Evaluation of Attacking and Placing Strategies in the Battleship Game without Considering Opponent Models

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ABSTRACT

A battleship game that uses 10x10 boards and five ships of the sizes from 2 to 5 has been chosen as a target domain for a representative testbed of the popular board games with incomplete information. The effective strategies for the attack or the placement of ships have been investigated without considering opponent models. As to the attacking strategies, a strategy, which is based on the checkerboard strategy and selects a square such that the number of possible placements is minimized when the attack is assumed to be missed, has shown the most excellent results. The accuracy of the attacking strategy has turned out to get better as the enumeration of possible placements is performed earlier. Considering the time limitation of experiments, we have selected the attacking strategy that starts the enumeration when the number of remaining ships is less than or equal to 4 and the number of unidentified squares is less than or equal to 70. The examinations of some placing strategies, with the above attacking strategy fixed, have shown a placing strategy that places ships along the edges is superior. In addition, the above placing strategy with prohibiting the connected placements of ships has shown the most excellent results.

Keywords: battleship game, incomplete-information game, board game, attacking strategy, placing strategy

1 Introduction

Complete-information two-player games are known to get the high performance by the minimax search. The center of studies on such games has shifted to efficient or accurate searches. On the other hand, no definite guiding principles to get the high performance are known in many incomplete-information games. The studies of such games are largely remained as future subjects. Some of well-known games, for instance, card games such as Bridge [5] and Poker [3], have been well studied. In addition, in the domain of the simplest game 'RoShamBo' (Rock-Paper-Scissors), the world programming competitions were held [2]. However, there are fewer studies on the incomplete-information board games. These board games relate to other research topics that the card games and RoShambo do not, such as the point location problem. We have been taking an interest in the incomplete-information board games in which some information of a game is concealed from the players. In particular, the mating problems of screen shogi have been solved deterministically [7][8]. As the domain of our new study, we select the classic battleship game, because it is very simple and the most popular and played most in the world. In addition, strong computer players of this game have not yet studied. In this paper, we investigate the efficient attacking strategies and the reasonable placing strategies without considering opponent models.

2 Battleship Game

A battleship game, which is also known as a submarine game, is a two-player incomplete-information game that are played using two sets of two rectangular boards with squares, some ship pieces that are to be put in the squares on the boards, and some marks that represent the attacks. The ship pieces are narrow and long with the sizes 2, 3, 4, and 5. A set of two boards is used by a player, one of which is for the player's army and the other is for the opponent army. Before starting the attacks, each player places all of his/her ships on his/her board anywhere he/she likes. After that, the attacks are played by turns. Suppose that the player to move is Player A and the opponent is Player B. Player A selects one empty square in the opponent board and put an attack mark in the square with telling the coordinate of the square to Player B. Player B tells Player A "Hit" if there is a ship on the square, or "Miss" otherwise. When all squares in a ship are hit, the ship sinks. The player that has sunk all opponent ships wins.

In the rules we have adopted, the board size is 10×10 and the each army consists of five ships: an aircraft carrier (size 5), a battleship (size 4), a cruiser (size 3), a submarine (size 3), and a destroyer (size 2), which is accordant with the rules in the Web sites [1] and [6]. Incidentally, other sites such as [4] use 16×16 boards. In addition, there is an amusing page [9] with excellent graphics and sound. The below are the detailed rules we have adopted in this study, which are considered as the most general among several variations.

1. A player can place the ships freely anywhere he/she likes.
2. A turn alternates even if an attack hits a ship.
3. Only the ship type is informed to the opponent when a ship has sunk, such as "A battleship sank".

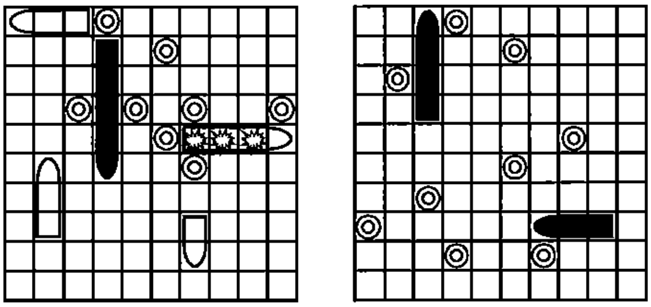

Figure 1: A player's view of a position in a battleship game

Note that the precise location of a sunk ship may be unclear for the opponent when a ship has sunk because only the type of a sunk ship is informed to the opponent.

