To appear in *Advanced Robotics*
Vol. 00, No. 00, August 2017, 1–18

FULL PAPER

A Survey of Robotic Caging and Its Applications

Satoshi Makita[1] and Weiwei Wan[2*]

[1] *National Institute of Technology, Sasebo College, Japan*
[2] *National Institute of Advanced Industrial Science and Technology (AIST), Japan*
(*Received 00 Month 201X; accepted 00 Month 201X*)

This paper presents a brief survey of robotic caging and its applications. Caging is a kind of grasping methods, which can be accomplished geometrically by position-controlled robotic agents, and has advantages over conventional manipulation which requires to fulfill mechanical conditions. Due to the advantages, caging was extensively studied and applied in robotics. This paper reviews the robotic literature related to caging ranging from its historical background, state-of-the-art developments, to practical applications. It provides our insights on some open problems and promising research and application directions. We hope the paper could help researchers quickly catch the strength and limitations of caging, and make impacting contributions to the research community.

Keywords: Caging; Caging Grasps; Forceless Grasping; Envelope Grasping;

1. Introduction

Caging is a kind of grasping methods, which can be applicable to various tasks that need capturing, ranging from pregrasping by robotic fingers to formation-controlled transport by mobile robots. The difference from conventional grasping [1, 2] is that caging is assumed to be accomplished by position-controlled robots without considering mechanical properties like contacts and forces. This merit relaxes robot control during manipulation.

The original concept of caging was raised by Kuperberg as a problem of finding a formation of points that prevented a polygon from moving arbitrarily far from its original position in the planar space [3]. This concept was further discussed in [4] from the viewpoint of mathematics, where problems related to caging were treated as *holding* problems. Discussions about holding problems existed long before the concept of caging. One early study was done by Besicovitch [5] who discussed the conditions that allow a net of inextensible strings to enclose a sphere and stop it from slipping out. A similar study was performed by Croft [6] who not only considered about the condition, but also discussed the minimum length of the holding net. The problem of holding an unit sphere using a polyhedron was studied [7], where the difference from a net was that the edges were undeformable line segments. A similar problem was discussed by Sphephard [8]. More generally, Zamfirescu [9] and Fruchard [10] studied the problem of holding convex bodies like triangular pyramids [11]. These studies were all about the caging problems outside robotics.

The seminal work that introduced caging into robotics was done by Rimon et al. [12]. The merits of caging made it a compelling tool to deal with uncertainties in robotics, which induced researchers to apply caging to multi-finger pregrasping [13], and multi-robot cooperative transport [14]. In the field of robotic grasping, caging can be regarded as the extension of *form closure* [15], where an object caged by fingers is allowed to move within a closed region, instead of being firmly grasped. Likewise, in multi-robot cooperative transport, an object caged by mobile robots

*Corresponding author. Email: wan-weiwei@aist.go.jp

can move within a closed region without interfering with robot bodies. The closed region adds flexibility to conventional methods and empowers caging the ability to deal with uncertainty. When the closed region shrinks as the finger formation or the mobile robot formation changes, the object converges to a certain position and orientation. This converged situation is known as *immobilization* [16] or *immobilizing grasps* [13, 17].

This paper presents a survey of robotic caging and its applications. It starts the discussion from static grasping, studies the relationship between caging and static grasping, and organizes the review of caging considering the relationship. Particularly the paper summarizes an overview of the studies related to caging ranging from its historical background, state-of-the-art developments, to practical applications, and provides our insights on some open problems and promising research and application directions. In Section 2, we discuss the relationship between grasping and caging. Section 3 summarizes previous work related to algorithmic caging with focus on different dimensions are summarized. Section 4 introduces various practical applications of caging performed in the real world, including grasping, manipulation, and multi-robot cooperative transport. Section 5 presents the open problems of caging and possible future applications.

2. Caging and Grasping

Known with much literature such as [1, 2, 18], the goal of robotic grasping problems are to determine contact points between a target object and robot fingers where satisfying force equilibrium could be applied to the target body. The analysis is performed in wrench space and is phrased as form closure, force closure, and fixturing. Form and force closures were extensively studied in mechanics and design of machines [19]. The concepts were initially discussed in robotic grasping by [20], and had been followed by many robotic researchers [21–23]. In form closure grasps, which was firstly introduced to robotics research by [20], contact wrenches (forces and torques) exerted by robots along surface normals are analyzed to generate firm grasp formation. In force closure grasps, similar contact wrenches exerted by robots are considered, but the applied wrenches could be in arbitrary magnitude and direction. Since there are no essential differences in the problems on "closure", some studies notably [24, 25] used the term of "force closure" as "form closure", as pointed by [15]. We differentiate form closure and force closure by friction forces. Form closure only considers the normal forces on contact points. In contrast, force closure considers both the normal forces and friction forces. Rimon et al. [26][27] initially pointed out that the early form closure theory failed to take into account object's surface curvature. The early form closure theory required 4 contact points on 2D polygons, which was beyond our intuition (the intuitive number is 3). Based on the discovery, Rimon et al. proposed the concept of 2nd-order form closure and 2nd-order immobility (see also [16]). He pointed out that immobilization was essentially performed based on geometrical constraint where *any local motion of the target object should be prevented by the rigidity of the object and the robots* [28]. Together with Blake, Rimon [12][13] published the work on caging a planar concave object by using two robotic fingertips, which was recognized as the seminal study on robotic caging. In the work, Rimon et al. considered *immobilization grasp* and *puncture points* for the target object. *Immobilization grasp* is a subset of the equilibrium grasps considering 2nd-order immobility, which ensures that the object can be grasped with fewer fingers than form closure grasps. *Puncture points* compose a subset of immobilization grasps, which represents a boundary of set of *caging*. The 2nd-order immobility was thoroughly discussed in [29, 30], and was expanded for immobilization of 3D objects in [31].

In this paper, we view force closure and immobilization as the results of analysis in different spaces. Force closure is done in wrench space where the goal is to ensure the convex hull formed by wrenches encloses the origin of the space. Although wrench space simplifies the analysis of force closure, it cannot take into account the curvature of object's surface (Fig.1). Wrench space analysis only meets the definition of early form and force closure theories. It is 1st order analysis and requires redundant fingers to grasp. On the other hand, immobilization is essentially done in the configuration space of object [32], where the object is constrained to a single point at if it was immobilized. Analyzing in the configuration space of object relaxes the early definition of form closure. If an object has outward-bending 2nd-order surface curvature at contact points, it would be constrained into a single point in the configuration space of object, and is immobilized (Fig.2). A interesting discussion about immobilization could be found in [33], where Stappen et al. computed all form closure grasps and immobilization grasps of a polygonal part with suitable number of frictionless pointed fingers [34]. Immobilization is a key to bridge form closure and caging theories. Sudsang et al. presented algorithms for immobilizing a 3D object by using several contact points and a plane [35], and also algorithms for immobilizing a planar object by using disc shaped robots [14, 36]. The algorithms were applied to in-hand manipulation in [37]. Gopalakrishnan et al. [38] studied grasping a planar deformable object by two fingertips considering its model of elasticity and potential energy. As a result of deformation, the shape of the gripped object became concave, and the grasps for the deformed object, which was called *deform closure*, looked more similar to caging instead of immobilizing. The same authors also proposed *contracting and expanding v-grip*, where a parallel jaw gripper was planned to grip a polyhedron at its concave sections [39]. The *contracting v-grip* was equivalent to planar immobilizing grasp using a two-fingered hand [13], which is also called as *squeezing caging* [40]. The *expanding v-grip* was equivalent to the concept of *dispersion caging* presented in [41, 42] or *stretching caging* in [43].

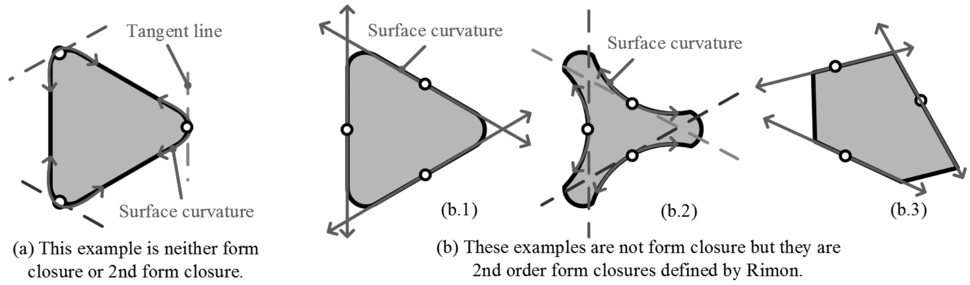

Figure 1. Curvatures of contact points. The contact points in (a) have negative 2nd-order surface curvature. Its contact surfaces are bending inward the contact tangent lines. The object is neither in form closure nor 2nd-order form closure. The contacts in (b.1), (b.2), and (b.3) have zero or positive 2nd-order surface curvature. Their contact surfaces are along or bending outward the contact tangent lines. They are not form closure but fulfill the requirements of 2nd form closure. The objects are immobilized.

