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FULL PAPER

Evaluation of Quality of Partial Caging by a Planar Two-Fingered Hand

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In partial caging, an object is partially constrained by robots and is able to escape from there. Although complete caging ensures the hand never releases the confined object, insufficient degrees of freedom of robots does not often satisfy the conditions for caging. Partial caging, however, can be accomplished even by robots having such mechanical restriction. We consider a case that an object moves in the semi-closed region formed by a planar robot hand with two fingers, as an example of partial caging in two dimensional space. Then the parameter of fingers: joint angles interferes in the object motion to escape from the hand through the gap between the fingertips. Some simulation results show differences of difficulty of escaping according to arrangement of fingers, and a factor interfering in the difficulty is analyzed. Additionally we also evaluate ease of entering the hand through the gap and define an ability index of robot hand for partial caging with the above two evaluation scores. Then a high index score indicates that the hand assumed to be able to capture objects easily and confine it without any finger motion. It can be utilized for mechanical design and controlling strategies of robots in capturing objects.

Keywords: Partial Caging, Two-fingered hand, Capturing, Grasping

1. Introduction

This paper discusses on posture of robot hand for *partial caging*, which is a geometrical strategy to capture an object. In partial caging, the robot hand incompletely confines the object by locating robotic fingers around it and there are some gaps through which the object can escape from the constraint (Figure 1). Then the arrangement and posture of fingers, especially those of fingertips, affects the difficulty of the object escaping from the robotic "cage" and also ease of entering there, like a fish trap (Figure 2). In this paper, we consider a simplified scenario of partial caging in planar space. An appropriate configuration for partial caging can facilitates robot hands having mechanical restriction to capture objects.

Robotic manipulation by multifingered hands has been variously studied to accomplish several tasks instead of humans. Most of these manipulation are with grasping [1, 2], in which the robot hand makes

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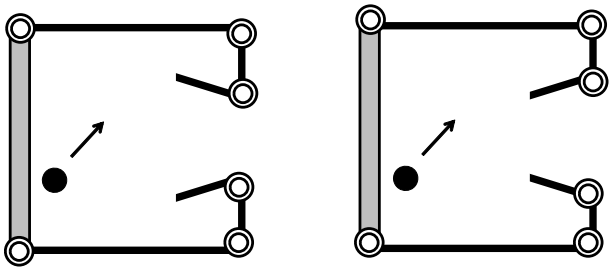


Figure 1. Differences of difficulty of escaping according to the finger posture.

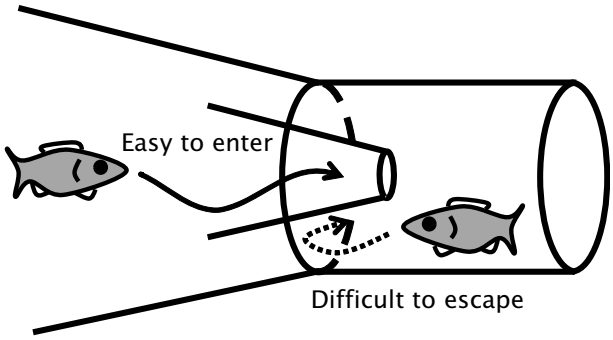


Figure 2. Fish trap (A case of partial caging).

contacts with objects and applies forces to them. Robotic grasping to hold target objects is very essential because stable manipulation to move and put the objects appropriately can be generally accomplished with determining the object's position and orientation. Thus *force closure*, which is a mechanical constraint including force equilibrium, is mainly discussed in grasp planning. However, it usually requires various mechanical parameters for force analysis.

In contrast, caging defined as a method to capture an object by robots geometrically does not require such mechanical conditions. In caging, the robots just surround the object to preclude it escaping from the caged region [3]. Thus the caged object can be restricted to move only in the closed region, and then the robots behave as obstacles to prevent the object from changing its position and orientation freely. Thus caging can be regarded as an idea of constraint expanded from *form closure* [4], in which a target object is relatively fixed to robots. And then even position-controlled robots are able to capture objects without excessive internal forces, which often occur in robotic contact tasks [5]. This is an advantage over conventional robotic grasping and fixturing, which usually require appropriate force control and force sensing. Consequently several approaches of caging have been proposed both two and three dimensional space. Caging in 2D is often preformed by multiple circular or pointed robots as [3], but 3D caging are not only by pointed fingers [6] but also by practical robot hands [7, 8].

On the other hand, it is not always necessary for robots to form entirely closed region to confine target objects. For instance, in puzzle rings, there exist paths to untangle them although the paths are usually very narrow to go through. Hence it is difficult for a ring to escape from another, and the ring is assumed to be substantially captured. As for robot hands, *complete caging*, in which a target object is absolutely constrained not to escape from the hand, cannot be always accomplished due to limitation of robots such as lack of degrees of freedom (DOFs) and/or less number of fingers. It gives the constrained object some paths to reach outside of the hand. Nevertheless the object hardly escapes from the constraint in the cases that the paths are too narrow and/or complicated. We call the situation *partial caging* (partial cage in [9]), in which the object is geometrically confined but the confinement is incomplete. Partial caging can sometimes be substituted for conventional complete caging, and be accomplished even by robot hands having mechanical restriction.

We consider cases that the robot hand cannot confine an object completely due to mechanical limitation of the hand as Figure 3. Although grasping the object is a reasonable choice in this situation, partial caging can be useful when the robot hand has to capture the object without grasping, for example, storing the object in the hand to capture one more another object.

In this paper, a simplified scenario of partial caging is studied, in which an object is almost confined by a planar robot hand (Figure 1, 3). The hand has two fingers, and there exists a gap between their fingertips. Thus the object is not completely constrained by the hand and is able to pass through the gap to escape from the constraint. There are some factors affecting the difficulty of escaping from the hand such as the width of the gap and the posture of fingers. This situation can be regarded as a simplified robotic application inspired fish traps (Figure 2). In addition, opposite directions to enter the hand from outside is also considered. The analysis and evaluation of partial caging may lead efficient arrangements of robot fingers to confine objects geometrically, and they can be applied to design of the hands and motion strategies for caging and enclosing.

First we describe formulas about physical phenomena about caged object and motion of the manipulator driving the hand. Next, we show some simulation results with changing the finger parameters that affect to motion of the object in the partially closed region. Additionally we discuss on factors interfering in the quality of partial caging from the simulation results.

An original concept of caging or capturing by coordinating robots is considered to be proposed by Kuperberg [10], and caging has been studied not only as geometrical capturing [11, 12] but also as preshaping of grasping or fixturing by robots in 2D plane independently. 2D caging as preshaping by two circular fingertips as a gripper in the plane was studied [3, 13]. Similarly 2D caging by circular or pointed fingers has been variously studied such as [14] for concave objects. Two-fingered caging can be usually performed by squeezing at a concave part of the object and dispersion of fingers at a hollow part [15–17]. Three or more fingertips can accomplish caging for convex objects [18–20]. Manipulation via caging by multiple mobile robots has been also proposed [21] and its mathematical condition of caging was derived in [22]. Caging manipulation by both robots and walls has been also proposed [23] for less number of robots.

Immobilizing grasp [3] can be achieved by shifting robots in caging to make contacts with the captured object [17] . This approach is applicable to deformable objects [24]. In addition, since caging as geometrical capturing allows some positioning errors caused by object recognition and/or control of robots, it can work as margins for uncertainties in grasping and manipulation [25]. Similarly Su et al. performed caging grasps of polyhedron-like workpieces by a gripper with vision system [26]. Maeda et al. proposed *Caging-based grasping*, in which a robot hand covered by soft materials makes contact with an object and accomplishes caging for the object by the inner limbs [27].

While many studies of caging focus on planar scenes, the followings proposed caging in 3D space and performed by actual robot hands. Similarly to planar caging, pointed fingers in 3D space can accomplish caging for polytopes [6]. Caging by a multifingered hand for some primitive shapes of objects was also proposed, and each sufficient condition has been derived [8] for motion planning [28]. A manipulator and a humanoid robot executed some manipulation tasks using caging by an attached end tool instead of grasping [7].