An example position of a game is shown in Figure 1, which is a view of a player. The left is a board for the player and the right is one for the opponent. The player knows the entire placements of ships on his/her board while he/she knows only the squares that he/she has attacked on the opponent board. A double circle '◎' denotes a square that is attacked but missed, while a mark of explosion denotes a hit square. A sunk ship of which location is settled is shown with painting out in black.

3 Attacking Strategies

In this study, we do not set up the opponent models but purely and simply investigate the efficient attacking strategies. Here we define an *unidentified square* as a square that is unknown whether the attack on it is to be hit or missed. The *squares to be missed*, namely, the squares on which no ship obviously exists are not included in the unidentified squares. An example of the squares to be missed is shown in Figure 2. In the figure, no ship obviously exists on the shaded squares if the minimum size of remaining ships is 3. We consider some levels of attacking strategies shown below.

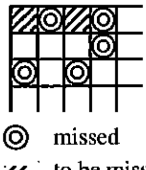

Figure 2: Squares to be missed

1. **Random (R)**
   The strategy selects an unidentified square randomly.

2. **Random with the neighboring attack (RNB)**
   When a square is hit, the neighboring squares of the hit square are attacked until a ship sinks. In Figure 3, the gray squares are the targets of the neighboring attack.

Since the neighboring attack enhances the performance absolutely in a battleship game, it is incorporated into all the strategies below.

3. **Checkerboard (CB)**
   In the early stage, it attacks the squares such that a large checkerboard pattern is formed. Next, it selects the squares between the former squares such that a small pattern is formed. The strategy is illustrated using 8×4 squares in Figure 4. First, the squares marked with 'A' are attacked. Then, the squares marked with 'B', 'C', and 'D' are attacked successively. Strictly speaking, random offsets from 0 to 3 are added to the horizontal and vertical coordinates in order to avoid the same sequence of attacks in each game.

4. **Minimum possible squares (MP)**
   In the early stage, the checkerboard strategy (CB) is used. If the number of remaining ships is less than or equal to 4 and the number of unidentified squares is less than or equal to a certain value, all the possible placements of ships are enumerated. Then, a square that has the maximum number of placements, namely, a square that has the largest hit probability is selected. If the number of unidentified squares when the enumeration starts is x, the strategy is denoted by MPx.

   If the number of remaining ships is 5, the number of possible placements has the order 5 of the number of unidentified squares. On the other hand, if the number of remaining ships is 4, the number of possible placements has the order 4 of the number of unidentified squares. Therefore, the condition that the number of remaining ships is less than or equal to 4 is included in order to limit the computation time. See Section 4.

5. **Minimum possible squares with the knight-checkerboard strategy (KMP)**
   We can use other strategies than CB for a uniform attack in the early stage. In KMP, the **knight-checkerboard strategy (KCB)** is used, which is illustrated in Figure 5. First, the squares marked with 'A' are attacked. Then, the squares marked with 'B', 'C', and 'D' are attacked successively. If the number of remaining ships is less than or equal to 4 and the number of unidentified squares is less than or equal to a certain value, KMP performs the same as MP. If the number of unidentified squares when the enumeration starts is x, the strategy is denoted by KMPx. Other strategies such as the large-knight-checkerboard strategy (LKCB) are possible for the uniform attack. However, LKCB results in CB or KCB after all.

6. **Lookahead by a few plies for the minimum possible squares (LA)**
   In the early stage, the checkerboard strategy is used. If the number of remaining ships is less than or equal to 4 and the number of unidentified squares is less than or equal to a certain value, all the possible placements of ships are enumerated. Then, a square that has the maximum number of placements after a certain number of attacks assuming that those attacks are missed, namely, a square that has the largest hit probability

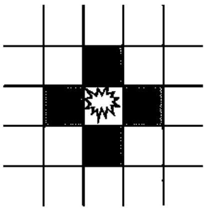

Figure 3: Attack on neighboring squares

	A		C		A		C
D		B		D		B	
	C		A		C		A
B		D		B		D	

Figure 4: Attacking strategy CB (checkerboard)

through a certain number of attacks is selected. If the lookahead ply is x, the number of moves selected in a position is y and the number of unidentified squares when the enumeration starts is z, the strategy is denoted by LAx_y_z.