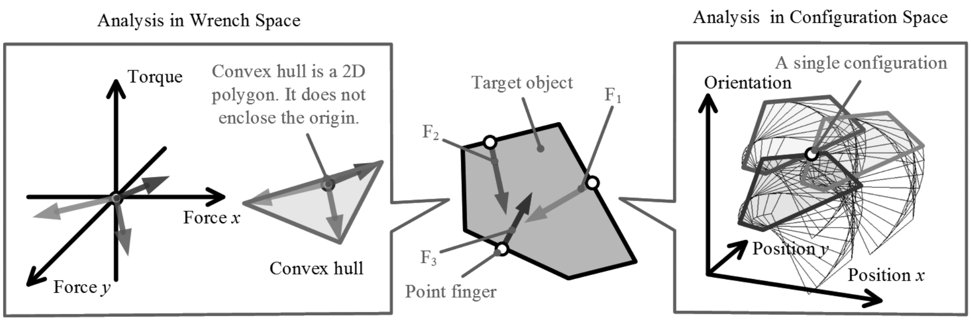

Figure 2. Details of Fig.1(b.3). In wrench space, the wrenches of Fig.1(b.3) form a planar polygon, which doesn't enclose the origin. It is not in form closure. In configuration space, the object is constrained to a single configuration. It is immobilized.

Caging is the expansion of immobilization, which could also be analyzed in configuration space. Fig.3 shows the relationship between immobilization and caging. At immobilization state, the object is constrained to a single configuration in configuration space. As the fingers retract from the contact surface, the single configuration expands into a subspace, allowing the object to move inside a caged area. If the retraction is too large, cages break. The constrained subspace (the block separated from the surrounding obstacles) in Fig.3(b) and (c) correspond to closed regions in workspace where the object could move inside but could not escape out of. The closed region adds flexibility to conventional methods and empowers caging the ability to deal with uncertainty.

Note that although caging is the expansion of immobilization, the robotic agents of cages are not limited to fingers. It could be mobile robots, environmental obstacles, tools, etc.

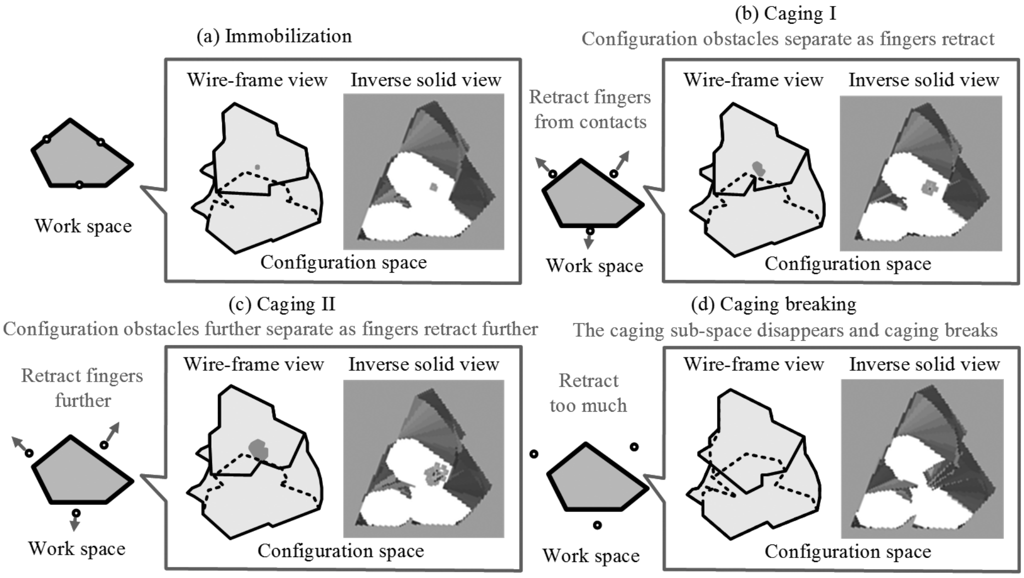

Figure 3. The relationship between immobilization and caging. They are both analyzed in Configuration space. (a) At immobilization state, the object is constrained to a single configuration in configuration space. (b, c) As the fingers retract from the contact surface, the single configuration expands into a subspace, allowing the object to move inside a caged area. (d) If the retraction is too large, cages break.

Following the description, immobilization and caging can be formally defined as follows. Suppose $\mathcal{C}_{\text{free\_obj}} \in \mathrm{R}^m$ denotes a configuration subspace in which the object is allowed to move freely, $\mathcal{C}_{\text{free\_inf}} \in \mathrm{R}^m$ denotes a configuration subspace that includes a point at infinity, $\boldsymbol{q}_{\text{obj}} \in \mathrm{R}^n$ denotes a pose of an object in work space, n and m denote the number of dimension of work space and its corresponding configuration space respectively, that is, $m = 3$ when $n = 2$, and $m = 6$ when $n = 3$. When $\mathcal{C}_{\text{free\_obj}} = \boldsymbol{q}_{\text{obj}}$, the object is immobilized. When $(\mathcal{C}_{\text{free\_obj}} \neq \boldsymbol{q}_{\text{obj}}) \cap (\mathcal{C}_{\text{free\_obj}} \neq \emptyset) \cap (\mathcal{C}_{\text{free\_obj}} \cap \mathcal{C}_{\text{free\_inf}} \neq \emptyset)$, the object is caged [44].

A simple form of caging is envelope caging, where redundant number of agents constrain a target object by enveloping it [45][46]. Since the number of agents are redundant, envelope caging looks like real cages or fences, and the caged object may rotate freely inside the fence. In contrast to envelope caging with redundantr number of agents, using non-redundant number of agents to cage is complicated. For example, two agents could only cage objects with concavities, although it was already solved decades ago [12]. Three-agent caging is still an unsolved problem. Only some variations with reduced degree of freedoms or special shapes of objects would be tackled [13, 47, 48]. Caging using more than three robotic agents is similar to three-agent caging, and could only be solved partially [42, 49]. Note that the condition to judge caging is applicable to any robots and objects in 2D work space. It is the most intuitive way to judge a cage in 2D work space. In 3D work space, the caging problems are complicated. We will review the developed algorithms in state-of-the-art literature to solve the problems of caging using redundant and non-redundant number of agents in both 2D and 3D work spaces in next section, and discuss about the open problems in Section 5.1.

As a summary of this section, the relationship between force closure to caging is:

$$Force\ closure \xrightarrow{frictionless} Form\ closure \xrightarrow{curvature} 2nd\text{-}order\ Form\ closure$$
$$\xrightarrow{configuration\ space} Immobilization \xrightarrow{non\text{-}single\ configuration} Caging \quad (1)$$

3. Algorithmic Caging

The goal of caging algorithms is to solve two problems: (1) *Caging test*: Given a formation of robotic agents and an object, decide whether the formation can cage the object. (2) *Finding caging sets*: Given an object and some robotic agents, find some configuration sets where the robotic agents cage the object. In most studies, the two problems are not differentiated. We therefore review them together, and divide the contents of this section according to the agents and the dimensions of target objects, instead of the two problems.

3.1 *2D and 2.5D Caging*

This section gives an overview of planar caging and its theoretical approaches. It includes the studies related to both 2D and 2.5D caging. 2.5D refers to a surface which is a projection of a plane into 3rd dimension. The object is 3-dimensional, but there are no overhanging elements possible. 2.5D objects are often represented by a contour map that gives the height of the object at each point. 2.5D caging means "planar caging practically performed by robotic systems in the three-dimensional real space". Its related algorithms follow the same principle as 2D objects and we review it together with the algorithms of 2D caging.

The literature of this section is organized based on the following classification. For two-agent and three-agent caging, the number of agents is non-redundant. For the others, the number of agents could be either redundant or non-redundant. Fig.4 exemplifies these problems with examples.

- Two-agent caging
- Three-agent caging
- Caging by more than three agents
- Caging with environments
- Incomplete caging

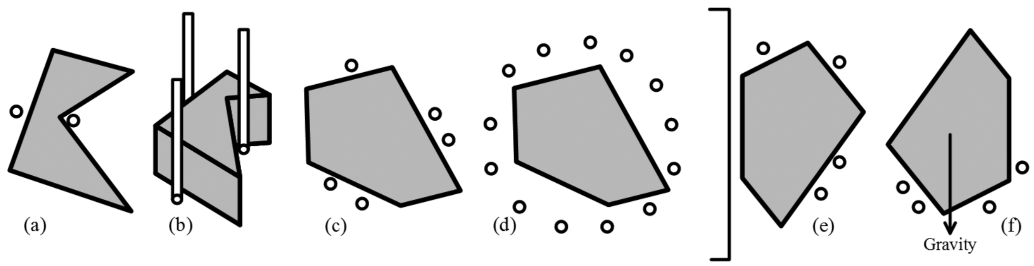

Figure 4. Classification of the caging problems: (a) Two-agent caging. (b) 2.5D three-agent caging. (c) Caging by more than three agents (non-redundant). (d) Caging by more than three agents (redundant). (e) Caging with environments (f) Incomplete caging.