Moreover other geometrical constraints has been derived from caging approaches. Jiang et al. proposed *gravity caging* to place grasped objects on a particular location [29]. This idea uses geometrical constraint partially and the gravitational force prevents the placed object from escaping toward unconfined direction. Makapunyo et al. studied quality of *partial cage*, in which pure geometrical constraint by robots is applied to an object incompletely and the object has paths to escape from the caging formation [9, 30]. They used probabilistic motion planners to test how much time elapses until the object escapes from the robots formation.

In our previous work [31], we also studied quality of partial caging as [9, 30] in two-dimensional space in cases that a circular object is partially surrounded by a robot hand (Figure 1) as simplified problems of partial caging. In addition two factors interfering in motion of the caged object are analyzed. This paper expands this previous work to apply the concept of partial caging to more practical robot hands and also to a stick-shaped object as another shape. We introduce a planar robot hand attached to a planar manipulator as Figure 3 and evaluate quality of partial caging by simulations.

2. Model of Partial Caging by a Robot Hand

2.1 *Assumptions and Notations*

In this paper, we select a circular object as a simplified target object and make the following assumptions.

- Simulations are performed in the two-dimensional space, and the objects moves in a planar two-fingered hand attached to a manipulator.
- Generally in this paper, *limbs* represent both the fingers and the palm of the hand.
- Only gravitational force is applied to the object, and any viscous and frictional force are not considered.

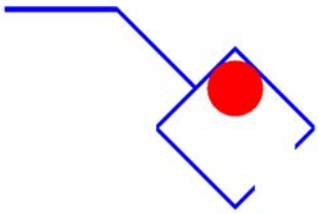


Figure 3. Partial caging for a circular object by a robot hand.

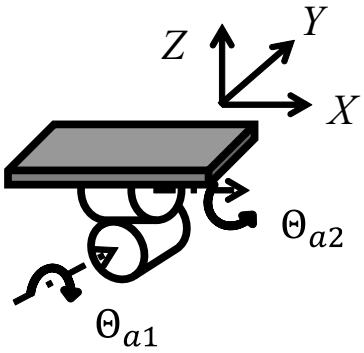


Figure 4. A two DOF manipulator inclining the hand.

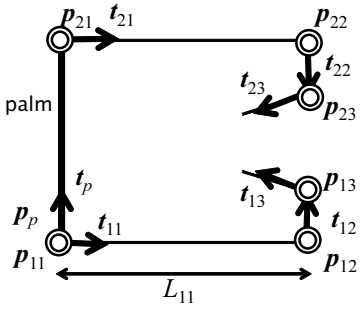


Figure 5. Setting of the hand.

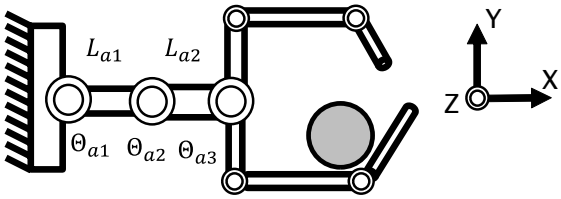


Figure 6. Setting of the robot arm.

- Every volume of the joint is negligible.
- The volume of the stick-shaped object is also negligible.

We define the following notations. In this paper, although we deal with simulations only in two-dimensional space, the theory can be expanded to that in three-dimensional space.

- $\boldsymbol{p}_o \in \mathbb{R}^3$: the position of the mass point of object.
- $\boldsymbol{v}_o \in \mathbb{R}^3$: the velocity of the object.
- $\boldsymbol{\omega}_o \in \mathbb{R}^3$: the angular velocity of the object, which is considered only for stick-shaped object.
- $r_o \in \mathbb{R}^1$: the radius of the circular object or the half length of the stick-shaped object.
- $M_o \in \mathbb{R}^1$: the mass of the object.
- $\boldsymbol{p}_{ij} \in \mathbb{R}^3$: the position of the j-th joint of the i-th finger.
- $\boldsymbol{t}_{ij} \in \mathbb{R}^3$: the unit tangential vector along the j-th limb of the i-th finger.
- $\boldsymbol{n}_{ij} \in \mathbb{R}^3$:the unit normal vector on the contact point of the j-th limb of the i-th finger. $\boldsymbol{t}_{ij} \cdot \boldsymbol{n}_{ij} = 0$
- $L_{ij} \in \mathbb{R}^1$: the length of the j-th limb of the i-th finger.
- $\theta_{ij} \in \mathbb{R}^1$: the joint angle of the j-th joint of the i-th finger.
- $L_{aj} \in \mathbb{R}^1$: the length of the j-th limb of the arm.
- $\Theta_{aj} \in \mathbb{R}^1$: the joint angle of the j-th joint of the arm.

When $\theta_{ij} = 0$ (or $\Theta_{aj} = 0$), the j-th limb is parallel to the $(j-1)$-th limb of the i-th limb.

The limbs of the hand are numbered as Figure 5 ($ij = p, 11, \ldots 21, \ldots$).

2.2 *Equations of the Moving Object's Motion*

As the assumptions, only the gravitational force is applied to the circular object in the hand. Here we call the center of the mass of the object *moving point*. For randomized movement of the hand, two situations

of robotic components are considered:

(1) a flat plane with walls as a hand (Figure 5) that confines the object is manipulated and inclined by a manipulator with 2 DOF like a universal joint as Figure 4 [31]. We call the manipulator "wrist manipulator".
(2) a planar robotic manipulator with an attached end effector manipulates the object as Figure 3, 6.

In the former case about the wrist manipulator, the relative rotation matrix between the frame of the hand and the reference frame defined as 1st joint of the manipulator can be calculated from each rotation matrix about Θ_{a1} rotating around $Y-$axis of the reference frame, which is also wrist joint of the manipulator, and Θ_{a2} rotating around $X-$axis of the next frame. And then the gravitational acceleration on the hand, $\boldsymbol{g}_P \in \mathbb{R}^3$, can be calculated with that on the reference frame, $\boldsymbol{g}_O := [0, 0, -g]^T \in \mathbb{R}^3$, as,

$$\begin{aligned}\boldsymbol{g}_P(\Theta_{a1}, \Theta_{a2}) &= {}^O_P\boldsymbol{R}^{-1}\boldsymbol{g}_O \\ &= \left[g\sin\Theta_{a1} \; -g\cos\Theta_{a1}\sin\Theta_{a2} \; -g\cos\Theta_{a1}\cos\Theta_{a2}\right]^T. \end{aligned} \tag{1}$$

Note that only the 1st and 2nd components of the vector are used in two dimensional space in practice.

In the latter case that a planar robotic manipulator and a hand are considered (Figure 6), the gravitational acceleration on the hand can be simply defined as $\boldsymbol{g}_P := [0, -g, 0]^T$, and it does not depend on the joint angles of the manipulator.

With the gravitational acceleration defined above, the equation of the moving object's motion can be expressed as:

$$M_o\ddot{\boldsymbol{p}}_o = M_o\boldsymbol{g}_P(\Theta_{a1}, \Theta_{a2}). \tag{2}$$

2.3 *Rebound Between the Moving Object and the Fingers*

2.3.1 Circular Object

The captured circular object makes collisions with the limb of the hand when the moving point, which is the center of the mass of the object, reaches the distance of r_o from the limb (Figure 7). The collisions can be examined by the following series of inequality:

$$\boldsymbol{v}_o \cdot \boldsymbol{n}_{ij} < 0, \tag{3}$$

$$0 \le \left(\boldsymbol{p}_o - \boldsymbol{p}_{ij}\right) \cdot \boldsymbol{n}_{ij} \le r_o, \tag{4}$$

$$0 \le \left(\boldsymbol{p}_o - \boldsymbol{p}_{ij}\right) \cdot \boldsymbol{t}_{ij} \le L_{ij}. \tag{5}$$

(3) expresses that the object is in direction toward a limb to make collision, not leaving. (4) and (5) express that the object locates within the range to be able to have contact with the limb. The object can have contact with each end of fingertip. A unit normal vector of the end point of the fingertip, $\boldsymbol{n}_{i,e}$ is derived as,

$$\boldsymbol{n}_{i,e} = \frac{\boldsymbol{p}_o - \boldsymbol{p}_{i,e}}{\|\boldsymbol{p}_o - \boldsymbol{p}_{i,e}\|}, \tag{6}$$

where $\boldsymbol{p}_{i,e}$ $(= \boldsymbol{p}_{i,\mathrm{tip}} + L_{i,\mathrm{tip}}\boldsymbol{t}_{i,\mathrm{tip}})$ is the position vector of the end point of the fingertip.

