3D Multifingered Caging: Basic Formulation and Planning

Satoshi MAKITA and Yusuke MAEDA

***Abstract*— In this paper, three-dimensional caging by a multifingered hand (3D multifingered caging) is studied. *Caging* is a method of object constraining in which robot bodies surround an object and make it inescapable from the *"cage"* composed of the robot bodies. In 3D multifingered caging, position-controlled robot hands can manipulate objects, and force control is not necessary. Furthermore, even a robot hand with low degrees of freedom can constrain an object to manipulate. We show some examples of 3D multifingered caging. Then, we derive sufficient conditions for caging objects, and propose a method to plan finger configurations for caging with the sufficient conditions.**

I. Introduction

Caging is a kind of object constraining in which robot bodies surround an object and make it inescapable from the *"cage"* composed of the robot bodies [1]. The surrounded object can move freely only in a closed configuration space constrained by the robot bodies. Caging can be regarded as a generalized extension of *form closure*, because form closure is a special case of caging where the closed configuration space is a point. Once we achieve caging the object, that is, the object is captured in a closed configuration space formed by the robot bodies, we can move the object without precise force control, only keeping the caging formation, although the position and orientation of the object is not determined exactly in the closed configuration space.

Rimon and Blake proposed two-dimensional caging, using a hand with two disk fingers. The fingers constrain a concave object [1], [2]. Wang and Kumar also presented 2D caging by multiple mobile robots [3]. Some mobile robots surround an object and manipulate the object keeping the cage formation of the robots. Only planar caging is considered in these studies.

Pipattanasomporn and Sudsang also studied caging a concave object by two fingers. They considered not only planar concave polygons [4] but also spatial concave polyhedra [5]. However, their method can deal with only three-dimensional caging problems that can be reduced to two-dimensional caging.

In this paper, we study **"3D multifingered caging"**: completely three-dimensional caging by a multifingered hand (Fig. 1). In 3D multifingered caging, force sensing and force control of robot fingers are not necessary to constrain an object, and the fingers and the palm of the hand should be

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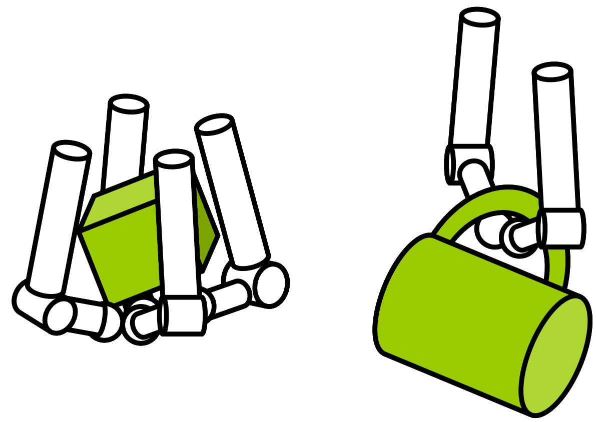

Fig. 1. 3D multifingered caging

position-controlled. This is an advantage over conventional robotic grasping because of its easy execution on actual robot hands. In addition, even a robot hand with low degrees of freedom can constrain an object (See the right figure of Fig. 1). Thus, multifingered caging will broaden the applications of robotic manipulation.

As the first step to construct the theory of 3D multifingered caging, the caging problem is mathematically formulated. Then, sufficient conditions to achieve 3D multifingered caging in a concrete form are derived. Finally, we propose a method to plan finger configurations for caging with the sufficient conditions. We show planning results of 3D multifingered caging in some cases.

II. Assumptions and Notations

To formulate conditions for 3D multifingered caging, we make some assumptions and define some notations.

A. Assumptions

The objects and bodies of the robot hand are rigid.

B. Notations

- $\mathcal{C}$: configuration space (C-space) of the object.
- $\mathcal{A}_{\rm obj}$: region of the object in the real space.
- $\mathcal{A}_{\rm plm}$: region of the palm of the hand in the real space.
- $\mathcal{A}_{ij}$: region of jth body of ith finger in the real space.
- $\boldsymbol{q}_{\rm obj}$: position and orientation of the object.
- $\boldsymbol{q}_{\rm plm}$: position and orientation of the palm of the hand.
- N: number of robot fingers.
- L_i: number of joints of the ith finger.
- $L := \sum_{i=1}^{N} L_i$: total number of joints.
- θ_{ij}: joint variable of the jth joint of the ith finger.
- $\boldsymbol{\theta}_i := [\theta_{i1}, \ldots, \theta_{iL_i}]^T \in \Re^{L_i}$.

- $\boldsymbol{\theta} := [\boldsymbol{\theta}_i^T, \ldots, \boldsymbol{\theta}_N^T]^T \in \Re^L$.