Two-agent caging: Two-agent caging is the basic problem of caging. As mentioned in the previous section, the work of Rimon et al. [12, 13] was the initial study on robotic caging algorithms. It formulated the two-finger caging problem and solved the problem using immobilizing grasps and puncture grasps. Using Morse theory, Rimon et al. proved that puncture grasps are equivalent to equilibrium grasps, and represented boundaries of *caging sets* using immobilizing grasps.

Besides Rimon, Pipattanasomporn et al. studied the problem of planar two-fingered caging for polygons with concavity [40]. A key idea of two-fingered caging, *squeezed trajectory*, was introduced in the work, in which two point fingers were placed at concave sections of the object, and the trajectories of squeezing are tested for caging. Pipattanasomporn et al. later applied the algorithm of squeezing trajectories to the cases of 3D polyhedra with two concave sections [50], which was basically as a similar approach to planar caging. Another name of two-finger caging is *dispersion control* [41], in which each finger is located at one opposite concave section and disperses from each other. The names *stretching operation* [40] and *stretching caging* [43] are also available in literature and represent similar things. A good summary of the algorithms for two-finger caging of both 2D and 3D objects, was done in the review section of [51]. An extension to non-convex objects was presented by Thomas et al. [52, 53], which introduced the method of contact space search for two-finger caging. Note that in the literature, robotic finger agents are assumed to be points. Fingers with radius, e. g., disc-shaped fingers, can be treated as pointed fingers by enlarging target objects using the Minkovski sum.

Three-agent caging: Compared with two-agent caging in which target objects are limited to polygons concavity, the target objects of three-agent caging may include convex shapes and redundant finger(s), which make caging problems more difficult. Researchers usually fix some degrees of freedoms to simplify the problem. For example, Ponce et al. [16] studied the *capture region* that prevented an arbitrary convex object from escaping from the cage formed by two fixed fingers and a movable finger. Given two fixed fingers, the goal of the problem was to find a region for the third finger where the three fingers together cage the target object. The problem was later solved by Erickson et al. in [54, 55] using z-buffer, and further was developed by Vahedi et al. in [56–59]. Wan et al. also studied the problem. He not only developed algorithms to find caging regions for the third finger [60], but also extended it to caging optimization or finding the most robust caging configurations [49].

Beyond two fixed fingers, Davidson et al. studied caging planar object using a three-finger one-parameter hand [61, 62]. The work was essentially a direct extension of the two-finger caging studied by Rimon et al. [12, 13], which found the boundaries of caging sets using puncture grasps. Bunis et al. [48] studied a similar one-parameter three-finger caging problem. Different from Davidson et al., Bunis et al. analyzed the problem in contact space and found the caging sets using *caging graph search*. Some other work like [63][64] studied the caging problems of four-finger one-parameter hand, which basically had the same complexity as three-finger one-parameter hands. More flexibly, Sudsang et al. used mobile robots as robotic agents to transport polygons [36][65][66]. He allowed free motion of each mobile robot, and proposed several rules to push target objects as well as maintain cages.

Caging by more than three agents: Caging by more than three robotic agents is also widely studied for practical applications like multi-robot transport. By far, the most effective way to judge a cage formed by more than three agents is using $(\mathcal{C}_{\text{free\_obj}} \neq \boldsymbol{q}_{\text{obj}}) \cap (\mathcal{C}_{\text{free\_obj}} \neq \emptyset) \cap (\mathcal{C}_{\text{free\_obj}} \cap \mathcal{C}_{\text{free\_inf}} \neq \emptyset)$. By expanding the condition using CC (configuration space of configuration space), Wang et al. proposed several caging test algorithms for more than three mobile robots [44, 67–73]. Wang et al.'s algorithms could solve caging problems with more than three but non-redundant number of agents (although the number of employed agents might not be smallest). Similar to Wang et al., Pereira et al. studied multi-robot caging using non-redundant number robots in [74–76]. He proposed the concept of *object closure conditions* for multi-robot cooperative transport. The conditions were maintained while transporting an object to a desired goal. To fairly contribute to the payload, the object closure condition was allowed to be broken. Conditional (force) closure, in which the object was partially caged, was introduced to tackle these cases. Suarod et al. presented an heuristic approach to compute caging formation of multi-robots from given loose region within which the object can move [77]. Their number of mobile robots was non-redundant too. Dai et al. [78] proposed symmetric caging formation for convex polygon to reduce the number of agents and to decrease rotation of the object. Wan et al. also reduced the number of mobile robots to accomplish caging the object [49]. Rodriguez et al. [79, 80] proposed the concept of F-cage, which presented an independent definition of multi-agent cages using topological analysis.

In contrast to the studies about non-redundant number of agents, there are lots of studies about redundant agents. The difference between redundant and non-redundant number of agents are as follows. Non-redundant number of agents: The agents cage objects using their geometric features. Objects cannot rotate freely. Redundant number of agents: The agents cage objects by forming a fence. Objects may rotate freely inside the fence. Trinckle et al. [45] and Eberman et al. [46] proposed envelope grasp, which was essentially caging using redundant number of fingers or links. Fink et al. proposed a fence transport [81], where a large number of mobile robots trapped and transported objects. Vongmasa et al. [82] studied coverage diameter, which actually indicated the largest gap between agents, and could help compute the smallest number of redundant agents. More recently, Varasa et al. [83] proposed herding by caging, which also used redundant number of robots to move objects.

Caging with environments: Caging an object with least number of robotic agents is a difficult problem. As the number of robots decreases, the cage becomes small and the flexibility in object positions and orientations becomes limited. To reduce the number of agents as well as to make the most of cages, researchers proposed two methods: (1) Taking the advantages of environments around the robots and object, and (2) allowing incompleteness of caging.

Caging by multiple mobile robots and environments (which are often called as walls) had been independently studied by [84, 85]. Yokoi et al. studied transporting planar objects along the walls while the robots maintaining the condition of object closure together with walls. Banyassady et al. formulated the problem of caging polygons by a point and a line [85], where the line could be considered as a part from the environment. Varkonyi [86] proposed a caging feeder to reorient parts. In the theoretical section, Varkonyi developed an algorithm to find a range of circle radius that led to monostatic behavior using alpha hull. The circles that exhibited monostatic behavior were a kind of environmental cages.

Incomplete caging: Incomplete caging, which is also known as *partial caging* [87, 88] or *conditional object closure* [44], is important in reducing the number of robots. Incomplete caging is practical since (1) most real-world robotic agents (robot arms, fingertips) are interconnected with each other instead of distributed, and (2) the gravity of objects cannot be ignored. Consequently, researchers studied incomplete caging to make use of the interconnected degree of freedoms and ignorable gravity. For one thing, compared with grasping, incomplete caging still constrains objects using a cage. Objects have some freedom inside the cage. For the other, the cage in incomplete caging is virtual and is not as strict as complete caging. Objects may escape when their energy is beyond certain thresholds.

Incomplete cages allow some escaping paths through the gap between the robots. As the robotic agents change their configurations, the number of escaping paths and their reachability differ. Therefore, studying the escaping paths and the difficulties of escaping is usually the goal of related work. For example, Makapunyo et al. discussed a framework to measure the quality of partial caging [87] based on the concept of motion planning of rigid bodies. They evaluated the quality of partial cages with several probabilistic path searching trials [89], in which the evaluation index was represented by elapsed time to find an escaping path. Makita et al. considered a planar two-fingered hand for partial caging. In their work, the interconnected robotic fingers had a gap between its fingertips where the captured object could escape [88]. They evaluated the possibility of escaping from the gap considering dynamics. The quality of their partial caging was the elapsed time until the captured object escaped from the constraint. In a later work [90], Makita et al. further examined two object primitives, a single point and a line segment. More recently, Mahler et al. proposed the concept of *energy-bounded caging*, in which the target object partially caged by robots can be prevented by gravitational force from escaping from traps [91].

Others: Instead of pure caging, several researchers developed theories and algorithms to use caging for pre-grasp. Rodriguez et al. discussed the relationship between caging and grasping with examples of two-finger caging, and generalized the problem to the cases of n fingers in [79, 80]. Wan et al. proposed *grasping by caging* for tolerance of pose and shape uncertainty caused by mechanical error of robots and sensory noises of perception [92], although they only considered convex objects. Wan et al. [47, 93] also improved their algorithm to find caging regions and immobilization points by using *space mapping* between work space and configuration space. The method based on space mapping could deal with both convex and concave shapes. It can quickly test planar caging formations.