4 Investigation of Possible Placements

When five ships of the sizes 5, 4, 3, 3 and 2 are placed in a starting 10×10 board of 100 unidentified squares, a rough upper limit of the number of possible placements is:

$$(100-40)\cdot 2 \times (100-30)\cdot 2 \times (100-20)\cdot 2 \times (100-20)\cdot 2 \\ \times(100-5-4-3-3)\cdot 2 \approx 7.3\times 10^{10}.$$

More generally speaking, when the five ships are placed on x unidentified squares, a rough upper limit of the number of possible placements is given as below.

$$x\cdot 2\times(x-5)\cdot 2\times(x-5-4)\cdot 2\times(x-5-4-3)\cdot 2 \\ \times(x-5-4-3-3)\cdot 2 < 2^5x^5$$

In addition, when the number of remaining ships is 4, a rough upper limit is 2^4x^4.

We have performed the enumeration of all possible placements in a starting position using a computer program. The results obtained by the program are shown in Figure 6. The total number of possible placements is 1.5×10^{10}. The squares shown in the figure represent the upper left side (5×5) of a

	A					D			
			C					B	
D					A				
		B					C		
				D					A
	C					B			
			A					D	

Figure 5: Attacking strategy KCB (knight-checkerboard)

board. The number of possible placements on each square is denoted by 'A' (center) to 'O' (corner). Since the numbers of possible placements distribute symmetrically, only the numbers of the squares in the upper triangle are shown. As we have expected, the numbers are the largest in the center squares and get smaller toward the outside and the smallest in the corner. However, the number in the center is only 2.7 times as large as that in the corner.

5 Evaluations of Attacking Strategies

First, each attacking strategy has been evaluated with the random placing strategy, in which ships are placed in a random coordinate and in a random direction. A roundrobin tournament has been performed using all attacking strategies listed in Section 3. Each match consists of 100,000 games. The results are shown in Table 1. The columns of 'firstp' and 'secondp' denote the attacking strategies of the first and the second players. R, RNB and CB represent Random, Random with the neighboring attack and Checkerboard strategies, respectively. MP70 represents the strategy of Minimum possible squares in condition the enumeration starts when the number of unidentified squares is less than or equal to 70. LA2_8_70 represents the strategy of Lookahead by a few plies for the minimum possible squares in condition the lookahead ply is 2, the number of moves selected in a position is 8 and the number of unidentified squares when the enumeration starts is 70. Since R strategy is too weak compared to others, only the results against RNB strategy are shown. The columns of 'winrf' and 'winrs' denote the winning ratios of the first and the second players by percentages. Assuming the normal distribution of the sample ratios by the random sampling, the two-digit values of increase or decrease in the 95% confidence intervals are shown in parentheses. For instance, 52.31(31) represents the 95% confidence interval is from 52.00 to 52.62. Since the variances are the same both in 'winrf' and 'winrs', only the cases in 'winrf' are shown. The columns of 'hitrf' and 'hitrs' denote the hit ratios of the first and the second players by percentages. The 95% confidence intervals are shown similarly. The column of 'seconds' denotes the total execution time by seconds. Here, all experiments in this paper were performed using a computer of Athlon 1.2 GHz with 512MB RAM under Windows 2000.

The hit ratios of R strategy are 20.8% and 20.6% for the first and the second players in the match R vs. R. These values are a little larger than the theoretical value of the random attack: $[(5+4+3+3+2)/100]\times 100 = 17\%$. This is probably because even R strategy skips the attacks on the squares to be missed. The strength of strategies increases in order of R, RNB, and CB. MP70 strategy is stronger than CB, however LA2_8_70 has no significant predominance over MP70 though the computational expenses in LA2_8_70 much increases than those in MP70.

