2014.5.18(福工大)

消散型波動方程式のCauchy問題の解の拡散現象

早稲田大政経
西原健二

10:00 − 11:30
14:00 − 15:30
(A棟 A11)

$$\begin{cases} u_{tt} - \Delta u + b(t,x)u_t = f(u), & (t,x) \in \mathbb{R}_+ \times \mathbb{R}^N \\ (u, u_t)(0,x) = (u_0, u_1)(x), & x \in \mathbb{R}^N \end{cases}$$

1. Model

(1) Cattaneo model $\begin{cases} u_t + q_x = 0 \\ q(t+\tau, x) = -\kappa u_x(t,x), \ 0 < \tau \ll 1 \end{cases}$

(2) System of 1-D compressible flow in porous media

$$\begin{cases} v_t - u_x = 0 & v > 0 : \text{specific vol.},\ u : \text{velocity} \\ u_t + p(v)_x = -\alpha u, & p(v) = v^{-\gamma} : \text{pressure } (\gamma \geq 1) \end{cases}$$

2. Linear damped wave equation — structure of solutions —

$$\begin{cases} v_{tt} - \Delta v + v_t = 0, \\ (v, v_t)(0,x) = (0, g)(x), \end{cases} \quad (t,x) \in \mathbb{R}_+ \times \mathbb{R}^N \Rightarrow v(t,x) =: [S_N(t) g](x)$$

$$\Rightarrow [S_3(t) g](x) = \frac{e^{-t/2}}{4\pi t} \partial_t \int_{|z| \leq t} I_0\left(\frac{1}{2}\sqrt{t^2 - |z|^2}\right) g(x+z)\, dz$$

$$= e^{-t/2} \cdot \frac{t}{4\pi} \int_{S^2} g(x + t\omega)\, d\omega + \frac{e^{-t/2}}{8\pi} \int_{|z| \leq t} I_1\left(\frac{1}{2}\sqrt{t^2 - |z|^2}\right) \frac{g(x+z)}{\sqrt{t^2 - |z|^2}}\, dz$$

$$=: e^{-t/2} \cdot [W_3(t) g](x) + \underbrace{[J_{03}(t) g](x)}_{\sim [e^{t\Delta} g](x) \text{ as } t \to \infty}$$

3. Semilinear damped wave equation

(1) 解表示 · N

(2) フーリエ変換 · Hayashi-Kaikina-Naumkin
· Ogawa, Hosono, Takeda, Yoshikawa, ⋯
($\Rightarrow$ system, 4-th order eq. ⋯)
· Wirth, Reissig, Narazaki, ⋯
($\Rightarrow$ time dependent damping, structural damping)

(3) 重み付エネルギー法 · Ikehata - Todorova - Yordanov, Li-N-Zhai, Wakasugi, ⋯
($\Rightarrow$ space(time) dependent damping, ⋯)

4. Damping with time or space dependent coefficient

$$\begin{cases} u_{tt} - \Delta u + \underset{\sim\sim\sim\sim}{b(t,x)\, u_t} = f(u), & (t,x) \in \mathbb{R}_+ \times \mathbb{R}^N \\ (u, u_t)(0,x) = (u_0, u_1)(x), & x \in \mathbb{R}^N \end{cases}$$

where $b(t,x) = \langle x \rangle^{-\alpha}, (1+t)^{-\beta}, \langle x \rangle^{-\alpha}(1+t)^{-\beta}$ etc.

(1) non-effective damping ($\Rightarrow$ Wave structure)
$\alpha > 1$ (望月), $\beta > 1$ (Wirth)

(2) effective damping ($\Rightarrow$ Diffusive structure)
$0 \leq \alpha < 1$; $-1 < \beta < 1$ … Ikehata et al.; Li-N-Zhai, Wakasugi etc.

(3) Critical case between wave and diffusive structures
$\alpha = 1$ … Ikehata et al.
$\beta = 1$ … D'Abicco, Wakasugi etc.

5. System of damped wave equations

(1) Weak coupling (Sun-Wang, N, N-Wakasugi, Narazaki)

$$\begin{cases} u_{tt} - \Delta u + u_t = |v|^p \\ v_{tt} - \Delta v + v_t = |u|^q \end{cases} \Rightarrow \quad \alpha := \max\left(\frac{p+1}{pq-1}, \frac{q+1}{pq-1}\right) = \frac{N}{2}$$

: critical

($p, q > 1$; $q \geq p > 1$ WLOG)

(2) General nonlinear term (Ogawa-Takeda)

$$u_{tt} - \Delta u + u_t = F(u),$$

where

$$u = \begin{pmatrix} u_1 \\ \vdots \\ u_m \end{pmatrix}, \quad F(u) = \begin{pmatrix} F_1(u) \\ \vdots \\ F_m(u) \end{pmatrix}, \quad F_j(u) = \prod_{k=1}^{m} |u_k|^{p_{jk}}$$

$\Rightarrow$ $\alpha := (P - E_m)^{-1}\begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}$, $P = (p_{jk})$ のとき,

$$\max(\alpha_1, \cdots, \alpha_m) = \frac{N}{2} \; : \text{critical}$$