3.2 *3D Caging*

Extending from 2D and 2.5D caging to 3D caging is challenging. The first difficulty is high dimensionality. The configuration space of 2D and 2.5D problems is 3D. It is possible to recover the complete configuration space using the ability of modern competitors. In contrast, the configuration space of 3D problems is 6D. Recovering the 6D configuration space and investigating the caged subspaces in it are computationally infeasible. In motion planning, researchers used probabilistic methods [94, 95] to build probabilistic roadmaps and search valid paths in high dimensional spaces. While the probabilistic methods could overcome the curse of dimensionality in a certain degree, they are not applicable to caging problems like caging test and finding caging sets. The caging problems require complete analysis, instead of probabilistic results. Another difficulty of 3D caging is the mechanism of robotic agents. The robotic agents in 2D and 2.5D are assumed to be distributed point fingers and point mobile robots, without considering interconnections. Implementing these distributed mechanisms for 2D and 2.5D caging is possible[96]. In contrast, robotic agents in 3D work space are difficult to be distributed: Fingertips cannot be considered independently, floating agents like drones are limited by nonholonomic constraints, etc. Researcher studying 3D caging have to use the interconnections of agents to solve practical problems, although challenging the high dimensionality without considering mechanical implementation is also an interesting topic.

Caging for 3D polyhedral objects was firstly studied as an extension of planar caging. These work focused on challenging the high dimensionality without considering mechanical implementations. For example, Pipattanasomporn et al. discussed caging rigid polytopes using two pointed fingers in [50]. It was the seminal work of 3D two-finger caging. Pipattanasomporn et al. later improved their algorithms to tackle caging problems with any number of fingers in any dimensional workspace [42]. Allen et al. also studied the two-finger caging problems of 3D polytopes [97] based on caging graph search in contact space [52].

There are also several studies which used multi-finger hands to cage 3D objects. These research takes advantages of the interconnections between finger links to solve practical problems. For example, Makita et al. studied caging for four primitive shapes of object using a skeletal multi-finger hand [98, 99]. They divided sufficient conditions for 3D caging using a multi-finger hand, which was named *3D multi-finger caging*, into three types: (1) Caging by surrounding (envelope-type caging), (2) caging using concave sections (waist-type caging), and (3) caging by hooking or knotting (ring-type caging). Pokorny et al. presented a method to plan caging grasps on objects with holes such as bags, mugs, etc., by using topological expression of loops of both the objects and the robot hands [100, 101]. The assumption was that objects with holes could be caged when a hopf link is formed between the robot hands and the hollow parts of the object [98]. Kwok et al. [102] studied rope caging, where a rope was stretched on the surface of 3D objects following topological rib graphs. Their algorithms were applicable to 3D objects with constricted parts [103] or necks [104]. Zarubin et al. used geodesic balls to determine caging regions of several complicated objects. Their approaches were named *circle caging* or *sphere caging* [105]. Circle caging let the robot hands wind the of neck of the object as *waist-type caging*. Sphere caging let the robot hand wrap a part of the object as *envelope-type caging*. In addition to the complete caging on objects with holes, Stork et al. studied hooking the loop [106] using partial caging. Partial caging in 3D work space could be considered with applied external forces to prevent the captured object from escaping. Jiang et al. proposed the concept of *gravity caging* [107]. He derived placing areas in which objects could be suitably held by a few supports under gravitational force, e. g., the inner side of pen-holders is the placing area for pens.

4. Practical caging systems

This section reviews the practical robotic systems that use the principle of caging. Following the types of robotic agents, the section is divided into grasping and multi-robot cooperative transport. Some examples are shown in Fig.5.

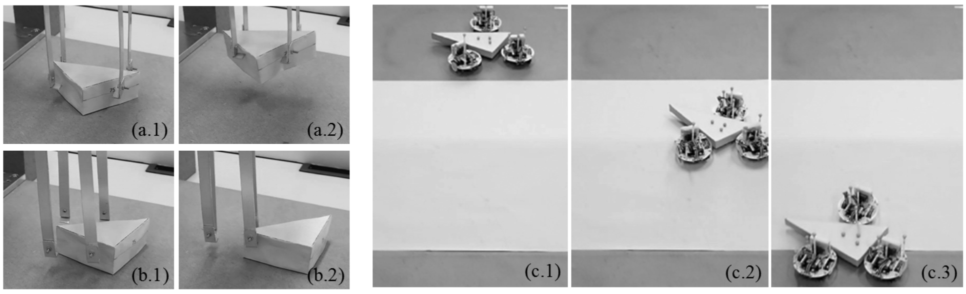

Figure 5. Using caging to conduct grasping and multi-robot cooperative transport. (a.1, a.2) Using caging as pregrasp. (b.1, b.2) A failure case where caging was not used as pregrasp. (c.1, c.2, c.3) Using caging to transport objects.

4.1 *Caging in Robot Grasping*

Most practical caging-based robot grasping systems are based on the 2D and 2.5D caging algorithms. The robotic agents are usually parallel jaw gripper with multiple sticks as fingers. Some algorithmic research such as [47, 48, 92, 93] tested their algorithms using computational dynamics engines and self-built robot platforms. These platforms appropriately demonstrated the advantages of caging grasps, in which the robots prevent the object from escaping without precise force control and accurate object perception. They are some of the practical caging-based grasping systems. More specifically, Wan et al. developed a four-finger one parameter hand using repeated caging optimization. The optimized design was claimed to be able to cage large number of objects using one actuator. Su et al. studied 2.5D caging of polyhedron considering the width of inner polygons [64]. He demonstrated their algorithms by caging several industrial parts using a gripper composed of four sticks and a vision system. Vakornyi's caging feeder is also a 2.5D practical caging grasping system [86].

Using 2D and 2.5D caging to manipulate objects in the micro world is also a popular research field. In micro scale, obtaining fine sensor data for positioning and force control of manipulators is difficult. There are lots of uncertainty like perception noises, control noises, and unexpected forces (e.g. electrostatic force, surface tension, van der Waals, casimir, etc) [108, 109]. Several practical caging systems were developed to deal with the uncertainty. For example, Grier et al. discussed about optical tweezers, which used energy of photons to trap beads or cells [110]. The optical tweezer is a practical caging system. Hu et al. [111] proposed electro-thermally activated cell manipulator, which used thermal energy to trap cells. It is also a practical caging system. The optical tweezers and the electro-thermally activated manipulators are energy bounded caging. They are not directly related to the algorithms discussed in previous sections. More near to the previous algorithms, Cappelleri et al. presented an application of planar caging grasps in micro manipulation/assembly by using multiple single micro probes [112, 113]. The system was improved to do both 2D and 3D micro-manipulation later in [114] [115].

There are also researchers who used caging to do in-cage manipulation. Blind et al. [116] used actuated piston pin array attached to a pachinko machine to manipulate objects. Their essential technique was to construct the capture regions using the pin array and perform manipulation inside the capture regions. This capturing approach is similar to the idea of modular fixturing of the vise toolkit [117]. Ma et al. [118] and Maeda et al. [119] respectively studied in-cage manipulation using two linked fingers, which could be viewed as a general form of in-hand manipulation.

Caging grasps in the real 3D workspace is not as widely developed since there remains several open problems in algorithmic foundations. However, practical systems using redundant number of agents to cage can still be found in literature. Diankov et al. presented a pose planing system by taking the advantages of 3D caging and probabilistic motion planning [120]. The goal nodes of the motion planning were cages of 3D objects using a three-finger hand. The goal of the whole work was to find an optimal pose of a mobile manipulator that can reach and cage target objects. Cages provided multiple goals and added flexibility to motion planning and robustness to perception and motion control. Makita et al. presented a practical system which used 3D two-finger caging to pick up concave objects with using AR markers [99, 121]. The geometric features of their objects for caging were retrieved from database by matching the markers. Fukui et al. applied caging grasp to hand-over tasks using a container case [122], in which the mechanism could be regarded as a robot hand performing sphere caging. Caging-based grasping presented by Meada et al. [123], in which inner rigid parts of the hand cage an object and outer soft parts make contact with the object, was performed both by planar multi-agents and by a 3D robotic hand.

4.2 *Caging in Multi-robot Transport*

Popular control methods of multi-robot object transport include: (1) formation control [124][71][125], (2) coordinated impedance control [126][127][128], and (3) task sequencing [129][130][131]. Formation control uses a formation of robots to enclose the object. The caging-based multi-robot transport is a kind of formation control. Compared with classical formation control methods which require precise localization of each agent [124], caging provides more flexibility. In the caging-based formation control, mobile robots are taken as points, objects are taken as polygons, the problem is formulated as 2D or 2.5D caging problems. An advantage of caging-based formation control is it doesn't need explicit force control and motion sequencing. Leading and pushing robots are passively shifted during the motion of the whole formation. Caging-based formation control helps to avoid complicated pushing analysis like [132–134].

Examples of practical caging-based multi-robot transport systems are as follows. Pereira et al. experimentally performed caging manipulation by multiple decentralized mobile robots in [74, 75], where polygonal mobile robots were controlled using visual feedback [76]. Sudsang et al. presented transporting planar objects by three disc-shaped robots [65, 66] based on their theoretical work of a motion planner [135]. More generally, Becker et al. [136] used ensemble control to perform manipulation using redundants robots. Their robot number is up to 100 in real time. Similar techniques like flocking [137] and distributed centroid estimation [138] were also demonstrated with real-world systems. They were promising applications of caging-based multi-robot transport.

