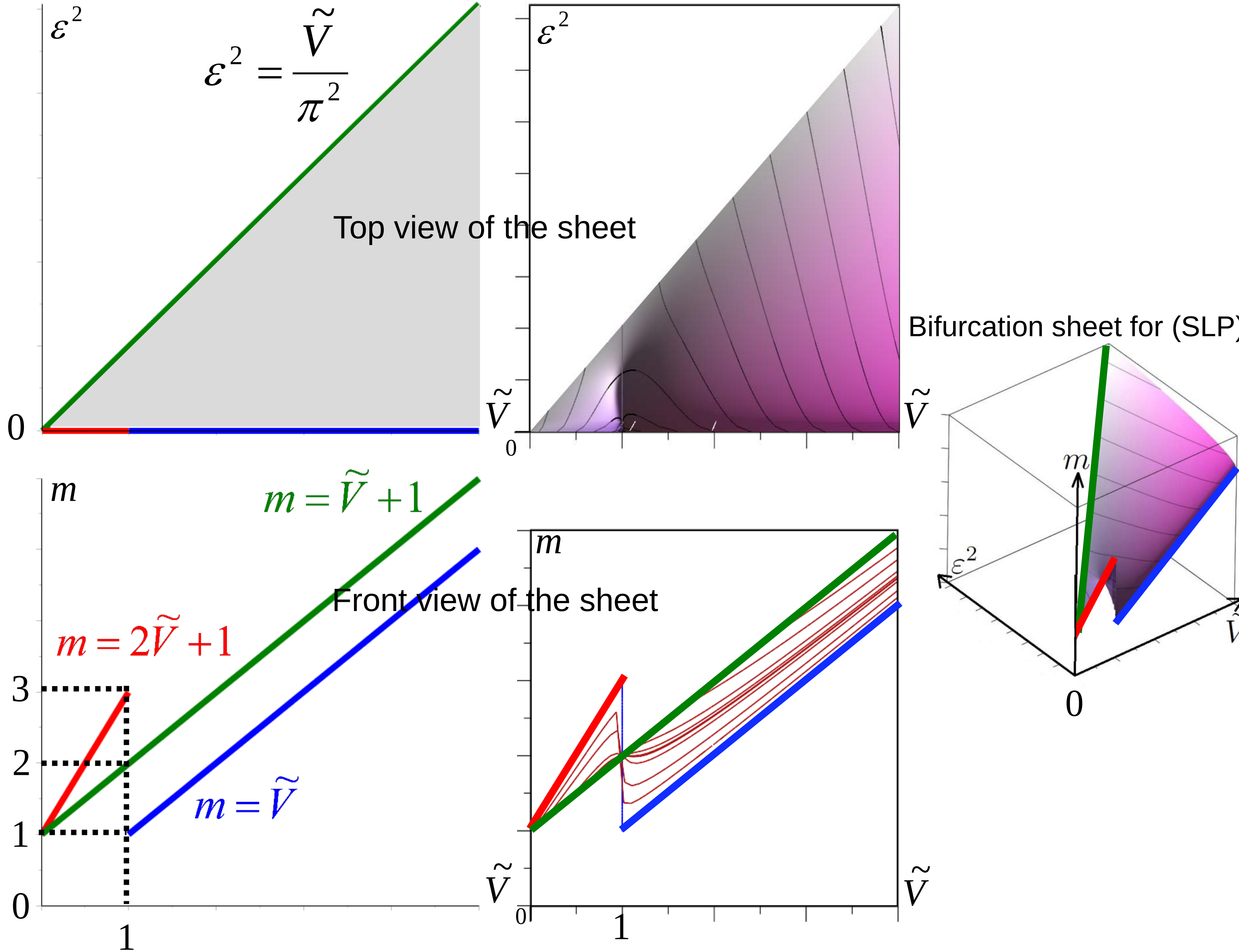
ε^2
$\varepsilon^2 = \frac{\tilde{V}}{\pi^2}$
0
$\tilde{V}$
Top view of the sheet
ε^2
0
$\tilde{V}$
m
$m = \tilde{V} + 1$
Front view of the sheet
$m = 2\tilde{V} + 1$
$m = \tilde{V}$
3
2
1
0
1
$\tilde{V}$



ε²
ε² = Ṽ/π²
0
Top view of the sheet
ε²
Ṽ
0
Ṽ
Bifurcation sheet for (SLP)
m
ε²
Ṽ
0
m
m = Ṽ + 1
Front view of the sheet
m = 2Ṽ + 1
3
2
1
0
m = Ṽ
Ṽ
1



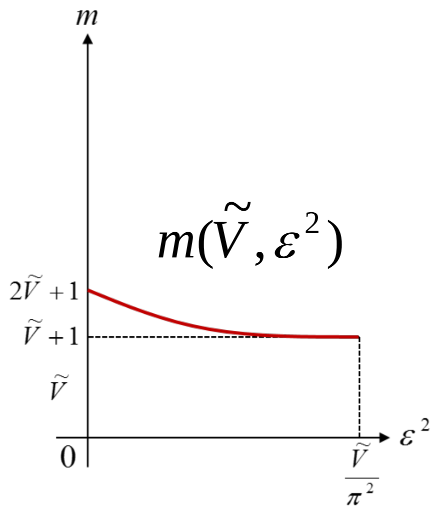
ε^2
$\varepsilon^2 = \frac{\tilde{V}}{\pi^2}$
0
$\tilde{V}$
Top view of the sheet
ε^2
0
$\tilde{V}$
Bifurcation sheet for (SLP)
m
ε^2
$\tilde{V}$
0
m
$m = \tilde{V} + 1$
Front view of the sheet
$m = 2\tilde{V} + 1$
3
2
1
0
$m = \tilde{V}$
$\tilde{V}$
1