Similarly when the object may have collision with a joint, the collision test using (3), (4) and (5) is examined. And also both sides of the finger is possible to make collision with the object if the fingertip is toward inner of the hand as Figure 5. Hence we set an additional normal vector on the opposite side of the limb for the above examination.

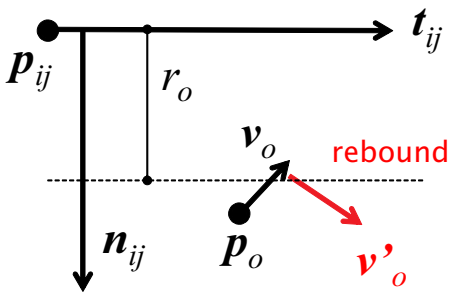


Figure 7. A rebound of the moving point.

Changed velocity of the moving object after rebound, $\boldsymbol{v}'_o$ (Figure 7) can be written as:

$$\boldsymbol{v}'_o = \boldsymbol{v}_o - (1 + e_r)(\boldsymbol{v}_o \cdot \boldsymbol{n}_{ij})\boldsymbol{n}_{ij}, \tag{7}$$

where e_r is the coefficient of restitution between the moving object and every limb.

Since (2) is an ordinary differential equation, it can be solved numerically by a one-step method such as the 4th-order Runge-Kutta Method. Additionally (4), (5) and (7) can be also solved discretely, with avoiding occupation of the robot by the object that is caused by computational errors.

2.3.2 Stick-shaped Object

The captured stick-shaped object makes collisions with the limb of the hand when the segment of the object and that of the limb have interpolation. Then r_{oa} denotes the distance between the center of the mass of the object and collision point on the object, and l_{ija} denotes the distance from the j-th joint of the i-th finger to the collision point on the j-th limb of the i-th finger. If any collisions occur, the following equations are satisfied:

$$\left(\left(\boldsymbol{p}_{ij} - \boldsymbol{p}_o\right) + l_{ija}\boldsymbol{t}_{ij}\right) \times \boldsymbol{t}_o = \boldsymbol{0}, \tag{8}$$

$$\left(\left(\boldsymbol{p}_o - \boldsymbol{p}_{ij}\right) + r_{oa}\boldsymbol{t}_o\right) \times \boldsymbol{t}_{ij} = \boldsymbol{0}, \tag{9}$$

$$0 \leq l_{ija} \leq L_{ij}, \tag{10}$$

$$-r_o \leq r_{oa} \leq r_o, \tag{11}$$

where $\boldsymbol{t}_o$ denotes an unit direction vector toward the end of the stick from the center of mass. When $\boldsymbol{t}_{ij}$ and $\boldsymbol{t}_o$ are parallel, l_{ija} and r_{oa} cannot be determined. Thus the following set of inequality is examined for collision check.

$$\left(\boldsymbol{p}_o - \boldsymbol{p}_{ij}\right) \cdot \boldsymbol{n}_{ij} = 0 \tag{12}$$

$$-r_o \leq \left(\boldsymbol{p}_o - \boldsymbol{p}_{ij}\right) \cdot \boldsymbol{t}_{ij} \leq L_{ij} + r_o \tag{13}$$

Changed velocity and angular velocity of the moving object after rebound, $\boldsymbol{v}'_o$ and $\boldsymbol{\omega}'_o$, can be expressed with the coefficient of rebound, e_r as:

$$(\boldsymbol{v}'_o + r_{oa}(\boldsymbol{\omega}'_o \times \boldsymbol{t}_o)) \cdot \boldsymbol{n}_c = -e_r(\boldsymbol{v}_o + r_{oa}(\boldsymbol{\omega}_o \times \boldsymbol{t}_o)) \cdot \boldsymbol{n}_c. \tag{14}$$

where $\boldsymbol{n}_c$ denotes an unit normal vector pointing in the direction to the object from the limb at the contact point.

They can be also calculated with impulse applied to the object, whose magnitude is denoted by Q, as

follows:

$$M_o\left(\boldsymbol{v}_o' - \boldsymbol{v}_o\right) = Q\boldsymbol{n}_c, \tag{15}$$

$$\boldsymbol{I}\left(\boldsymbol{\omega}_o' - \boldsymbol{\omega}_o\right) = r_{oa}\boldsymbol{t}_o \times Q\boldsymbol{n}_c, \tag{16}$$

where $\boldsymbol{I}$ denotes the inertia tensor of the object.

From (14), (15) and (16), we can obtain Q as:

$$Q = -\left(1 + e_r\right)\frac{\left(\boldsymbol{v}_o + r_{oa}\left(\boldsymbol{\omega}_o \times \boldsymbol{t}_o\right)\right)\cdot \boldsymbol{n}_c}{\frac{1}{M_o} + r_{oa}^2\left(\left(\boldsymbol{I}^{-1}\left(\boldsymbol{t}_o \times \boldsymbol{n}_c\right)\right) \times \boldsymbol{t}_o\right)\cdot \boldsymbol{n}_c}, \tag{17}$$

and calculate $\boldsymbol{v}_o'$ and $\boldsymbol{\omega}_o'$.

3. Evaluation of Partial Caging Quality

We evaluate effects of finger arrangements for partial caging. A purpose of partial caging is to capture an object even by a mechanically-restricted robot hand that cannot confine the object completely as Figure 3. Thus difficulty of escaping from the hand by passing through the gap between the fingertips (DoEs) is emphasized in this paper.

On the other hand, the object can pass through the gap to enter the semi-closed region. Then ease of entering the hand (EoEn) can be an index of evaluation of finger arrangements. We call the gap as "goal" because each simulation in this paper finishes when the object reach the goal, that is, the object can escape from the hand or enter the hand.

3.1 *Indexes for Evaluation*

Difficulty of passing through the goal is evaluated with the following two indexes:

- *Index 1*: Average simulation time until the moving object reaches the gap of the hand.
- *Index 2*: Rate of cases that the moving object cannot reach the gap within a limitation time.

The index 1 has been also proposed in [9], and the index 2 is introduced in our scheme due to limitation of computing time.

After simulations, we obtain both index scores of *EoEn* and *DoEs* under a particular condition of finger arrangement. And then we calculate a new index score:

$$\frac{Index\ score\ of\ DoEs}{Index\ score\ of\ EoEn}, \tag{18}$$

which indicates whether the arrangement of fingers has an advantage to capture an object without any finger motions and to confine it geometrically.

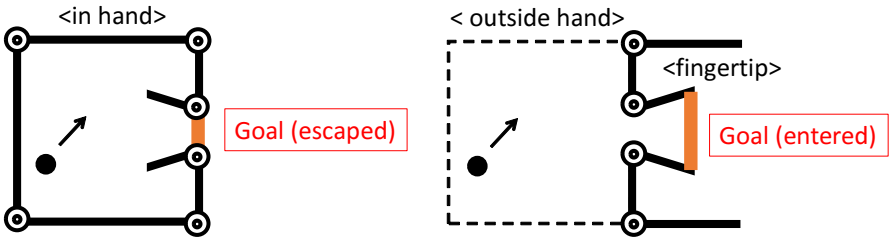
3.2 *Simulation Settings*

We defined the following notations as parameters of finger arrangement.

$$L_g := L_{i,\mathrm{tip}}, \tag{19}$$

$$W_g := \begin{cases} \|\boldsymbol{p}_{1,\mathrm{tip}} - \boldsymbol{p}_{2,\mathrm{tip}}\| & \text{(Figure 8(a))}, \\ \|\boldsymbol{p}_{1,e} - \boldsymbol{p}_{2,e}\| & \text{(Figure 8(b))}, \end{cases} \tag{20}$$

$$\theta_g := \frac{\pi}{2} - \theta_{i,\mathrm{tip}}. \tag{21}$$



(a) In evaluating difficulty of escaping. (b) In evaluating ease of entering.

