偏微分方程式レクチャーシリーズ
in 福岡工業大学　　2014. 5/17−18

Asymptotic behavior of solutions to nonlinear partial differential equations with dissipation

1. Introduction

generalized cubic double dispersion equation

(1) $$\begin{cases} (1-\Delta)u_{tt} + \Delta^2 u - \mu\Delta u_t - a^2\Delta u = \Delta f(u) \\ u(0) = u_0, \quad u_t(0) = u_1 := \Lambda v_1 \qquad t \geq 0,\ x \in \mathbb{R}^n \end{cases}$$

$\mu > 0$, $a > 0$: constants,

$f(u) = O(u^2)$ for $u \to 0$, $f(u)$: smooth

$\Lambda u = \mathcal{F}^{-1}[i|\xi|\hat{u}]$ $\qquad \Lambda = i(-\Delta)^{1/2}$, $\Lambda^2 = \Delta$

waveguide (導波管) wave propagation in waveguide

$$u_{tt} - u_{xx} = \frac{1}{4}(\gamma u^3 + 6u^2 + \alpha u_{tt} - \beta u_{xx} + \delta u_t)_{xx}$$

$u = v_x$: strain, v : longitudinal displacement
縦方向

Aim

1. Global existence and decay estimate

$n \geqq 1$, $s \geqq [\frac{n}{2}]$, $E_1 = \|(u_0, U_1)\|_{H^{s+1} \cap L^1}$: small

$\|\partial_x^k u(t)\|_{L^2} \leqq CE_1(1+t)^{-\frac{n}{4}-\frac{k}{2}}$, $0 \leqq k \leqq s+1$

2. Asymptotic behavior for $n \geqq 2$

$u \sim w^+ + w^-$

$w^\pm(x,t) = M_\pm K_\pm(x,t)$: linear diffusion waves

$K_\pm(x,t) = \mathcal{F}^{-1}[e^{-\frac{\mu}{2}|\xi|^2 t} e^{\pm a i|\xi| t}](x)$

$M_\pm = \frac{1}{2}\int_{\mathbb{R}^n} (u_0 + \frac{1}{a}U_1)(x)\,dx$

$w := K_\pm$ satisfies $w_t \mp a\Lambda w = \frac{\mu}{2}\Delta w$

3. Asymptotic behavior for $n=1$,

$f(u) = \frac{b}{2}u^2 + O(u^3)$, $u_1 = \partial_x U_1$

$u \sim w^+ + w^-$

$w^\pm(x,t) = \frac{1}{b_\pm}\frac{1}{\sqrt{t+1}}\Phi\left(\frac{x \pm a(t+1)}{\sqrt{t+1}};\ b_\pm M_\pm\right)$,

$b_\pm = \mp\frac{b}{2a}$ nonlinear diffusion waves

$w := w^{\pm}$ satisfies

$$w_t \mp a w_x + \left(\frac{1}{2} b_{\pm} w^2\right)_x = \frac{\mu}{2} w_{xx}$$

$z := \frac{1}{\sqrt{t}} \Phi\left(\frac{x}{\sqrt{t}}; M\right)$: self-similar sol of the viscous Burgers equation

$$z_t + \left(\frac{1}{2} z^2\right)_x = \frac{\mu}{2} z_{xx}$$

satisfying $\int_{-\infty}^{\infty} z \, dx = M$.