MP70 corresponds to LA1_x_70, which selects the squares that has the largest number of possible placements. In LA2_8_70,

	a	b	c	d	e
1	O	N	L	K	I
2		M	J	H	F
3			G	E	D
4				C	B
5					A

Total number of possible placements: 15046987768 $\approx 1.50 \times 10^{10}$
Number on each square from A (center) to O (corner): $\times 10^9$

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
3.21	3.14	3.06	3.00	2.92	2.77	2.77	2.67	2.51	2.49	2.39	2.16	2.15	1.73	1.20

Figure 6: Results of enumerating possible placements

Table 2: Average winning ratios in KMP vs. MP

vs	win average	vs	win average
KMP70	47.27(22)	MP70	52.73
KMP75	48.30(22)	MP75	51.70
KMP80	48.69(22)	MP80	51.32

it seems that there occur only a small number of refutations and those refutations do not much affect the outcomes of games even though the lookahead ply increases from 1 to 2. Actually, in the match LA2_8_70 vs. MP70, the ratio of refutations in LA2_8_70 against the MP70 selection is only about 1.6%.

The winning ratio of MP or LA strategy increases as the enumeration of possible placements is started earlier. Figure 7 shows the changes of winning ratios of MP strategy against CB strategy with increasing the starting number of unidentified squares (denoted by 'startnum') from ten to eighty by tens. The 'first' and 'second' lines indicate the ratios in case MP is for the first and the second players. However, while the execution time of MP70 is 650 seconds, that of MP80 is about 14 hours, which is too much time-consuming for many experiments.

We have compared KMPx with MPx by matching each other with 100,000 games ($x = 70, 75, 80$). The summary of results is shown in Figure 2. For all x, MPx is stronger than KMPx. This supports the superiority of CB as a uniform attacking strategy.

We have not adopted LA strategy because its execution times are several times longer than that of the corresponding MP strategy though LA only has the similar performance to MP. In MP strategies, MP80 is too time-consuming. Therefore, we have adopted MP70 for the good attacking strategy as a basis of the analysis hereafter.

6 Evaluations of Placing Strategies

In this section, the following strategies for placing ships are considered and examined in a certain attacking strategy. Here, we do not use any opponent models but simply apply the attacking strategies described in Section 3 to the players with certain placing strategies.

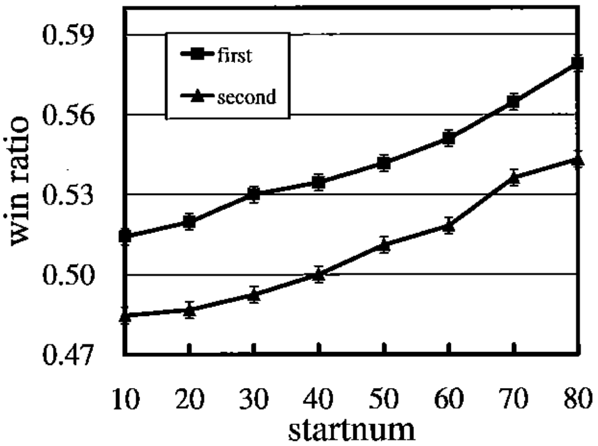

Figure 7: Winning ratio of MP against CB with changing the starting number of unidentified squares for enumeration

1. **Random placing (R)**
   All ships are placed in a random coordinate and in a random direction.

2. **Placing along edges (ED1)**
   Ships are placed randomly along the edges of the board. See Figure 8.

3. **Placing within outmost double lines (ED2)**
   Ships are placed randomly within the squares in outmost double lines of the board. See Figure 9.

4. **Packing the ships except one of the minimum size (PK)**
   All ships except one of the minimum size are packed within a rectangle of a certain size. A ship of the minimum size is placed anywhere else. When the size of a packing rectangle is $x \times y$ ($x < y$), the strategy is denoted by PKxXy. Here, the location and direction of a packing rectangle is determined randomly. An example of placements by PK2X8 is shown in Figure 10.