Figure 8. Goal judgment for simulations.

Thus $\theta_g = 0$ when the fingertip is perpendicular to the linked limbs. Hence the gate is closing while $\theta_g > 0$ and vice versa.

In evaluating the index *EoEn*, it is difficult for the object to enter the hand if the object is entirely free from any obstacles. Hence tests to evaluate *EoEn* are simulated under the same conditions as tests for *DoEs*. The index DoEs is evaluated under situations as Figure 8(a): the fingertips is toward inner of the hand. On contrast, the index EoEn is evaluated under situations as Figure 8(b): it is assumed to be that the object is outside of the hand but it is virtually confined.

The following conditions are commonly used in this section.

- The step size of the solver, $dt = 0.001$.
- The value of gravitational acceleration defined in Sec. 2.2, $g = 9.8$.
- The coefficient of restitution between the object and every limb, $e_r = 0.1$.
- The limitation of simulation steps is introduced.
- Initial position of the moving point in each simulation is randomly selected near either end of the palm. And every initial velocity is always 0.
- A random number generator of the GSL (GNU Scientific Library [32]) is used as the algorithm of *Mersenne Twister*.
- When the simulation steps reach the limitation defined above, the number of limitation steps is used to calculate the average steps, *index 1*.
- Trials for each simulation setting are executed 5,000 times with each different seed of random number generator.

3.3 *Simulation Results: A Flat Plane with Walls Inclined by a Robot Arm*

In this section, we consider cases that a circular object is partially confined in a flat plane with walls inclined by a wrist manipulator (Figure 4).

First we show evaluation results of difficulty of escaping from the hand (DoEs) as a quality index of partial caging, with changing parameters of the hand: width of the goal and angle of the fingertips. After the evaluations to verify validity of the defined indexes and the setting of simulations, evaluations for both the difficulty and ease of entering the hand (EoEn) will be shown, with changing the angle of the gate and the scale of the object.

3.3.1 Parameters Settings

Each finger of the hand has three joints and limbs, and the palm length is $L_p = 500$; and every length of the limbs is as follows: $L_{11} = L_{21} = 500$; L_{12}, L_{22} are changed by the parameters about the fingertip: L_g and W_g; $L_{13}, L_{23} = 0.2L_p$. Since the hand forms a rectangular shape, the joint variables, $|\theta_{11}| = |\theta_{12}| = |\theta_{21}| = |\theta_{22}| = \frac{\pi}{2}$.

The joint angles of the manipulator inclining the hand, Θ_{a1}, Θ_{a2} are changed within the following range $-\pi/6 < \Theta_{a1}, \Theta_{a2} < \pi/6$ [rad]. Each desired angle is randomly determined every 10,000 steps, and each angle changes to the desired value in uniform angular velocity in 1,000 steps. Thus each joint keeps

the desired joint variable in other 9,000 steps.

The limitation of simulation steps is 600,000.

3.3.2 Changing the Angle of the Fingertip and the Width of the Gap

This section shows the differences of difficulty of escaping from the hand related to the angle of the fingertips. In this analysis, we set the width of the goal, $W_g = 0.4L_p, 0.6L_p, 0.8L_p$, the length of the fingertip, $L_g = 0.2L_p$, and change the angle of the fingertip, θ_g. The radius of the object, $r_o = 0.01L_p$.

Figure 9(a) shows evaluation results of the index 1 and 2 related to angle of the fingertips in the case of $W_g = 0.4L_p$. Since both indexes has similar trends, we calculate coefficient of correlation between them, and then, it is 0.984. Therefore it is enough to present only results of index 1 in later discussion. Figure 9(b) shows changing of index 1 related to angle of the fingertips, with various width of the goal. From Figure 9 while $W_g = 0.4L_p$, the index score almost depends on the width of the gap determined by the angle of the fingertips except slight decrease of index 2 about $\theta_g = 40$ [deg]. When $W_g = 0.6L_p, 0.8L_p$, each index score increases while $\theta_g \leq 40, 50$ [deg], and decreases after the peak, respectively (Figure 9(b) while $W_g = 0.6L_p, 0.8L_p$). From these results, some effects other than the width of the goal should be considered.

3.3.3 Changing the Angle of the Fingertips and the Radius of the Object

This section shows the differences of both difficulty of escaping from the hand and ease of entering the hand related to both the angle of fingertips and the radius of the object. In this analysis, we fix the width of the goal, $W_g = 0.4L_p$ and the length of the fingertips, $L_g = 0.2L_p$, and change the angle of the fingertips, θ_g for several radius of the object: $r_o = 0.01L_p, 0.05L_p, 0.1L_p$.

As concerns difficulty of escaping from the hand related to angle of the fingertips for several sizes of objects (Figure 10), every case has same trend of scores that the difficulty increases according to the angle θ_g as described in Sec. 3.3.2.

Figure 11 shows that evaluation score of *EoEn* is low when the angle of the fingertips, $\theta_g \leq 10$ [deg]. It indicates that the fingertips at that time seldom interfere in the object entering the hand if only it enters the gap between the fingertips, whose width is $(W_g - 2L_g \sin\theta_g) \geq W_g$ while $\theta_g \leq 0$.

On the other hand, $\theta_g \geq 0$, the width of the gap is $(W_g - 2L_g \sin\theta_g) \leq W_g$. Then collisions between the object and the tapered fingertips prevent the object from reaching the goal as the results of Figure 9.

3.4 *Result: Evaluation Indexes of Partial Caging Quality*

We defined an index score to evaluate partial caging quality with (18). A high score represents that the configuration of the hand including finger posture has an ability to capture an object easily without any finger motions and to confine it geometrically. The indexes related to various finger postures and the object's sizes are calculated from the above two results in Figure 10 and 11, and they are shown in Figure 12.

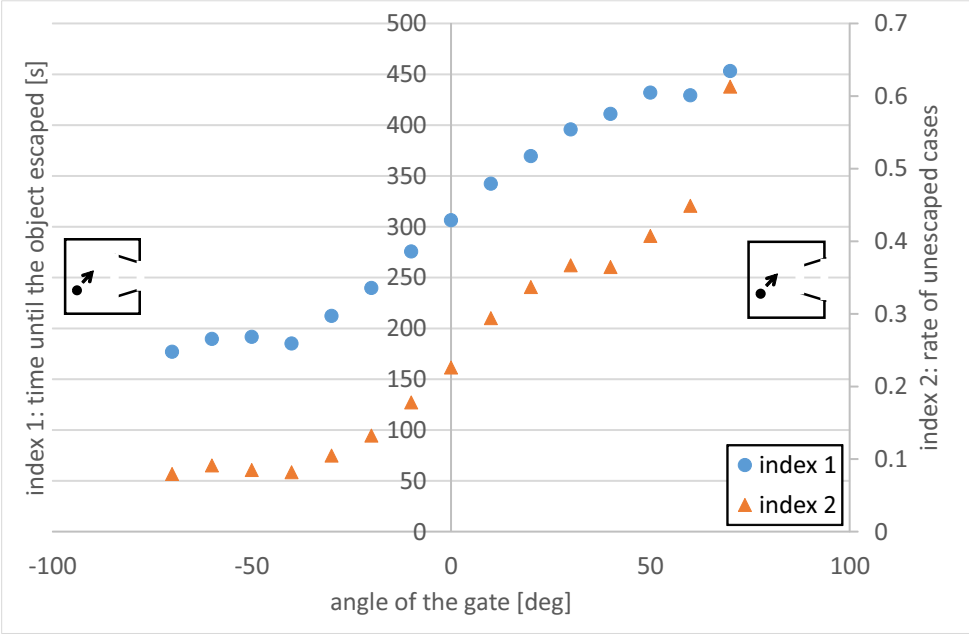
The result shows that the index score is high about $\theta_g = 10$ [deg] for every size of object. Hence it means that a robot hand under the condition, $\theta_g = 10$ [deg] is assumed to be in an appropriate configuration to capture an circular object and confine without any finger motion.