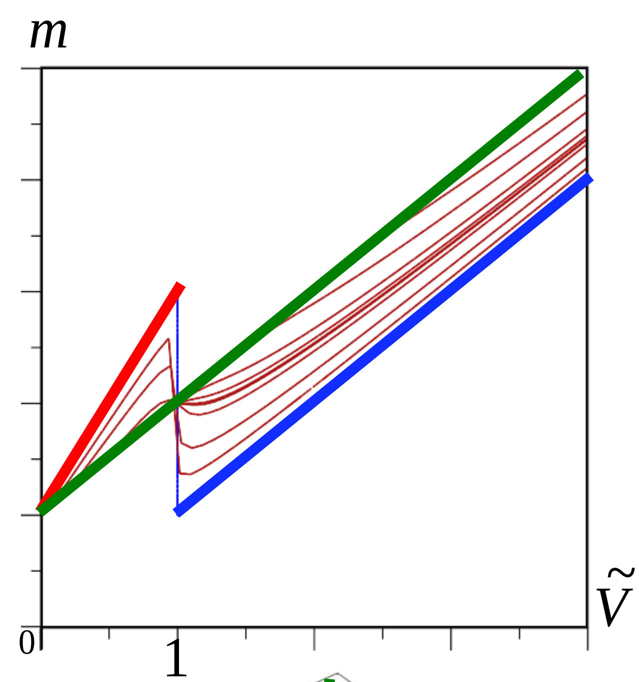
ε^2
$\varepsilon^2 = \frac{\tilde{V}}{\pi^2}$
Top view of the sheet
Bifurcation sheet for (SLP)
$m = \tilde{V} + 1$
$m = 2\tilde{V} + 1$
$m = \tilde{V}$
Front view of the sheet
m
$\tilde{V}$
0
1
2
3

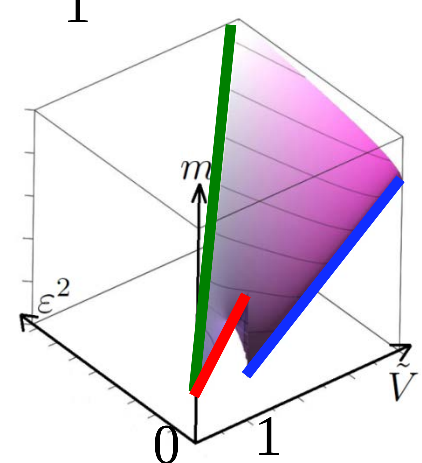
Theorem3 *Let* $\tilde{V} > 0$ *be fixed. It holds that*

$$\frac{\partial m(\tilde{V}, \varepsilon^2)}{\partial \varepsilon} < 0 \quad \textit{for} \quad \varepsilon^2 \in \left(0, \frac{\tilde{V}}{\pi^2}\right) \textit{ with } \tilde{V} \in (0, 1),$$



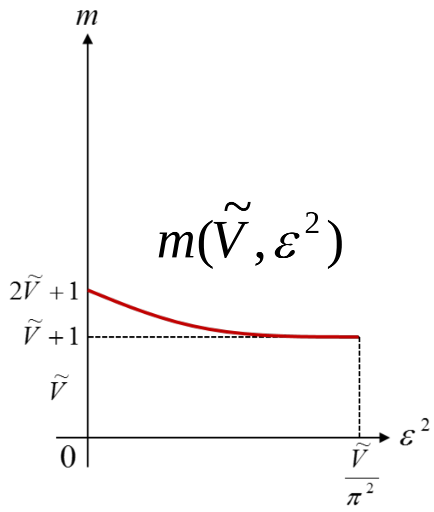
$\tilde{V} \in (0, 1)$ *be the fixed.*





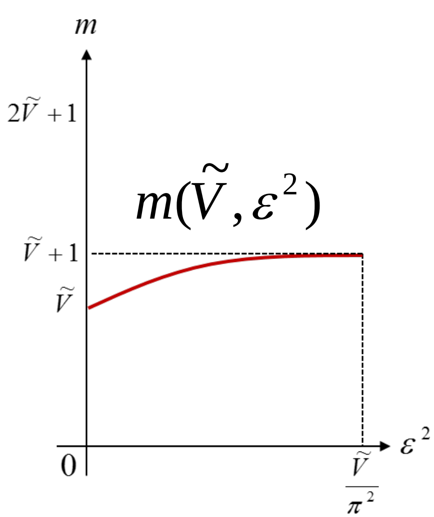
Theorem 3 *Let $\tilde{V} > 0$ be fixed. It holds that*

$$\frac{\partial m(\tilde{V}, \varepsilon^2)}{\partial \varepsilon} < 0 \quad \textit{for} \quad \varepsilon^2 \in \left(0, \frac{\tilde{V}}{\pi^2}\right) \quad \textit{with} \quad \tilde{V} \in (0, 1),$$



$\tilde{V} \in (0, 1)$ *be the fixed.*

$$\frac{\partial m(\tilde{V}, \varepsilon^2)}{\partial \varepsilon} > 0 \quad \textit{for} \quad \varepsilon^2 \in \left(0, \frac{\tilde{V}}{\pi^2}\right) \quad \textit{with} \quad \tilde{V} \in (1, \infty).$$



$\tilde{V} \in (1, \infty)$ *be the fixed.*