III. Formulation of 3D Multifingered Caging

Based on the formulation in [3], let us formulate conditions for 3D multifingered caging. We consider C-space obstacle region (C-obstacle), where a robot body interferes with an object and the object cannot exist. C-obstacle constructed by the jth body of the ith finger: $\mathcal{C}_{ij}$ and C-obstacle constructed by the palm: $\mathcal{C}_{\mathrm{plm}}$ are respectively written as follows:

$$\mathcal{C}_{ij}(\boldsymbol{q}_{\mathrm{plm}}, \boldsymbol{\theta}_i) := \{\boldsymbol{q}_{\mathrm{obj}} \in \mathcal{C} \mid \mathcal{A}_{\mathrm{obj}}(\boldsymbol{q}_{\mathrm{obj}}) \cap \mathcal{A}_{ij}(\boldsymbol{q}_{\mathrm{plm}}, \boldsymbol{\theta}_i) \neq \emptyset\} \quad (1)$$

$$\mathcal{C}_{\mathrm{plm}}(\boldsymbol{q}_{\mathrm{plm}}) := \{\boldsymbol{q}_{\mathrm{obj}} \in \mathcal{C} \mid \mathcal{A}_{\mathrm{obj}}(\boldsymbol{q}_{\mathrm{obj}}) \cap \mathcal{A}_{\mathrm{plm}}(\boldsymbol{q}_{\mathrm{plm}}) \neq \emptyset\}. \quad (2)$$

Thus, C-obstacle constructed by the robot hand: $\mathcal{C}_{\mathrm{rob}}$ can be written as follows:

$$\mathcal{C}_{\mathrm{rob}}(\boldsymbol{q}_{\mathrm{plm}}, \boldsymbol{\theta}) := \left(\bigcup_{i=1}^{N} \bigcup_{j=1}^{L_i} \mathcal{C}_{ij}(\boldsymbol{q}_{\mathrm{plm}}, \boldsymbol{\theta}_i) \right) \cup \mathcal{C}_{\mathrm{plm}}(\boldsymbol{q}_{\mathrm{plm}}). \quad (3)$$

Since C-space free region: $\mathcal{C}_{\mathrm{free}}$, where the object can move freely without interference by the robot bodies, is a complement set of the C-obstacles. Therefore, $\mathcal{C}_{\mathrm{free}}$ is written as follows:

$$\mathcal{C}_{\mathrm{free}}(\boldsymbol{q}_{\mathrm{plm}}, \boldsymbol{\theta}) = \mathcal{C} \setminus \mathcal{C}_{\mathrm{rob}}(\boldsymbol{q}_{\mathrm{plm}}, \boldsymbol{\theta}). \quad (4)$$

Then, we divide $\mathcal{C}_{\mathrm{free}}$ into two subsets: $\mathcal{C}_{\mathrm{free\_obj}}$ and $\mathcal{C}_{\mathrm{free\_inf}}$. The object is in $\mathcal{C}_{\mathrm{free\_obj}}$:

$$\boldsymbol{q}_{\mathrm{obj}} \in \mathcal{C}_{\mathrm{free\_obj}}. \quad (5)$$

$\mathcal{C}_{\mathrm{free\_inf}}$ includes a point at infinity: $\boldsymbol{q}_{\mathrm{inf}}$:

$$\boldsymbol{q}_{\mathrm{inf}} \in \mathcal{C}_{\mathrm{free\_inf}}. \quad (6)$$

When $\mathcal{C}_{\mathrm{free\_obj}}$ is not an empty set and completely surrounded by $\mathcal{C}_{\mathrm{rob}}$, that is, $\mathcal{C}_{\mathrm{free\_obj}}$ is not connected with $\mathcal{C}_{\mathrm{free\_inf}}$, caging the object by the robot bodies is achieved. Therefore, the necessary and sufficient condition for 3D multifingered caging is written as follows:

$$\mathcal{C}_{\mathrm{free\_obj}} \neq \emptyset \quad (7)$$

$$\mathcal{C}_{\mathrm{free\_obj}} \cap \mathcal{C}_{\mathrm{free\_inf}} = \emptyset. \quad (8)$$

IV. Sufficient Conditions for Caging

We derived the necessary and sufficient condition for 3D multifingered caging in an abstract form in Section III. Then, we have to convert it to a concrete form that is applicable to actual caging problems. However, it is difficult to derive the necessary and sufficient condition for caging in a concrete form. It is because there are various patterns of caging (See Fig. 1) and concrete representation of $\mathcal{C}_{\mathrm{free\_obj}}$ is highly complex. Thus, we derive not the necessary and sufficient condition but sufficient conditions for caging in some cases.

A. Assumptions

To make it easy to derive sufficient conditions for caging, we make some assumptions as follows.