On the other hand, the score for the smaller object decreases in inverse relation to θ_g because of difficulty of entering the hand reported in Figure 11.

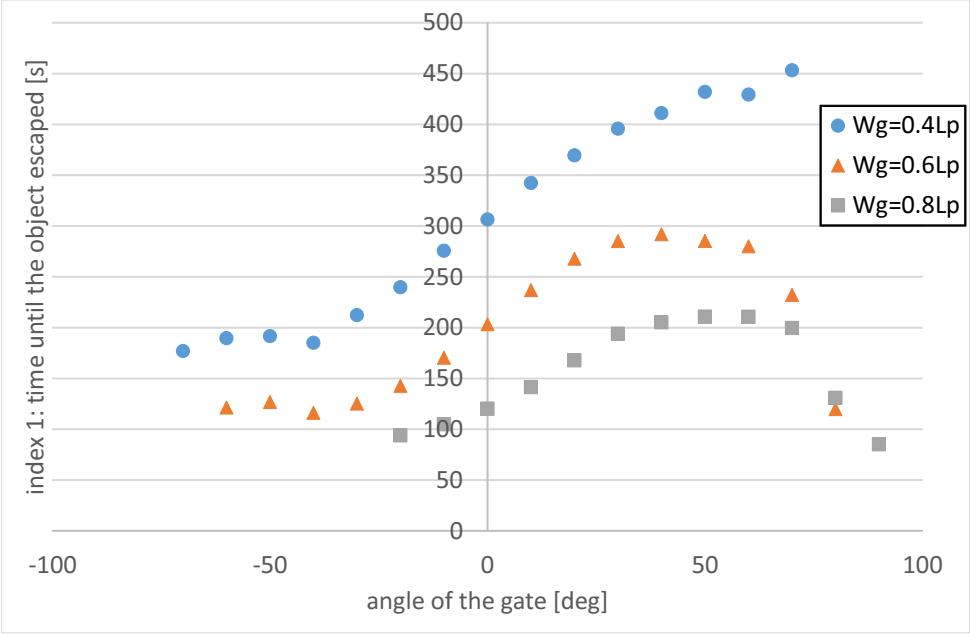
3.5 *Discussion on the Effects of Fingertip Posture for Partial Caging*

From the results in Sec. 3.3.2, it can be said that the difficulty of escaping from partial caging by the hand for a circular object generally decreases according to the width of the gap between the fingertips W_g, which is determined by angle of the fingertip, θ_g. However increase θ_g causes narrow W_g does not always decrease the score as cases that $W_g = 0.6L_p, 0.8L_p$.

As for the evaluation of ease of entering the hand (Sec. 3.3.3), the fingertips with their angle $\theta_g \leq 10$ [deg] seldom interfere index scores of the evaluation. While $\theta_g \geq 10$ [deg], tapered fingertips and



(a) Differences of index 1 and index 2 while $W_g = 0.4L_p$



(b) Differences of index 1 while $W_g = 0.4L, 0.6L, 0.8L$

Figure 9. Difference of difficulty of escaping from the hand related to angle of the fingertips.

narrower gap prevent the object from reaching the goal.

We have discussed the factors in our previous work [31]. One factor is obviously width of the gap between fingertips, from the above results. Moreover we discuss another factor interfering in object's behavior: retention of the object caused by wedge-shaped space between a fingertip and its linked limb.

According to the results illustrated by Figure 9(b) and 10, difficulty of escaping, that is, quality of partial cage increases according to the angle of the fingertips, where larger angle makes the gap between the fingertips narrower. It implies that the quality can be estimated by the ratio between object's radius

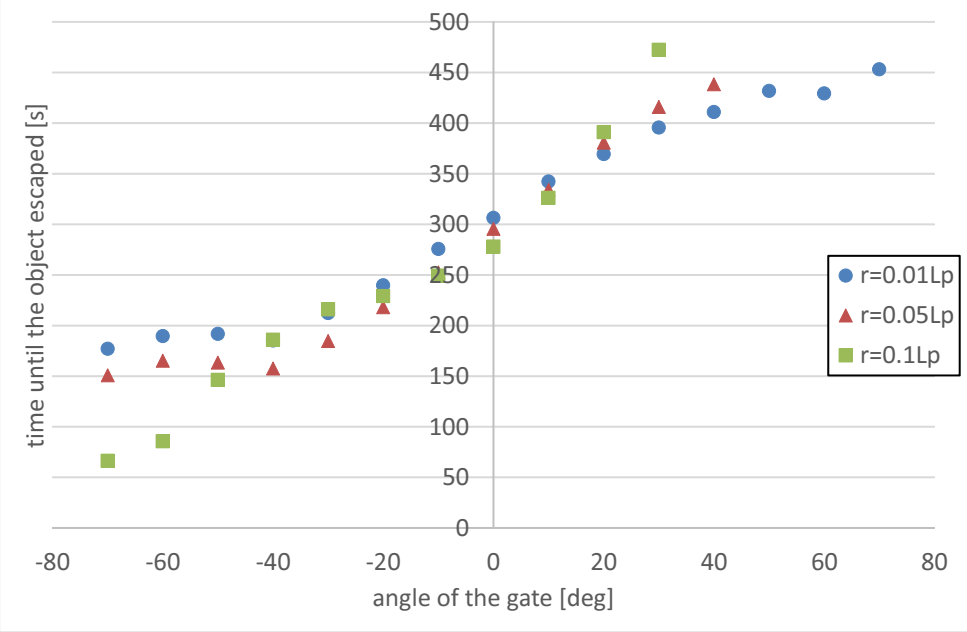


Figure 10. Differences of difficulty of *escaping from the hand* related to angle of the fingertips.

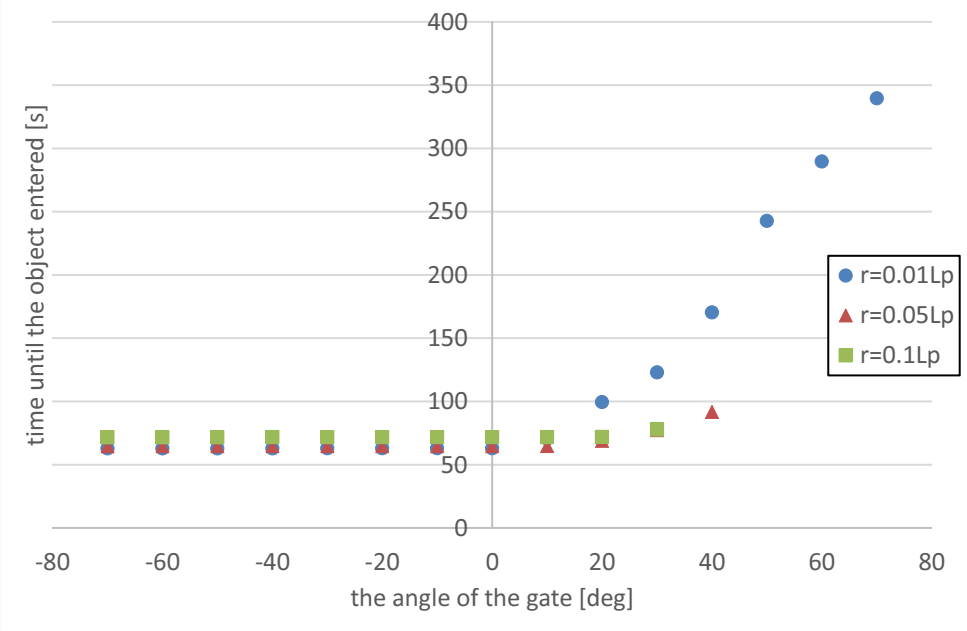


Figure 11. Differences of difficulty of *entering the hand* related to angle of the fingertips.

and narrowest passage gap. However, while the angle is negative, where entrance of the gap become wider with same narrowest gap, the quality of partial cage decreases according to the angle. Thus the quality of partial cage is not always determined only by the width of the narrowest gap but is also affected by the angle of the gate, that is, evaluation by simulations performed in the manuscript is necessary to investigate effect of the robot finger configuration in partial caging.

