A Discrete Exploration of Object Configuration for Three-dimensional Caging Grasps

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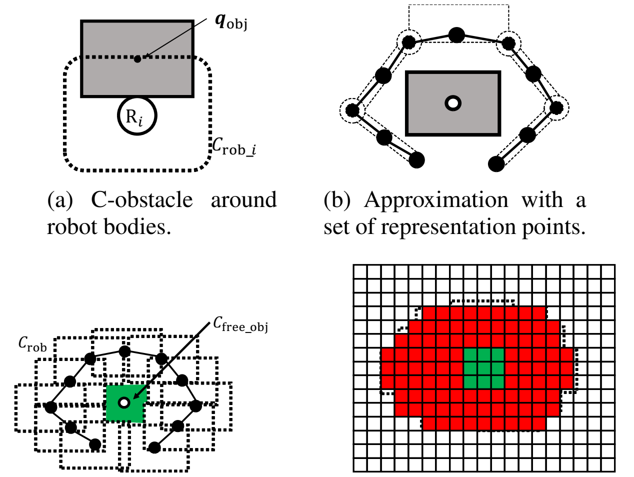
1 Introduction

In caging grasps, an object captured by the robot hand never escapes from the geometric constraint structured by the robot bodies. From the viewpoint of practical robotic manipulation, as in some previous studies, each series of sufficient conditions should be derived for the combination of an object and a robot hand. On the other hand, the necessary conditions expressed mathematically have attracted some researchers. Caging by multiple isolated robots in two- and three-dimensional spaces has been studied [1, 2]. Additionally, caging by a realistic robot hand with linkages in the three-dimensional space is still an open problem [3, 4].

A difficulty of such caging grasps lies in the complexity of exploring closed free space for the object confined to the robotic hand. Since the robotic hand can have various postures with various joint variables, it makes the mathematical descriptions of the closed free space intricate. Therefore, we proposed an approach to assess caging constraints in the three-dimensional scenes with a multifingered hand. Since mathematically describing the volume of obstacles in the configuration space of the object is complicated, we divide the configuration space with finite voxels and classify all the voxels into two patterns: occupied by the robot bodies or accessible for the object.



(a) C-obstacle around robot bodies.
(b) Approximation with a set of representation points.
(c) Object Closure (Caging)
(d) Voxelization and labeling process (Red cells: $\mathcal{C}_{rob}$).

Fig. 1: Object closure and its discretization.

2 Examination of Conditions for Caging Grasps

When a robotic hand completes caging grasps for a target object, the object never escapes from the *cage* structured by the hand. Thus, the problem of caging in the three-dimensional space is to prove the non-existence of escape paths for the caged object in the six-dimensional configuration space for the object's position and orientation [4].

On the other hand, the robot bodies in this configuration space play as obstacles surrounding the object, and their occupying region changes according to the robot configuration with the joint variables. Let us consider a multifingered hand with L joints, and then, the robot configuration is composed of the joint variables, $\boldsymbol{\theta} \in \mathbb{R}^L$, and the generalized coordinate (position and orientation) of the hand, $\boldsymbol{q}_{\rm rob} \in \mathbb{R}^6$. Therefore, obtaining the necessary conditions for caging by the multifingered hand is to examine the non-existence of escape paths for the caged object in the $(L+6)$-dimensional configuration space. The joint angles determines the hand posture and change the corresponding C-obstract regions.

2.1 Definition of Multifingered Caging based on Object Closure

Object Closure represents a situation where an object is confined by surrounding robots and is movable only in the bounded space. We consider C-obstacle (or C-Closure Object in [1]), where robot bodies interfere with the object (Figure1a) and express it as follows:

$$\mathcal{C}_{rob}(\boldsymbol{q}_{\rm rob}, \boldsymbol{\theta}) := \left\{ \boldsymbol{q}_{\rm obj} \in \mathcal{C} \mid \mathcal{A}_{\rm obj}\left(\boldsymbol{q}_{\rm obj}\right) \cap \mathcal{A}_{rob}\left(\boldsymbol{q}_{\rm rob}, \boldsymbol{\theta}\right) \neq \emptyset \right\}, \tag{1}$$

where $\mathcal{C}$ denotes the configuration space (C-space) of the object. $\mathcal{A}_{\rm obj}$ and $\mathcal{A}_{\rm rob}$ denote the regions of the object and the robot bodies in the real space, respectively. $\boldsymbol{q}_{\rm obj} \in \mathbb{R}^6$ and $\boldsymbol{q}_{\rm rob} \in \mathbb{R}^6$ denote positions and orientations of the object and the robot, respectively. $\boldsymbol{\theta} \in \mathbb{R}^L$ denotes all the joint variables of the robot, and L is the number of joints of the robot hand.