Some other studies used tools instead of robotic bodies for caging and transport. For example, Kim et al. studied manipulation of multiple objects by multiple mobile robots using cables [139, 140], in which the robot trajectories were planned to separate each category of target objects. Yamashita et al. demonstrated cooperative manipulation by mobile robots using either a stick or a string attached to the robot [141]. Donald et al. [142] and Maneewarn et al. [143] also developed similar systems.

5. The Open Problems

We discuss the open problems of caging from both the viewpoint of algorithms and the viewpoint of applications.

5.1 *The open problems in algorithmic caging*

In the algorithmic section, we discussed different algorithms following the number of fingers and the type of objects. A summary of the problems and their solutions are shown in Table.1. The rows of the table show the types of agents, including point fingers, mobile point robots, polygon fingers, mobile polygonal robots, multi-joint hands, walls, and tools (see the footnote of the table). 2, 3, and >3 indicate two-agent caging, three-agent caging, and caging using more than three agents (but non-redundant). "inf" means caging with redundant number of agents. The markers ○ indicates the problem corresponding to the row and column of that grid has been completely solved. The △ indicates the correspondent problem was partially solved. The × indicates the correspondent problem was not studied. For example, the review in previous sections told us point fingers have been extensively studied, and the 2D and 3D caging problem using 2 and redundant number of point fingers could be completely solved. The grids corresponding to these problems, (2D workspace, ptf(2)) and (2D workspace, ptf(inf)), are filled with ○. The 2D caging problem using 3 fingers could only be solved partially. Researchers need some conditions, e. g., removing some degrees of freedoms, to simplify the problem. The grid corresponding to the problem, (2D workspace, ptf(3)), is therefore filled with △. Completely solving the problems with △ markers is the first open problem.

Table 1. Different problems and their solutions

Agent types	**ptf**				**ptf+w**				**ptf+t**				**plf**				**mjh**			
# of agents	2	3	>3	inf	1	2	>2	inf	1	2	>2	inf	2	3	>3	inf	1	2	>2	inf
2D workspace	○	△	△	○	○	△	△	○	△	△	△	○	△	△	△	○	△	△	×	×
3D workspace	○	△	△	○	×	×	×	×	×	×	×	×	×	×	×	×	△	△	△	△
Agent types	**ptr**				**ptr+w**				**ptr+t**				**plr**				**mjr**			
# of agents	2	3	>3	inf	1	2	>2	inf	1	2	>2	inf	2	3	>3	inf	1	2	>2	inf
2D workspace	○	△	△	○	○	△	△	○	△	△	△	○	△	△	△	○	×	×	×	×
3D workspace	×	×	×	×	×	×	×	×	△	×	×	×	×	×	×	×	△	×	×	×

Meanings of abbreviations: **ptf** = point fingers, **plf** = polygon fingers, **mjh** = multi-joint hands, **ptr** = mobile point robots, **plr** = mobile polygonal robots, **mjr** = multi-joint robots, ***+**w** = ***+wall, ***+**t** = ***+tool, **tool** = rigid bodies, rope, etc.

Partially solving the problems with × markers is the second open problem. To our best knowledge, completely solving the problems with × markers are impossible since the computational cost is beyond the computational capabilities of modern computers. However, finding partial solutions, e.g., constrain the problem by removing some degree of freedoms or by using redundant agents, is meaningful to actual work. For example, solving the problem corresponding to (3D workspace, mjr), or whole-body caging, is meaningful to relax the force analysis in whole-body manipulation [144].

Besides the problems shown in the table, an interesting and practical open problem is to take into account external forces (e.g. gravity force, frictional fingers, frictional objects). Some far-seeing researchers have started some seminal study [91] by considering gravity force. Extending the results to general external forces, discussing about completeness, or defining redundant conditions for this problem are promising directions.

5.2 *The promising caging applications*

From the viewpoint of application, an interesting problem is using caging to deal with in-hand manipulation. In-hand manipulation means to hold and move an object with one hand. It requires that the object to be constrained in the hand while being manipulated. One difficulty of in-hand manipulation is to maintain force closure at essential time points during operation, which in most cases impossible due to mechanical constraints and planning problems. Using caging instead of force closure to connect transit states during in-hand manipulation is a promising approach. Caging looses force closure into cages as well as maintains a connection to force closure. Some studies could already be found in most recent publications [91, 119].

Caging using tools is also widely needed in the real world. The cases usually appear in heavy industry where gantry robots clasp big objects using hooks or wrap them using metal chains. Planning tool cages and providing the results to gantry robot controllers would be very helpful. Kwok et al.'s rope caging planner is a seminal study in this field [102]. It is promising in these applications.

Micro-manipulation using optical tweezers is an interesting application, too. Optical tweezers use two highly focused laser beams to trap very small crystal beads. Using several beads to transport objects like cells is similar to the case of distributed multi-agent transport. Caging is expected to play an important role in it since the control of the trapped beads is uncertain. The beads are not firm and have lag in motion during formation control. Caging may help overcome the problems caused by uncertainty. Note that the application is not limited to optical tweezers, transport using other tweezer-like micro-robots, e. g. Ohta's bubble robots [111], is also practical applications to study.

6. Conclusions

This paper presented a brief survey of robotic caging and its applications studied in the past couple of years. First, it reviewed the history of caging and its relationship with traditional concepts like form and force closure in grasping. It showed that caging is the extension of 2nd-order form closure and immobilization in configuration space. Then, the paper presented the algorithmic development of caging in robotics, and the practical robotic caging systems. It summarized the open problems in both algorithmic and practical system development based on the review of contemporary theoretical studies and real-world applications. The paper provided our insights on some open problems and promising research and application directions. It is expected to help researchers quickly catch the strength and limitations of caging, and make impacting contributions to the community.

References

[1] Shimoga KB. Robot Grasp Synthesis Algorithms: A Survey. The International Journal of Robotics Research. 1996;15(3):230–266.

[2] Bicchi A, Kumar V. Robotic Grasping and Contact: A Review. In: Proceedings of IEEE International Conference on Robotics and Automation. 2000. p. 348–353.

[3] Kuperberg W. Problems on Polytopes and Convex Sets. In: Proceedings of DIMACS: Workshop on Polytopes. 1990. p. 584–589.

[4] O'Rourke J. Computational Geometry Column 9. SIGACT News. 1990;21(1):18–20.

[5] Besicovitch AS. A Net to Hold a Sphere. The Mathematical Gazette. 1957;41:106–107.

[6] Croft HT. A Net to Hold a Sphere. Journal of London Mathematical Society. 1964;39:1–4.

[7] Besicovitch AS. A Cage to Hold a Unit Sphere. In: Proceedings of Symposium in Pure Math. 1963. p. 19–20.

[8] Shephard GC. A Sphere in a Crate. Journal of London Mathematical Society. 1965;40:433–434.

[9] Zamfirescu T. How to Hold a Convex Body? Geometriae Dedicata. 1995;54(3):313–316.