The efficiency of derived parameter for partial caging in responsive usage is discussed. As results in Sec. 3.4, the defined index score is high about $\theta_g = 10$ [deg] for every size of circular object object in this situation. Hence it means that a robot hand under the condition, $\theta_g = 10$ [deg] is assumed to be in an appropriate configuration to capture an circular object and confine without any finger motion. For example, this box-shaped hand with a gate whose angle $\theta_g = 10$ [deg] and no degrees of freedom has ability to capture balls and prevent them from release without any robotic motion. It can be useful for

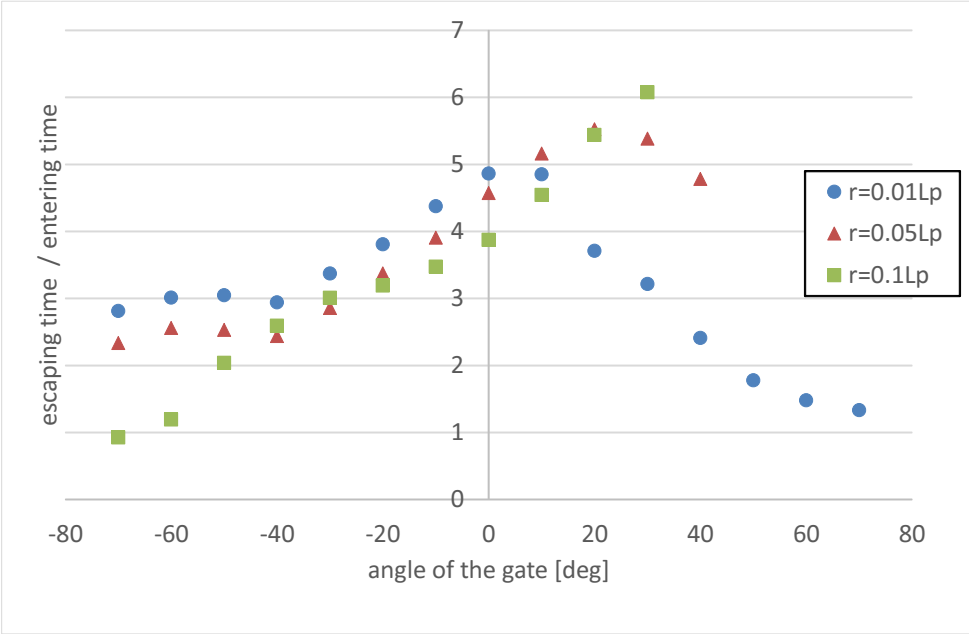


Figure 12. Difference of index evaluating partial caging quality related to the angle of the fingertips.

such as bin-picking without any active grasping or parts feeding mechanism.

3.6 *Simulation Results: Application to a Planar Robot Arm and Hand*

In this section, we apply our proposed partial caging to a planar robotic manipulator and a hand as Figure 3. We consider cases that the robot hand cannot confine a circular object completely due to mechanical limitation of the hand. In general grasping a small object such as by pinching is a reasonable strategy, but partial caging can be useful in situations where the hand has to store grasped object to capture multiple objects.

In addition, we also consider partial caging for a stick-shaped object, where it is generally difficult to confine it completely by robot hands.

3.6.1 Parameters Settings

The common settings for simulations are presented in Sec. 3.2, and others are determined as follows:

- The robot hand has two fingers, whose number of joints is two for each, and is attached to the manipulator with three DOFs. every length of the limbs of the arm and the hand is as follows: $L_{a1} = L_{a2} = 200$; the length of the palm, $L_p = 200$; $L_{11} = L_{21} = 200$; $L_{12}, L_{22} = 100$. The joint angles of the hand, $\theta_{11}, \theta_{12}, \theta_{21}, \theta_{22}$ are changed in each simulation.
- The joint angles of the arm, Θ_{aj}, Θ_{a2} are changed every time period in simulations with the following ranges, $\Theta_{a1,\mathrm{ini}} - \pi/4 < \Theta_{a1} < \Theta_{a1,\mathrm{ini}} + \pi/4$ [rad] and $\Theta_{a2,\mathrm{ini}} - \pi/4 < \Theta_{a2} < \Theta_{a2,\mathrm{ini}} + \pi/4$ [rad], where $\Theta_{i,\mathrm{ini}}$ is the initial angle of the i-th joint of the arm. In this section, we set $\Theta_{a1,\mathrm{ini}} = 0$, $\Theta_{a2,\mathrm{ini}} = \frac{\pi}{4}$ [rad], respectively, as Figure 3. And Θ_{a3} is always fixed at 0 [deg].
- The time period of changing the desired angles of the arm joints, is 10,000 steps, and joint angles change to the desired values during 5,000 steps at each constant angular velocity. These desired angles are randomly determined within each above range.
- Both the radius of the circular object and the half length of the stick-shaped object is $r_o = 50$.
- The limitation of simulation steps for a circular object is 300,000 and that for a stick-shaped object is 180,000.

Note that only the index score for difficulty of escaping from the hand is evaluated in this situation with *index 1*.

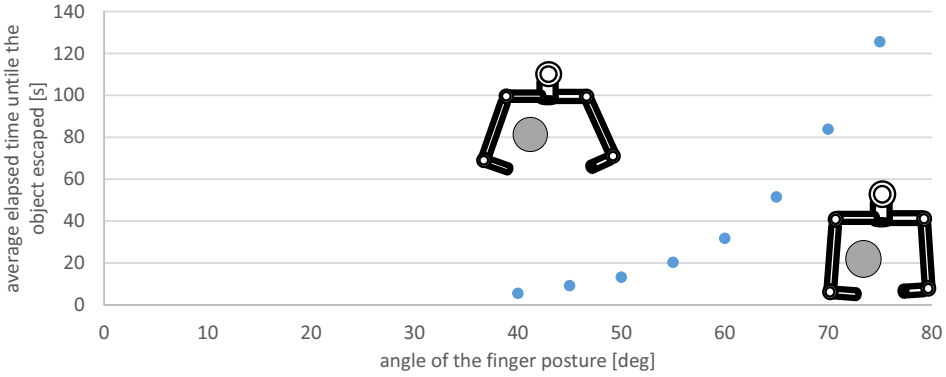


Figure 13. Difference of difficulty of escaping from the hand related to the posture of the fingertips.

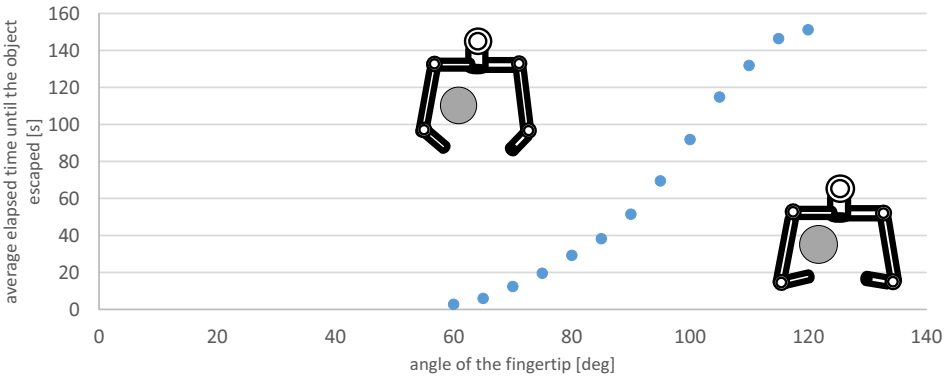


Figure 14. Difference of difficulty of escaping from the hand related to the angle of the fingertips.