$\mathcal{C}_{\rm free}$ denotes free configuration spaces for the object so that the object can move freely without any interference with the robot bodies. Thus, it is written as a complementary set of the C-obstacles:

$$\mathcal{C}_{\rm free}(\boldsymbol{q}_{\rm rob}, \boldsymbol{\theta}) = \mathcal{C} \setminus \mathcal{C}_{\rm rob}(\boldsymbol{q}_{\rm rob}, \boldsymbol{\theta}). \tag{2}$$

Then, we divide $\mathcal{C}_{\rm free}$ into two subsets: $\mathcal{C}_{\rm free\_obj}$ and $\mathcal{C}_{\rm free\_inf}$. The object is in $\mathcal{C}_{\rm free\_obj}$: $\boldsymbol{q}_{\rm obj} \in \mathcal{C}_{\rm free\_obj}$. $\mathcal{C}_{\rm free\_inf}$ contains a point at infinity: $\boldsymbol{q}_{\rm inf} \in \mathcal{C}_{\rm free\_inf}$.

When $\mathcal{C}_{\rm free\_obj}$ is not an empty set and is surrounded by $\mathcal{C}_{\rm rob}$, $\mathcal{C}_{\rm free\_obj}$ does not connect with $\mathcal{C}_{\rm free\_inf}$, and caging the object by the robot bodies is achieved. Therefore, the necessary and sufficient conditions for three-dimensional caging grasps are written as follows:

$$\mathcal{C}_{\rm free\_obj} \neq \emptyset, \tag{3}$$

$$\mathcal{C}_{\rm free\_obj} \cap \mathcal{C}_{\rm free\_inf} = \emptyset. \tag{4}$$

2.2 Closing Test of Bounded Space for the Object using Discretization of Configuration Space

We can discretely explore the configuration space for the object's position and orientation. The problem of planar caging is defined with three parameters of the objects as [1], and their three-dimensional configuration space can be simply divided by voxels. On the other hand, three-dimensional caging is defined with six parameters and the corresponding six-dimensional configuration space cannot be divided by hypervoxels as the above because orientation expression with three parameters is not consistent. Thus we discretize the configuration space separately for position and orientation. For the latter case, we use a four-dimensional hypersphere to express each set of quaternions.

2.2.1 Voxelization of Configuration Space for Position

Instead of a mathematical formulation for the six-dimensional configuration space, we propose a discrete exploring approach. We approximate the three-dimensional configuration space for the position

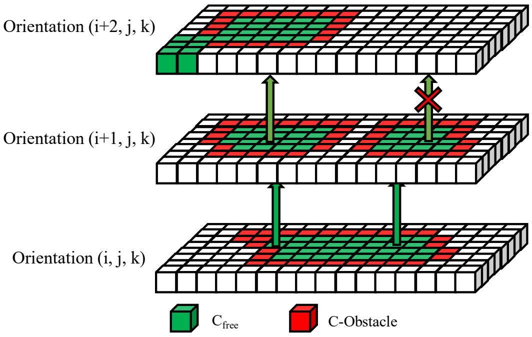


Fig. 2: Graph connection between sets of the accessible regions for the object in the configuration space.

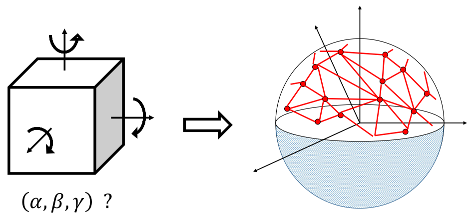


Fig. 3: Quaternions express the transition of object orientation.

of the objects with finite voxels and divide the whole configuration space into two subsets: C-obstacles of robots, $\mathcal{C}_{\mathrm{rob}}$, and freely accessible configuration space for objects: $\mathcal{C}_{\mathrm{free}}$ (Figure1d). This voxelization of configuration space allows us to adopt a labeling process, often utilized for image processing to divide pixels into clusters.