Since the risk of entanglement with the neighboring attack is high if ships are connected, we consider the placing strategy that forbids the connected placements of ships,

Table 1: Roundrobin results of every 100,000 games with changing attacking strategies

firstp	secondp	winrf	winrs	hitrf	hitrs	seconds
R	R	52.31(31)	47.69	20.814(29)	20.641(29)	56
RNB	RNB	51.12(31)	48.88	31.407(41)	31.452(42)	22
RNB	R	94.61(14)	5.39	30.918(39)	18.138(33)	32
R	RNB	6.26(15)	93.74	18.227(32)	30.953(39)	31
CB	CB	51.54(31)	48.47	36.683(46)	36.766(46)	21
CB	RNB	70.62(28)	29.38	36.430(45)	32.001(44)	21
RNB	CB	31.54(29)	68.46	31.863(43)	36.466(45)	22
MP70	MP70	51.67(31)	48.33	38.171(47)	38.250(48)	1253
MP70	CB	56.46(31)	43.54	37.971(47)	36.897(47)	661
MP70	RNB	74.44(27)	25.56	37.662(46)	32.089(44)	679
CB	MP70	46.38(31)	53.62	36.835(47)	38.123(47)	639
RNB	MP70	27.49(28)	72.51	31.999(44)	37.723(46)	671
LA2_8_70	LA2_8_70	51.67(31)	48.33	38.178(47)	38.257(48)	9054
LA2_8_70	MP70	51.71(31)	48.29	38.177(47)	38.250(48)	5239
LA2_8_70	CB	56.47(31)	43.53	37.980(47)	36.910(47)	4687
LA2_8_70	RNB	74.56(27)	25.44	37.710(46)	32.091(44)	4773
MP70	LA2_8_70	51.63(31)	48.37	38.172(47)	38.257(48)	5068
CB	LA2_8_70	46.39(31)	53.61	36.843(47)	38.118(47)	4521
RNB	LA2_8_70	27.40(28)	72.61	31.969(44)	37.764(46)	4737

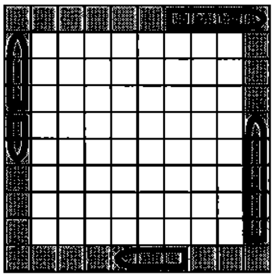

Figure 8: Placing strategy ED1

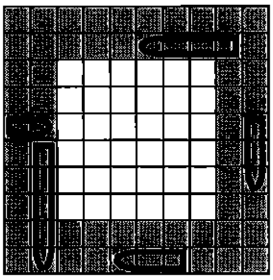

Figure 9: Placing strategy ED2

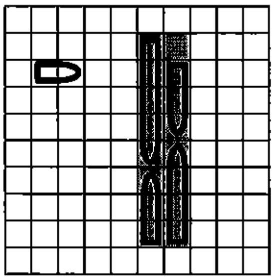

Figure 10: Placing strategy PK2X8

which we call the *separated placement*. As for the placing strategies R, ED1, ED2 and PK described above, the strategies of separated placements are defined as RSP, ED1SP, ED2SP and PKSP, respectively. Here, PKSP indicates the placing strategy PK in which a ship of the minimum size is separated from the packing rectangle.

First, some types of PK strategies were examined to determine which size of packing is appropriate. We performed a roundrobin tournament of PK strategies with changing the size of a packing rectangle: 2×8, 2×9, 2×10, 3×6, 3×7, 4×5, and 4×6. 100,000 games were played in a match. Here, the attacking strategy was fixed at MP70. The totaled winning ratios for each size are shown in Table 3, in which

Table 4: Average winning ratios on placing strategies (attacking strategy: MP70)

	R	ED1	ED2	PK2X10
winrf average	46.21(15)	60.00(15)	53.43(15)	46.98(15)
winrs average	42.79	56.84	50.07	43.68

the ratios are totaled separately for the first and the second players. Since the size 2×10 has the largest ratio both in the first and the second player, we have selected PK2X10 as a representative strategy of PK.

Next, a roundrobin tournament among the placing strategies R, ED1, ED2 and PK2X10 was performed. 100,000 games were played in a match. Here, the attacking strategy was fixed at MP70. The totaled winning ratios for each strategy are shown in Table 4, in which the ratios are totaled separately for the first and the second players. The ratios get larger in order of R, PK2X10, ED2 and ED1 both in the first and the second player. Though R and PK2X10 differ only a little, ED1 and ED2 has considerably large ratios. Among the two, ED1 is remarkably excellent.