3.6.2 Changing Angles of Fingers Joints for a Circular Object

This section presents the difference of difficulty of escaping from the hand related to the angles of fingers joints. First, θ_{11}, θ_{21} are changed and θ_{12}, θ_{22} are fixed at 90 [deg]. Next, θ_{11}, θ_{21} are fixed at 65 [deg], and θ_{12}, θ_{22} are changed.

The results that evaluated the scores of *index 1* while θ_{11}, θ_{21} are changed is shown in Figure 13. In this situation, since the distance between the fingertips decreases according to the angles of the finger, θ_{12}, θ_{22}, the result that difficulty of escaping increases according to the angles is reasonable as mentioned in Sec. 3.3.2.

In a case that θ_{12}, θ_{22} are changed, the scores of *index 1* increases according to the angle of joints (Figure 14), because the distance between the fingertips in this situation also decreases as the above case. Consequently, the scores of *index 1* obviously depends on the width between the fingertips (Figure 15).

In these situations (Figure 3 and 6), the direction of gravitational acceleration is often toward the gap between the fingertips. Additionally, the relative position and orientation between the object and the hand can be frequently changed not only by motion of the object but also by that of the hand. Thus retention of the object mentioned in Sec. 3.5 may not often occur.

3.6.3 Changing Orientation of a Stick-shaped Object and Fingertip Angle

This section deals with partial caging for a stick-shaped object (Figure 16), where we have to consider the orientation of the object against the gate because the difficulty of escaping from the cage strongly depends on it. The object can easily escape while the object is laid perpendicular to the goal. On the other hand, the object cannot sometimes escape while the object is laid parallel to the gap and is stuck to the limbs.

We investigate the difference of difficulty of escaping from the hand related to the angles of fingertip joints and the orientation of the stick. The joint angles, θ_{11}, θ_{21} are fixed at 65 [deg], and θ_{12}, θ_{22} are

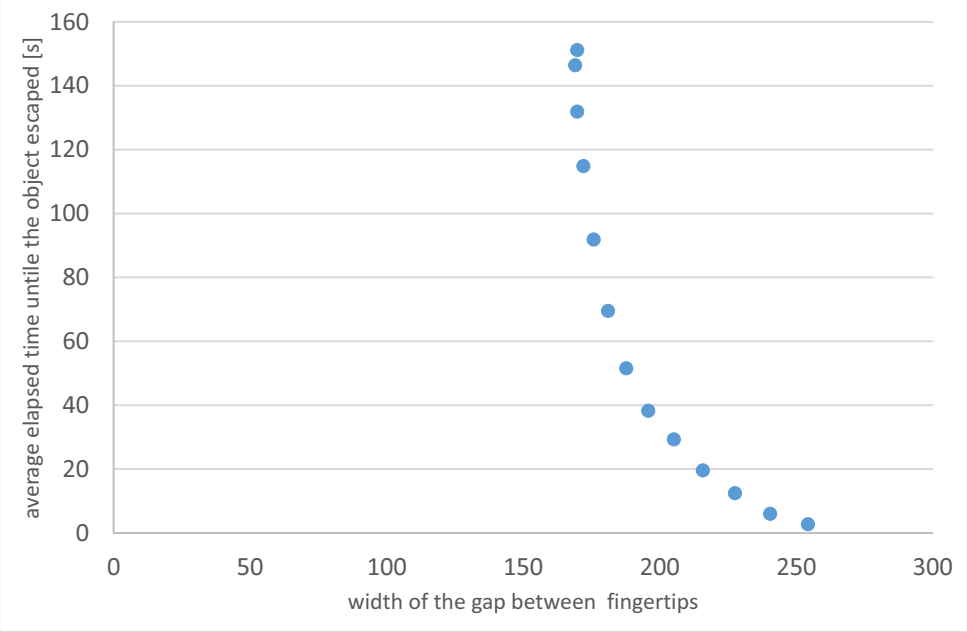


Figure 15. Difference of difficulty of escaping from the hand related to the width between the fingertips.

(a) $\theta_{12} = \theta_{22} = 60$ [deg] (b) $\theta_{12} = \theta_{22} = 120$ [deg]

Figure 16. Illustration of partial caging for a stick-shaped object.

changed from 60 to 120 [deg]. The initial orientation of the stick-shaped object is changed from 0 to 90 [deg].

The result of the simulations is illustrated by Figure 17. Note that a number of simulations that may include obviously inappropriate performances are removed from the results because some computational errors cause missing of collision check expressed in Sec. 2.3.2. That is, only seeds of random number generator contributing to calculation with no errors for every setting of simulation are applied. This result shows that difficulty of escaping for the stick-shaped object is slightly changed by its initial orientation. It is because the initial orientation can determine the first contact between the object and the hand, which strongly affects the behavior after the collision. Thus applying partial caging to any polygonal objects in further works or practical applications is suggested to consider initial orientation against the cage.

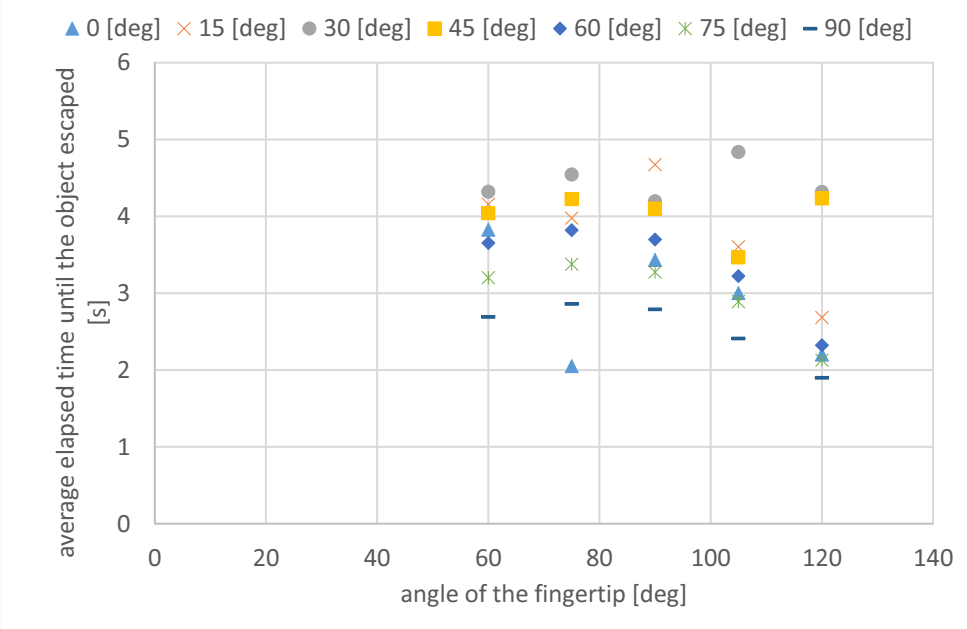


Figure 17. Difference of difficulty of escaping from the hand related to the angle of fingertips, for a stick-shaped object with changed initial orientation.

4. Conclusion

This paper discussed on quality of partial caging and efficient finger configuration for it. In partial caging, a robot hand surround an object geometrically with existing paths for the object to escape from the robotic cage. A goal of partial caging is to capture an object geometrically by position-controlled robots that have mechanical restriction such as insufficient degrees of freedom of the robots. Thus quality of partial caging can be defined as difficulty of escaping for the object from the hand confining it. On the other hand, ease of entering the hand from outside can be also evaluated by a similar way. From the two evaluation scores, we proposed an index score to evaluate the ability of robot hand for partial caging. It can be useful for design and caging strategies of robot hands that have mechanical restriction.