2.2.2 Connection Check of Voxels between Different Orientations of the Object

In Figure2, we consider the voxelized configuration space for the object with the orientation of (i, j, k). Then the voxel $v(x, y, z, i, j, k)$ denotes a single position and orientation of the object. When the voxel $v(x, y, z, i, j, k)$ is adjacent to $v(x, y, z, i+1, j, k)$, the object can rotate from the orientation (i, j, k) to $(i+1, j, k)$ at the position of (x, y, z). In addition, $\mathcal{C}_{\mathrm{free},l}(i, j, k)$ denotes a free region for the object with the orientation of (i, j, k). Thus, when at least one voxel in $\mathcal{C}_{\mathrm{free},1}(i, j, k)$ is adjacent to a voxel in $\mathcal{C}_{\mathrm{free},1}(i+1, j, k)$, the object can change its posture from $\mathcal{C}_{\mathrm{free},1}(i, j, k)$ in the configuration space. We continue the above process for all the connections between the nodes of the object's orientation shown in the next section.

2.2.3 Description of Discretized Orientation Space

To discretely explore the configuration space about orientation parameters changing as in the previous section, two constraint conditions are needed. One-to-one correspondence is required to determine one description for a particular orientation of the object with a set of parameters. Continuity of orientation ensures that various parameters describing the orientation of the object change continuously and produce no singular points.

In order to satisfy the above two conditions in orientation expression for discretized configuration space, we adopt the unit quaternion expression (Figure3) and its uniform sampling [5]. One-to-one correspondence can be satisfied by omitting negative unit vectors of quaternions. Since every uniformly sampled unit quaternion depends on the four-dimensional hypersphere, we connect them with Delaunay triangulation, which connects two sampled quaternions with the nearest Euclidean distance. Finally, we can obtain a graph of quaternions. High density of sampled points will ensure the connectivity between adjacent quaternions.

3 Simulation Results

In this section, we present some simulation results to verify our proposed test for the achievement of caging constraints by a multi-fingered hand. For the simulations, we assume some conditions. To simplify its calculation of the C-obstacles, every limb and joint of the robot hand has no volume. C-obstacles can be constructed on each joint and some finite points placed on each limb. The Voxelization resolution of configuration space for the position is 1 $[\mathrm{mm}]$. The number of sampling for unit quaternion is three. The size of bounded three-dimensional C-space for the position is $100 \times 100 \times 100$ $[\mathrm{mm}]$. The robot hand is a symmetric hand, which has a palm of a regular octagon inscribed in the circle with a radius of 14 $[\mathrm{mm}]$ and eight fingers attached to every vertex of the palm as Figure4. Each finger has four joints and moves only in the plane containing both the central axis and each vertex of the palm. Each length of the limbs of the finger is $12,\ 18,\ 12,$ and 12 $[\mathrm{mm}]$ respectively from its base. Each joint variable of the finger is set $50,\ 40,\ 40,$ and 40 $[\mathrm{deg}]$ respectively. The target objects are a 15 $[\mathrm{mm}]$ cube and a 20 $[\mathrm{mm}]$ cube.

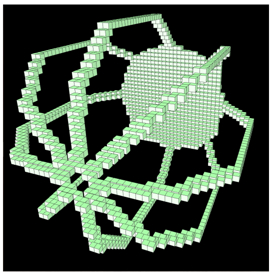


Fig. 4: A symmetric eight-fingered hand

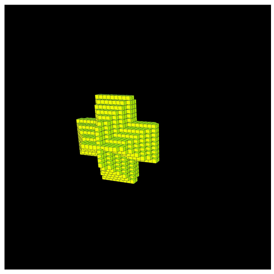


Fig. 5: An example of caged region for the cube inside the robotic hand.

Figure5 shows a set of three-dimensional accessible regions for the object with a particular orientation, which is caged by the multifingered hand with the above joint variables.

4 Conclusions

This paper studies achievement tests for three-dimensional caging, where robot fingers entirely geometrically capture an object. Since caging constraints can be accomplished when the accessible space for the object is completely confined by the robot bodies in the six-dimensional C-space, the procedure of the tests is constructing obstacle regions and exploring the non-existence of escape paths from the surrounded region for the object. To avoid the difficulty of describing caged regions mathematically, we discretize the six-dimensional C-space. The connectivity of the voxelized configuration space is explored with the labeling process and unit quaternions. Our simulation results show that our proposed algorithm can examine the accomplishment of caging grasps by a multifingered hand.

In future works, mathematically ,proper description and exploration of high dimensional configuration space should be considered to derive the necessary conditions for caging constraints.

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