Next, the effects of separated placements were evaluated. In the experiments, each placing strategy of R, ED1, ED2 or PK2X10 played a match with 100,000 games against the corresponding separated strategy of RSP, ED1SP, ED2SP or PKSP2X10, respectively. The results are shown in Table 5. Though ED1SP has a higher winning ratio than ED1, the strategies that do not forbid connected placements exceeds the corresponding separated strategies for R, ED2 and PK2X10. The cause of these seemingly curious results is supposed as follows. In case the separated placements are forbidden, the risk of entanglement with the neighboring attack is low. However, at the same time, the placements are likely to be so scattered that the numbers of possible placements in endgames are limited. The effect of evading the entanglement with the neighboring attack is larger in ED1, while the effect of limiting the number of possible placements is larger in the other

Table 3: Average winning ratios of PK placing strategies with changing the sizes of packing rectangles

rectangle size	2 × 8	2 × 9	2 × 10	3 × 6	3 × 7	4 × 5	4 × 6
winrf average	51.47(12)	52.08(12)	52.32(12)	50.58(12)	51.81(12)	51.69(12)	51.71(12)
winrs average	47.98	48.70	48.89	47.41	48.47	48.48	48.42

Table 5: Effects of separated placements (every 100,000 games, attacking strategy:MP70)

firstp	secondp	winrf	winrs	hitrf	hitrs	seconds
RSP	RSP	52.01(31)	47.99	38.725(48)	38.764(48)	1256
RSP	R	48.45(31)	51.55	38.241(47)	38.708(48)	1264
R	RSP	55.34(31)	44.66	38.675(48)	38.331(48)	1249
ED1SP	ED1SP	52.21(31)	47.79	34.401(44)	34.465(44)	2612
ED1SP	ED1	52.80(31)	47.20	34.950(44)	34.635(44)	2144
ED1	ED1SP	50.64(31)	49.36	34.547(44)	35.071(44)	2173
ED2SP	ED2SP	51.67(31)	48.33	37.116(46)	37.221(47)	1629
ED2SP	ED2	47.23(31)	52.77	36.529(46)	37.202(46)	1671
ED2	ED2SP	56.18(31)	43.82	37.103(46)	36.600(46)	1666
PKSP2X10	PKSP2X10	51.69(31)	48.31	37.992(47)	38.067(47)	1933
PKSP2X10	PK2X10	49.57(31)	50.43	37.766(47)	37.989(47)	1953
PK2X10	PKSP2X10	54.28(31)	45.72	37.973(47)	37.786(47)	1964

strategies. As supporting evidence, we refer to the results of similar experiments when the attacking strategy is fixed at RNB. For all placing strategies, the separated placements enhance the winning ratios. This is because there is only the effect of evading the entanglement with the neighboring attack in the RNB attacking strategy.

Finally, within our experiments, a combination of the attacking strategy MP70 and the placing strategy ED1SP is the most excellent.

7 Summaries and Future Works

In a battleship game that uses 10×10 boards and five ships of the sizes from 2 to 5, the effective strategies for the attack or the placement of ships have been investigated without considering opponent models. As to the attacking strategies, a strategy, which is based on the checkerboard strategy and selects a square such that the number of possible placements is minimized when the attack is assumed to be missed, namely, selects a square such that the hit probability is maximized, has shown the most excellent results. The accuracy of the attacking strategy has turned out to get better as the enumeration of possible placements is performed earlier. The problem is that the computational expenses get much larger as the enumeration is performed earlier. This could be solved by the incremental computation of the possible placements, which remains a future subject.

Considering the time limitation of experiments, we have selected the attacking strategy that starts the enumeration when the number of remaining ships is less than or equal to 4 and the number of unidentified squares is less than or equal to 70. The examinations of some placing strategies, with the above attacking strategy fixed, have shown a placing strategy that places ships along the edges is superior. In addition, the above placing strategy with prohibiting the connected placements of ships has shown the most excellent results.

Hereafter, we investigate the comprehensive strategies that include the excellent attacking and placing strategies with considering the opponent models of the attacking and placing strategies examined in this study. Since all the placing strategies except the random one are superior to the random one, it is reasonable to incorporate them into the opponent models.

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