[10] Fruchard A. Small Holding Circle. arXiv. 2010;math.MG:1004.5255v1.
[11] Tanoue Y. Regular Triangular Pylamids Held by a Circle. Journal of Geometry. 2009;94:151–157.
[12] Rimon E, Blake A. Caging 2D Bodies by 1-Parameter Two-Fingered Gripping Systems. In: Proceedings of IEEE International Conference on Robotics and Automation. 1996. p. 1458–1464.
[13] Rimon E, Blake A. Caging Planar Bodies by One-Parameter Two-Fingered Gripping Systems. International Journal of Robotics Research. 1999;18:299–318.
[14] Sudsang A, Ponce J. On Grasping and Manipulating Polygonal Objects with Disc-Shaped Robots in the Plane. In: Proceedings of IEEE International Conference on Robotics and Automation. 1998. p. 2740–2746.
[15] Trinkle JC. A Quantitative Test For Form Closure Grasps. In: Proceedings of IEEE International Conference on Intelligent Robots and Systems. 1992. p. 1670–1677.
[16] Ponce J, Burdick J, Rimon E. Computing the Immobilizing Three-Finger Grasps of Planar Objects. IEEE Transactions on Robotics and Automation. 1995;11:868–881.
[17] Czyzowicz J, Stojmenovic I, Urrutia J. Immobilizing A Shape. International Journal of Computational Geometry and Applications. 1999;9:181–206.
[18] Mason MT. Mechanics of Robotic Manipulation. The MIT Press. 2001.
[19] Reuleaux F. The Kinematics of Machinery. Macmillanv and Company. 1876.
[20] Salisbury JK. Kinematic and Force Analysis of Articulated Hands. [Ph.D. thesis]. Stanford University. 1982.
[21] Bicchi A. On the Closure Properties of Grasping. The International Journal of Robotics Research. 1995;14:319–334.
[22] Rimon E, Burdick J. On Force and Form Closure For Multiple Finger Grasps. In: Proceedings of IEEE International Conference on Robotics and Automation. 1996. p. 1795–1800.
[23] Cheong JS, Haverkort HJ, Stappen AFVD. Computing All Immobilizing Grasps of a Simple Polygon with Few Contacts. Algorithmica. 2006;44(2):117–136.
[24] Nguyen VD. Constructing Force-Closure Grasps. In: Proceedings of IEEE International Conference on Robotics and Automation. 1986. p. 1368–1373.
[25] Mishra B, Schwartz JT, Sharir M. On the Existence and Synthesis of Multifinger Positive Grips. Algorithmica. 1987;2:541–558.
[26] Rimon E, Burdick J. Mobility of Bodies in Contact-I: A New 2nd Order Mobility Index for Multiple-Finger Grasps. In: Proceedings of IEEE International Conference on Robotics and Automation. 1994. p. 2329–2335.
[27] Rimon E, Burdick J. Mobility of Bodies in Contact-II: How forces are Generated by Curvature Effects? In: Proceedings of IEEE International Conference on Robotics and Automation. 1994. p. 2336–2341.
[28] Rimon E, Stappen AFVD. Immobilizing 2-D Serial Chains in Form-Closure Grasps. IEEE Transactions on Robotics. 2012;28(1):32–43.
[29] Rimon E, Burdick JW. Mobility of Bodies in Contact – I: A New 2nd Order Mobility Index for Multiple-Finger Grasps. IEEE Transactions on Robotics and Automation. 1998;14:696–708.
[30] Rimon E, Burdick JW. Mobility of Bodies in Contact – II: How Forces are Generated by Curvature Effects. IEEE Transactions on Robotics and Automation. 1998;14:709–717.
[31] Rimon E. A Curvature-Based Bound on the Number of Frictinless Fingers Required to Immobilize Three-Dimensional Objects. IEEE Transactions on Robotics and Automation. 2001;17:679–697.
[32] Lozano-Perez T, Jones JL, Mazer E, O'Donnell PA. HANDEY: A Robot Task Planner. The MIT Press. 1992.
[33] Stappen AFVD. Immobilization: Analysis, Existence, and Output-sensitive synthesis. In: Computer-Aided Design and Manufacturing. AMS-DIMACS. 2005. p. 162–187.
[34] Stappen AFVD, WentinkMark C, Overmars H. Computing Immobilizing Grasps of Polygonal Parts. International Journal of Robotics Research. 2000;19(5):467–479.
[35] Sudsang A, Ponce J, Srinivasa N. Algorithms for Constructing Immobilizing Fixtures and Grasps of Three-Dimensional Objects. In: Algorithmic Foundations of Robotics II. A K Peters, Ltd.. 1997. p. 363–380.
[36] Sudsang A, Ponce J, Hyman M, Kriegman DJ. Manipulating Polygonal Objects with Three 2-DOF Robots in the Plane. In: Proceedings of IEEE International Conference on Robotics and Automation. 1999. p. 2227–2234.
[37] Sudsang A, Ponce J, Srinivasa N. Grasping and In-Hand Manipulation: Geometry and Algorithms. Algorithmica. 2000;26:466–493.

[38] Gopalakrishnan KG, Goldberg K. D-space and Deform Closure Grasps of Deformable Parts. The International Journal of Robotics Research. 2005;24(11):899–910.

[39] Gopalakrishnan KG, Goldberg K. Gripping Parts at Concave Vertices. In: Proceedings of IEEE International Conference on Robotics and Automation. 2002. p. 1590–1596.

[40] Pipattanasomporn P, Vongmasa P, Sudsang A. Two-Finger Caging of Concave Polygon. In: Proceedings of IEEE International Conference on Robotics and Automation. 2006. p. 2137–2142.

[41] Pipattanasomporn P, Vongmasa P, Sudsang A. Caging Rigid Polytopes via Finger Dispersion Control. In: Proceedings of IEEE International Conference on Robotics and Automation. 2008. p. 1181–1186.

[42] Pipattanasomporn P, Makapunyo T, Sudsang A. Multifinger Caging Using Dispersion Constraints. IEEE Transactions on Robotics. 2016;32(4):1033–1041.

[43] Rodriguez A, Mason MT. Two Finger Caging: Squeezing and Stretching. In: Algorithmic Foundations of Robotics VIII. Vol. 57. Springer. 2008. p. 119–133.

[44] Wang Z, Kumar V. Object Closure and Manipulation by Multiple Cooperating Mobile Robots. In: Proceedings of IEEE International Conference on Robotics and Automation. 2002. p. 394–399.

[45] Trinkle JM, Abel JM, Paul RP. An Investigation of Frictionless Enveloping Grasping in the Plane. The International Journal of Robotics Research. 1988;7:33–51.

[46] Eberman BS. Whole-Arm Manipulation: Kinematics and Control. The MIT Press. 1989.

[47] Wan W, Fukui R. Efficient Planar Caging Test Using Space Mapping. IEEE Transactions on Automation Science and Engineering. 2016;(99).

[48] Bunis HA, Rimon ED, Golan Y, Shapiro A. Caging Polygonal Objects Using Equilateral Three-Finger Hands. IEEE Robotics and Automation Letter. 2017;1:1164–1179.

[49] Wan W, Fukui R, Shimosaka M, Sato T, Kuniyoshi Y. Cooperative Manipulation with Least Number of Robots via Robust Caging. In: Proceedings of IEEE/AMSE International Conference on Advanced Intelligent Mechatronics. 2012. p. 896–903.

[50] Pipattanasomporn P, Vongmasa P, Sudsang A. Two-Finger Squeezing Caging of Polygonal and Polyhedral Object. In: Proceedings of IEEE International Conference on Robotics and Automation. 2007. p. 10–14.

[51] Pipattanasomporn P, Sudsang A. Two-Finger Caging of Nonconvex Polytopes. IEEE Transactions on Robotics. 2011;27(2):324–333.

[52] Allen T, Burdick J, Rimon E. Two-fingered Caging of Polygons Via Contact-Space Graph Search. In: Proceedings of IEEE International Conference on Robotics and Automation. 2012. p. 4183–4189.

[53] Allen TF, Burdick JW, Rimon E. Two-Finger Caging of Polygonal Objects Using Contact Space Search. IEEE Transactions on Robotics. 2015;31(5):1164–1179.

[54] Erickson J, Thite S, Rothganger F, Ponce J. Capturing a Convex Object with Three Discs. In: Proceedings of IEEE International Conference on Robotics and Automation. 2003. p. 2242–2247.

[55] Erickson J, Thite S, Rothganger F, Ponce J. Capturing a Convex Object with Three Discs. IEEE Transactions on Robotics. 2007;23:1133–1140.

[56] Vahedi M, Stappen AFVD. Caging Polygons with Two and Three Fingers. In: Proceedings of International Workshop on Algorithmic Foundations of Robotics. 2006. p. 71–86.

[57] Vahedi M, Stappen AFVD. Geometric Properties and Computation of Three-Finger Caging Grasps of Convex Polygons. In: Proceedings of IEEE Conference on Automation Science and Engineering. 2007. p. 404–411.

[58] Vahedi M, Stappen AFVD. Caging Convex Polygons with Three Fingers. In: Proceedings of IEEE/RSJ International Conference on Intelligent Robots and Systems. 2008. p. 1777–1783.

[59] Vahedi M, Stappen AFVD. Caging Polygons with Two and Three Fingers. The International Journal of Robotics Research. 2008;27:1308–1324.

[60] Wan W, Fukui R, Shimosaka M, Sato T, Kuniyoshi Y. On the Caging Region of a Third Finger with Object Boundary Clouds and Two Given Contact Positions. In: Proceedings of IEEE International Conference on Robotics and Automation. 2012. p. 4154–4161.

[61] Davidson C, Blake A. Caging Planar Objects with a Three-Finger One-Parameter Gripper. In: Proceedings of IEEE International Conference on Robotics and Automation. 1998. p. 2722–2727.

[62] Davidson C, Blake A. Error-Tolerant Visual Planning of Planar Grasp. In: Proceedings of International Conference on Computer Vision. 1998. p. 911–916.

[63] Wan W, Fukui R, Shimosaka M, Sato T, Kuniyoshi Y. How to manipulate an object robustly with only one actuator (An application of caging). In: Proceedings of IEEE/RSJ International Conference on Intelligent Robots and Systems. 2013. p. 2991–2998.

[64] Su J, Qiao H, Ou Z, Liu ZY. Vision-Based Caging Grasps of Polyhedron-Like Workpieces With a Binary Industrial Gripper. IEEE Transactions on Automation Science and Engineering. 2015; 12(3):1033–1046.