We considered a simplified scenario in which a planar two-fingered robot hand that has a gap between both fingertips confines an object, and the hand is manipulated by two types of manipulator. We defines evaluation indexes of quality of partial caging as elapsed time until the confined object escaped from the cage. The index scores of evaluation mainly depend on the width of the gap between fingertips, and also on retention of the object at wedge-shaped spaces formed by finger limbs. Width of the gap can be determined by two parameters of the hand: angles of the finger joints and length of the limbs. Retention of the object easily occurs when the angle of the joints is small or negative and a resultant space is narrow. The above two factors affecting behavior of the object are, however, trade-off with changing the angles of finger joints.

As for partial caging by a planar arm/hand robot, the width of the gap between the fingertips is principle of quality of partial caging for a circular object. It is because the gravitational force prevent stuck of the object at the foot of finger. On the other hand, difficulty of escaping from partial cage for a stick-shaped object depends on the object's initial orientation against the cage. It suggests that partial caging for polygonal object should take the relative orientation into consideration to increase its quality of confinement.

In future works, more variety of objects such as polygon shapes should be introduced to partial caging. Moreover, partial caging in three dimensional space by a multifingered hand is expected.

References

[1] Shimoga KB. Robot grasp synthesis algorithm: A survey. Int J of Robotics Research. 1996 June;15(3):230–266.

[2] Bicchi A, Kumar V. Robotic grasping and contact: A review. In: Proc. of IEEE int. conf. on robotics and automation. San Francisco, CA, U.S.A.. 2000 April. p. 348–353.

[3] Rimon E, Blake A. Caging planar bodies by one-parameter two-fingered gripping systems. Int J of Robotics Research. 1999 March;18(3):299–318.

[4] Trinkle JC. A quantitative test for form closure grasps. In: Proc. of IEEE/RSJ int. conf. on intelligent robots and systems. Raleigh, NC, USA. 1992 July. p. 1670–1677.

[5] Maeda Y. On the possibility of excessive internal forces on manipulated objects in robotic contact tasks. In: Proc. of IEEE int. conf. on robotics and automation. Barcelona, Spain. 2005 April. p. 1953–1958.

[6] Pipattanasomporn P, Sudsang A. Two-finger caging of nonconvex polytopes. IEEE Trans on Robotics. 2011 April;27(2):324–333.

[7] Diankov R, Srinivasa SS, Ferguson D, Kuffner J. Manipulation planning with caging grasps. In: Proc. of IEEE/RAS int. conf. on humanoid robots. Daejeon, Korea. 2008 December. p. 285–292.

[8] Makita S, Maeda Y. 3D multifingered caging: Basic formulation and planning. In: Proc. of IEEE/RSJ int. conf. on intelligent robots and system. Nice, France. 2008 October. p. 2697–2702.

[9] Makapunyo T, Phoka T, Pipattanasomporn P, Niparnan N, Sudsang A. Measurement Framework of Partial Cage Quality. In: Proc. of IEEE int. conf. on robotics and biomimetics. Guangzhou, China. 2012 December. p. 1812–1816.

[10] Kuperberg W. Problems on polytopes and convex sets. In: Dimacs workshop on polytopes. 1990 January. p. 584–589.

[11] Gopalakrishnan KG, Goldberg K. Gripping parts at concave vertices. In: Proc. of IEEE int. conf. on robotics and automation. Washington, DC, U.S.A.. 2002. p. 1590–1596.

[12] Sudsang A. A sufficient conditions for capturing an object in the plane with disc-shaped robots. In: Proc. of IEEE int. conf. on robotics and automation. Washington D.C., U.S.A.. 2002 May. p. 682–687.

[13] Rimon E, Blake A. Caging 2d bodies by 1-parameter two-fingered gripping systems. In: Proc. of IEEE int. conf. on robotics and automation. Minneapolis, MN, U.S.A.. 1996. p. 1458–1464.

[14] Pipattanasomporn P, Sudsang A. Two-finger caging of concave polygon. In: Proc. of IEEE int. conf. on robotics and automation. Orlando, FL, U.S.A.. 2006 May. p. 2137–2142.

[15] Pipattanasomporn P, Vongmasa P, Sudsang A. Two-finger squeezing caging of polygonal and polyhedral object. In: Proc. of IEEE int. conf. on robotics and automation. Rome, Italy. 2007 April. p. 205–210.

[16] Alberto Rodriguez MTM. Two finger caging: Squeezing and stretching. In: Chirikjian GS, Choset H, Morales M, Murphey T, editors. Algorithmic foundation of robotics viii. Vol. 57 of Springer Tracts in Advanced Robotics. Berlin Heidelberg: Springer. 2010. p. 119–133.

[17] Rodriguez A, Mason MT, Ferry S. From caging to grasping. The Int J of Robotics Research. 2012 Apr; 31(7):886–900.

[18] Davidson C, Blake A. Caging planer objects with a three-finger one-parameter gripper. In: Proc. of IEEE int. conf. on robotics and automation. Leuven, Belgium. 1998. p. 2722–2727.

[19] Erickson J, Thite S, Rothganger F, Ponce J. Capturing a convex object with three discs. IEEE Trans on Robotics. 2007 December;23(6):1133–1140.

[20] Vahedi M, van der Stappen AF. Caging polygons with two and three fingers. Int J of Robotics Research. 2008 November/December;27(11-12):1308–1324.

[21] Fink J, Hsieh M, Kumar V. Multi-robot manipulation via caging in environments with obstacles. In: Proc. of IEEE int. conf. on robotics and automation. 2008 May. p. 1471–1476.

[22] Wang Z, Kumar V. Object closure and manipulation by multiple cooperating mobile robots. In: Proc. of IEEE int. conf. on robotics and automation. Washington D.C., U.S.A.. 2002 May. p. 394–399.

[23] Yokoi R, Kobayashi T, Maeda Y. 2d caging manipulation by robots and walls. In: Proc. of IEEE int. symp. on assembly and manufacturing. Suwon, Korea. 2009 November. p. 16–21.

[24] Jia Y, Guo F, Lin H. Grasping deformable planar objects: Squeeze, stick/slip analysis, and energy-based optimalities. The Int J of Robotics Research. 2014 February;Published online before print(N/A).

[25] Wan W, Fukui R, Shimosaka M, Sato T, Kuniyoshi Y. Grasping by caging: A promising tool to deal with uncertainty. In: Proc. of IEEE int. conf. on robotics and automation. St. Paul, MN, USA. 2012 May. p. 5142–5149.

[26] Su J, Qiao H, Ou Z, Liu ZY. Vision-based caging grasps of polyhedron-like workpieces with a binary industrial gripper. IEEE Trans on Automation Science and Engineering. 2015 July;12(3):1033–1046.

[27] Maeda Y, Kodera N, Egawa T. Caging-based grasping by a robot hand with rigid and soft parts. In: Proc. of IEEE int. conf. on robotics and automation. St. Paul, USA. 2012 May. p. 5150–5155.
[28] Makita S, Okita K, Maeda Y. 3d two-fingered caging for two types of objects: Sufficient conditions and planning. Int J of Mechatronics and Automation. 2013 December;3(4):263–277.
[29] Jiang Y, Zheng C, Lim M, Saxena A. Learning to place new objects. In: Proc. of IEEE int. conf. on robotics and automation. St. Paul, MN, USA. 2012 May. p. 3088–3095.
[30] Makapunyo T, Phoka T, Pipattanasomporn P, Niparnan N, Sudsang A. Measurement framework of partial cage quality based on probabilistic motion planning. In: Proc. of IEEE int. conf. on robotics and automation. 2013 May. p. 1574–1579.
[31] Makita S, Nagata K. Evaluation of finger configuration for partial caging. In: Proc. of IEEE int. conf. on robotics and automation. Seattle, WA, USA. 2015 May. p. 4838–4844.
[32] Free Software Foundation, Inc. GSL (GNU Scientific Library). http://www.gnu.org/software/gsl/. 1996.