[65] Sudsang A, Rothganger F, Ponce J. Motion Planning for Disc-shaped Robots Pushing a Polygonal Object in the Plane. IEEE Transactions on Robotics and Automation. 2002;18:550–562.

[66] Sudsang A. A Sufficient Condition for Capturing an Object in the Plane with Disc-Shaped Robots. In: Proceedings of IEEE International Conference on Robotics and Automation. 2002. p. 682–687.

[67] Wang Z, Kumar V. A Decentralized Test Algorithm for Object Closure by Multiple Cooperating Mobile Robots. In: Asama H, Arai T, Fukuda T, Hasegawa T, editors. Distributed Autonomous Robotic Systems. Springer. 2002. p. 165–174.

[68] Wang Z, Hirata Y, Kosuge K. CC-Closure Object and Object Closure Margin of Object Caging by Using Multiple Robots. In: Proceedings of IEEE/ASME International Conference on Advanced Intelligent Mechantronics. 2003. p. 344–349.

[69] Wang Z, Kumar V, Hirata Y, Kosuge K. A Strategy and a Fast Testing Algorithm for Object Caging by Multiple Cooperative Robots. In: Proceedings of IEEE International Conference on Robotics and Automation. 2003. p. 2275–2280.

[70] Wang Z, Hirata Y, Kosuge K. Control Multiple Mobile Robots for Object Caging and Manipulation. In: Proceedings of IEEE/RSJ International Conference on Intelligent Robots and Systems. 2003. p. 1751–1758.

[71] Wang Z, Hirata Y, Kosuge K. An Algorithm for Testing Object Caging Condition by Multiple Mobile Robots. In: Proceedings of IEEE/RSJ International Conference on Intelligent Robots and Systems. 2005. p. 3022–3027.

[72] Wang Z, Hirata Y, Kosuge K. Dynamic Object Closure by Multiple Mobile Robots and Random Caging Formation Testing. In: Proceedings of IEEE/RSJ International Conference on Intelligent Robots and Systems. 2006. p. 3675–3681.

[73] Wang Z, Matsumoto H, Hirata Y, Kosuge K. A Path Planning Method for Dynamic Object Closure by using Random Caging Formation Testing. In: Proceedings of IEEE/RSJ International Conference on Intelligent Robots and Systems. 2009. p. 5923–5929.

[74] Pereira GAS, Kumar V, Spletzer JR, Taylor CJ, Campos MFM. Cooperative Transport of Planar Objects by Multiple Mobile Robots Using Object Closure. In: Experimental Robotics VIII. Springer. 2002. p. 275–285.

[75] Pereira GAS, Kumar V, Campos MFM. Decentralized Algorithms for Multirobot Manipulation via Caging. In: Proceedings of International Workshop on Algorithmic Foundations of Robotics. 2002. p. 242–258.

[76] Pereira GAS, Campos MFM, Kumar V. Decentralized Algorithms for Multi-robot Manipulation Via Caging. International Journal of Robotics Research. 2004;23:783–795.

[77] Suarod N, Boonpion N, Sudsang A. A Heuristic Method for Computing Caging Formation of Polygonal Object. In: Proceedings of IEEE International Conference on Robotics and Biomimetics. 2007. p. 823–828.

[78] Dai Y, Kim YG, Lee DH, Lee S. Symmetric Caging Formation for Convex Polygon Object Transportation by Multiple Mobile Robots. In: Proceedings of IEEE International Conference on Advanced Intelligent Mechatronics. 2015. p. 595–600.

[79] Rodriguez A, Mason MT, Ferry S. From Caging to Grasping. In: Proceedings of Robotics: Science and Systems. 2011. p. 37.

[80] Rodriguez A, Mason MT, Ferry S. From Caging to Grasping. In: The International Journal of Robotics Research. Vol. 31. 2012. p. 886–900.

[81] Fink J, Michael N, Kumar V. Composition of Vector Fields for Multi-Robot Manipulation via Caging. In: Proceedings of Robotics: Science and Systems. 2007. p. 352.

[82] Vongmasa P, Sudsang A. Coverage Diameters of Polygons. In: Proceedings of IEEE/RSJ International Conference on Intelligent Robots and Systems. 2006. p. 4036–4041.

[83] Varava A, Hang K, Kragic D, Pokorny FT. Herding by Caging: a Topological Approach towards Guiding Moving Agents via Mobile Robots. In: Proceedings of Robotics: Science and Systems. 2017.

[84] Yokoi R, Kobayashi T, Maeda Y. 2D Caging Manipulation by Robots and Walls. In: Proceedings of IEEE International Symposium on Assembly and Manufacturing. 2009. p. 16–21.

[85] Banyassady B, Davoodi M, Mohades A. Caging Polygons by a Finger and a Wall. In: Proceedings of EUroCG. 2015. p. 97–100.

[86] Varkonyi PL. Sensorless Part Feeding With Round Cages in Two and Three Dimensions. IEEE

Robotics and Automation Letters. 2016;1(2):724–731.

[87] Makapunyo T, Phoka T, Pipattanasomporn P, Niparnan N, Sudsang A. Measurement Framework of Partial Cage Quality. In: Proceedings of IEEE International Conference on Robotics and Biomimetics. 2012. p. 1812–1816.

[88] Makita S, Nagata K. Evaluation of Finger Configuration for Partial Caging. In: Proceedings of IEEE International Conference on Robotics and Automation. 2015. p. 4838–4844.

[89] Makapunyo T, Phoka T, Pipattanasomporn P, Niparnan N, Sudsang A. Measurement Framework of Partial Cage Quality based on Probabilistic Motion Planning. In: Proceedings of IEEE International Conference on Robotics and Automation. 2013. p. 1574–1579.

[90] Makita S, Nagata K. Evaluation of Quality of Partial Caging by a Planar Two-fingered Hand. Advanced Robotics. 2016;30(3):178–189.

[91] Mahler J, Pokorny FT, Mccarthy Z, Stappen AFVD, Goldberg K. Energy-Bounded Caging : Formal Definition and 2-D Energy Lower Bound Algorithm Based on Weighted Alpha Shapes. IEEE Robotics and Automation Letters. 2016;1(1):508–515.

[92] Wan W, Fukui R, Shimosaka M, Sato T, Kuniyoshi Y. Grasping by Caging: A Promising Tool to Deal with Uncertainty. In: Proceedings of IEEE International Conference on Robotics and Automation. 2012. p. 5142–5149.

[93] Wan W, Fukui R, Shimosaka M, Sato T, Kuniyoshi Y. A New "Grasping by Caging" Solution by Using Eigen-shapes and Space Mapping. In: Proceedings of IEEE International Conference on Robotics and Automation. 2013. p. 1566–1573.

[94] Kavraki LE, Švestka P, Latombe JC, Overmars MH. Probabilistic Roadmaps for Path Planning in High-Dimensional Configuration Spaces. IEEE Transactions on Robotics and Automation. 1996 August;12(4):566–580.

[95] Lavalle SM. Rapidly-exploring random trees: A new tool for path planning. Computer Science Dept., Iowa State University. 1998. Tech Rep TR98-11.

[96] Fukui R, Kadowaki K, Niwa Y, Wan W, Shimosaka M, Sato T. Design of Distributed End-Effectors for Caging-Specialized Manipulator. In: Desai JP, Dudek G, Khatib O, Kumar V, editors. Experimental Robotics: The 13th International Symposium on Experimental Robotics. Springer. 2013. p. 15–26.

[97] Allen T, Burdick J, Rimon E. Two-fingered Caging of 3D Polyhedra Using Contact-Space Graph Search. In: Proceedings of IEEE International Conference on Robotics and Automation. 2014. p. 2005–2012.

[98] Makita S, Maeda Y. 3D Multifingered Caging: Basic Formulation and Planning. In: Proceedings of IEEE/RSJ International Conference on Intelligent Robots and Systems. 2008. p. 2697–2702.

[99] Makita S, Okita K, Maeda Y. Motion Planning for 3D Multifingered Caging with Object Recognition using AR Picture Markers. In: Proceedings of IEEE International Conference on Mechatronics and Automation. 2012. p. 2158–2164.

[100] Pokorny FT, Stork JA, Kragic D. Grasping Objects with Holes: A Topological Approach. In: Proceedings of IEEE International Conference on Robotics and Automation. 2013. p. 1100–1107.

[101] Stork JA, Pokorny FT, Kragic D. Integrated Motion and Clasp Planning with Virtual Linking. In: Proceedings of IEEE/RSJ International Conference on Intelligent Robots and Systems. 2013. p. 3007–3014.

[102] Kwok TH, Wan W, Pan J, Wang CCL, Yuan J, Harada K, Chen Y. Rope Caging and Grasping. In: Proceedings of IEEE International Conference on Robotics and Automation. 2016. p. 1980–1986.

[103] Tsuji T, Uto S, Harada K, Kurazume R, Hasegawa T, Murooka K. Grasp Planning for Constricted Parts of Objects Approximated with Quadric Surfaces. In: Proceendings of ieee international conference on intelligent robots and systems. ????

[104] Varava A, Kragic D, Pokorny FT. Caging Grasps of Rigid and Partially Deformable 3-D Objects With Double Fork and Neck Features. IEEE Transaction on Robotics. 2016;32(6):1479–1497.

[105] Zarubin D, Pokorny FT, Toussaint M, Kragic D. Caging Complex Objects with Geodesic Balls. In: Proceedings of IEEE/RSJ International Conference on Intelligent Robots and Systems. 2013. p. 2999–3006.

[106] Stork JA, Pokorny FT, Kragic D. A Topology-based Object Representation for Clasping, Latching and Hooking. In: Proceedings of IEEE/RAS International Conference on Humanoid Robots. 2013. p. 138–145.

[107] Jiang Y, Zheng C, Lim M, Saxena A. Learning to Place New Objects. In: Proceedings of IEEE International Conference on Robotics and Automation. 2012. p. 3088–3095.

[108] Sato T. Micro/nano Manipulation World. In: Proceedings of IEEE/RSJ International Conference on Intelligent Robots and Systems. 1996. p. 834–841.
[109] Chronis N, Lee LP. Electrothermally Activated SU-8 Microgripper for Single Cell Manipulation in Solution. Journal of Microelectromechanical Systems. 2005;14:857–863.
[110] Grier D. A Revolution in Optical Manipulation. Nature. 2003;424:810–816.
[111] Hu W, Ishii KS, Ohta AT. Micro-assembly using Optically Controlled Bubble Microrobots. Applied Physics Letters. 2011;99:094103–094103–3.
[112] Cappelleri DJ, Fatovic M, Shah U. Caging Micromanipulation for Automated Microassembly. In: Proceedings of IEEE International Conference on Robotics and Automation. 2011. p. 3145–3150.
[113] Cappelleri DJ, Fatovic M, Fu Z. Caging Grasps for Micromanipulation & Microassembly. In: Proceedings of IEEE/RSJ International Conference on Intelligent Robots and Systems. 2011. p. 925–930.
[114] Cappelleri DJ, Fu Z, Fatovic M. Caging for 2D and 3D Micromanipulation. Journal of Micro-Nano Mechatronics. 2012;7:115–129.
[115] Cappelleri DJ, Fu Z. Towards Flexible, Automated Microassembly with Caging Micromanipulation. In: Proceedings of IEEE International Conference on Robotics and Automation. 2013. p. 1427–1432.
[116] Blind SJ, McCullough CC, Akella S, Ponce J. Manipulating Parts with an Array of Pins: A Method and a Machine. International Journal of Robotics Research. 2001;20(10):808–818.
[117] Wallack AS, Canny JF. Modular Fixture Design for Generalized Polyhedra. In: Proceedings of IEEE International Conference on Robotics and Automation. 1996. p. 830–837.
[118] Ma RR, Bircher WG, Dollar AM. Toward Robust, Whole-hand Caging Manipulation with Underactuated Hands. In: Proceedings of IEEE International Conference on Robotics and Automation. 2017.
[119] Maeda Y, Asamura T. Sensorless In-Hand Caging Manipulation. In: Chen W, Hosoda K, Menegatti E, Shimizu M, Wang H, editors. Intelligent Autonomous Systems 14: Proceedings of the 14th International Conference. Springer. 2017. p. 255–267.
[120] Diankov R, Srinivasa S, Ferguson D, Kuffner J. Manipulation Planning with Caging Grasps. In: Proceedings of IEEE/RAS International Conference on Humanoid Robots. 2008. p. 285–292.
[121] Makita S, Okita K, Maeda Y. 3D Two-fingered Caging for Two Types of Objects: Sufficient Conditions and Planning. International Journal of Mechatronics and Automation. 2013;3:263–277.
[122] Fukui R, Mori T, Sato T. Application of Caging Manipulation and Compliant Mechanism for a Container Case Hand-over Task. In: Proceedings of IEEE International Conference on Robotics and Automation. 2010. p. 4511–4518.
[123] Maeda Y, Kodera N, Egawa T. Caging-Based Grasping by a Robot Hand with Rigid and Soft Parts. In: Proceedings of IEEE International Conference on Robotics and Automation. 2012. p. 5150–5155.
[124] Spletzer J, Das AK, Fierro R, Taylor CJ, Kumar V, Ostrowski JP. Cooperative localization and control for multi-robot manipulation. In: Proceedings of IEEE International Conference on Robotics and Automation. 2001. p. 631–636.
[125] Fink J, Hsieh MA, Kumar V. Multi-Robot Manipulation via Caging in Environments with Obstacles. In: Proceedings of IEEE International Conference on Robotics and Automation. 2008. p. 1471–1476.
[126] Cho HK, Lee BH, Ko MS. Time-optimal Motion of a Cooperating Multiple Robot System along a Prescribed Path. IEEE Transactions on Systems, Man, and Cybernetics. 1995;25(12):1648–1657.
[127] Kosuge K, Hirata Y, Asama H, Kaetsu H, Kawabata K. Motion Control of Multiple Autonomous Mobile Robots Handling a Large Object in Coordination. In: Proceedings of IEEE International Conference on Robotics and Automation. 1999. p. 2666–2673.
[128] Luiz Chaimowicz VK, Campos M. A Mechanism for Dynamic Coordination of Multiple Robots. Autonomous Robots. 2004;17:7–21.
[129] Parker LE. ALLIANCE: An Architecture for Fault Tolerant Multi-Robot Cooperation. IEEE Transactions on Robotics and Automation. 1998;14:220–240.
[130] Gerkey BP, Mataric MJ. Pusher-watcher: An Approach to Fault-tolerant Tightly-coupled Robot Coordination. In: Proceedings of IEEE International Conference on Robotics and Automation. 2002. p. 464–469.
[131] Cheng P, Fink J, Kumar V. Abstractions and Algorithms for Cooperative Multiple Robot Planar Manipulation. In: Proceedings of Robotics, Science and Systems. 2008. p. 143–150.
[132] Mason MT. Mechanics and Planning of Manipulator Pushing Operations. International Journal of

Robotics Research. 1986;5(3):53–71.
[133] Lynch KM, Mason MT. Stable Pushing: Mechanics, Controllability and Planning. International Journal of Robotics Research. 1996;15(6):533–556.
[134] Li Q, Payandeh S. Manipulation of Convex Objects via Two-agent Point-contact Push. International Journal of Robotics Research. 2007 April;26(4):377–403.
[135] Sudsang A, Ponce J. A New Approach to Motion Planning for Disc-shaped Robots Manipulating a Polygonal Object in the Plane. In: Proceedings of IEEE International Conference on Robotics and Automation. 2000. p. 1068–1075.
[136] Becker A, Habibi G, Werfel J, Rubenstein M, McLurkin J. Massive Uniform Manipulation: Controlling Large Populations of Simple Robots with a Common Input Signal. In: Proceedings of IEEE/RSJ International Conference on Intelligent Robots and Systems. 2013. p. 520–527.
[137] Rubenstein M, Cabrera A, Werfel J, Habibi G, McLurkin J, Nagpal R. Collective Transport of Complex Objects by Simple Robots: Theory and Experiments. In: Proceedings of International Conference on Autonomous Agents and Multi-agent Systems. 2013. p. 47–54.
[138] Habibi G, Kingston Z, Xie W, Jellins M, McLurkin J. Distributed Centroid Estimation and Motion Controllers for Collective Transport by Multi-robot Systems. In: Proceedings of IEEE International Conference on Robotics and Automation. 2015. p. 1282–1288.
[139] Kim S, Bhattacharya S, Heidarsson H, Sukhatme GS, Kumar V. A Topological Approach to Using Cables to Separate and Manipulate Sets of Objects. In: Proceedings of Robotics: Science and Systems. 2009. p. 46.
[140] Bhattacharya S, Kim S, Heidarsson H, Sukhatme GS, Kumar V. A Topological Approach to Using Cables to Separate and Manipulate Sets of Objects. International Journal of Robotics Research. 2015 May;34(6):799–815.
[141] Yamashita A, Sasaki J, Ota J, Arai T. Cooperative Manipulation of Objects by Multiple Mobile Robots with Tools. In: Proceedings of Asia-Europe Congress on Mechatronics. 1998. p. 310–315.
[142] Donald B, Gariepy L, Rus D. Experiments in constrained prehensile manipulation: Distributed manipulation with ropes. In: Experimental Robotics VI. Lecture Notes in Control and Information Sciences. Vol. 250. Springer. 2000. p. 25–36.
[143] Maneewarn T, Detudom P. Mechanics of Cooperative Nonprehensile Pulling by Multiple Robots. In: Proceedings of International Conference on Intelligent Robots and Systems. Edmonton, Canada. 2005 August. p. 1319–1324.
[144] Harada K, Kaneko M. Whole Body Manipulation. In: Proceedings of IEEE Conference on Robotics, Intelligent Systems and Signal Processing. 2003. p. 190–195.