1) A robot hand has N fingers, and each finger has $\bar{L}$ joints ($\therefore L_i = \bar{L} \quad (i = 1, \ldots, N)$).
2) All the joints are revolutionary and can be approximated by points.
3) The jth body of the ith finger can be approximated by a line segments with length l_j.
4) The palm of the hand is a regular N-gonal plane.
5) Each finger is attached to each vertex of the palm.
6) Each finger can move only in the plane passing through both the vertical center axis of the palm and each vertex of the palm.

In addition, the movements of the fingers have rotational symmetries through $N/360$ degrees about the vertical center axis of the palm. Thus,

$$\theta_{1j} = \theta_{2j} = \cdots = \theta_{Nj} (= \bar{\theta}_j) \qquad (j = 1, \ldots, \bar{L}) \quad (9)$$

$$\therefore \boldsymbol{\theta}_i = \bar{\boldsymbol{\theta}} \qquad (i = 1, \ldots, N) \quad (10)$$

In spite of the above assumption, note that the sufficient conditions derived in the following can be applied not only to the cases where the finger bodies can be approximated by line segments but also to the cases where the finger bodies include the line segments (Fig. 2).

B. Caging a Sphere

Let us derive a sufficient condition for caging a sphere (Fig. 3). The radius of the sphere is r_{sphere}. The configuration space of the object is three-dimensional because of the symmetry of the sphere. We assume that $N \geq 3$.

In this case, it is necessary that the target object cannot escape among the fingers of the hand. In the three-dimensional C-space, the C-obstacles are constructed by expanding the robot bodies as follows:

- The jth body of the ith finger approximated by a line segment is expanded to a cylinder with radius r_{sphere}, which corresponds to $\mathcal{C}_{ij}$.
- A hemisphere with radius r_{sphere} is attached to every fingertip to form $\mathcal{C}_{i\bar{L}}$.
- The region within r_{sphere} from the palm is $\mathcal{C}_{\mathrm{plm}}$

Then, all the faces whose vertices are finger joints or fingertips should be completely covered by C-obstacles. The faces can be classified according to their vertices as follows:

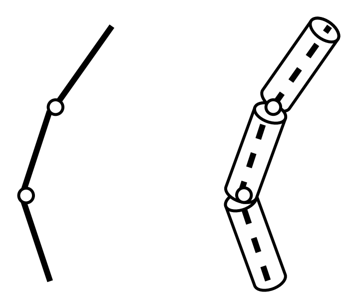

Fig. 2. Finger bodies include the approximated line segments

1) A face formed between the jth bodies of two adjacent fingers.
2) A face formed among the palm and the first bodies of two adjacent fingers.
3) The face whose vertices are fingertips.

Let us derive a sufficient condition that each of faces can be completely covered by the C-obstacles.

The face formed between the jth bodies of the ith finger and the $(i+1)$th finger, which are adjacent to each other, is shaped into a symmetric trapezoid, which has two base sides (Fig. 4, 5). Length of the base sides are the distances between the jth (or the $(j+1)$th) joints of the ith finger and the $(i+1)$th finger. If $\mathcal{C}_{ij}$ and $\mathcal{C}_{(i+1)j}$ cover both of the base sides, the sphere cannot escape from the face. We denote the distance between the jth joints of the ith finger and the $(i+1)$th finger by $d_j(\bar{\boldsymbol{\theta}})$, and the distance between the jth joint of the ith finger and the intersection point of $\mathcal{C}_{ij}$ with one base side by r'_j. Then the following is a sufficient condition that the sphere cannot escape from the face:

$$r'_j = \frac{r_{\text{sphere}}}{\sqrt{1-\left(\frac{d_j(\bar{\boldsymbol{\theta}})-d_{j+1}(\bar{\boldsymbol{\theta}})}{2l_j}\right)^2}} > \max\left(\frac{d_j(\bar{\boldsymbol{\theta}})}{2}, \frac{d_{j+1}(\bar{\boldsymbol{\theta}})}{2}\right). \quad (j = 2, \dots, \bar{L}) \qquad (11)$$

Note that the $(\bar{L}+1)$th joints correspond to the fingertips, and the $(N+1)$th finger corresponds to the first finger.

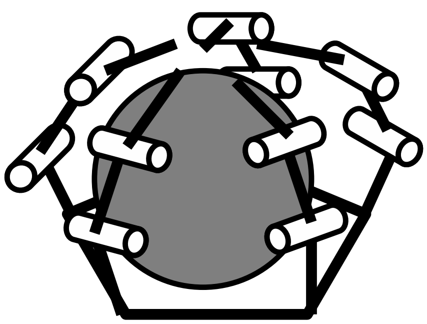

Fig. 3. Caging a sphere

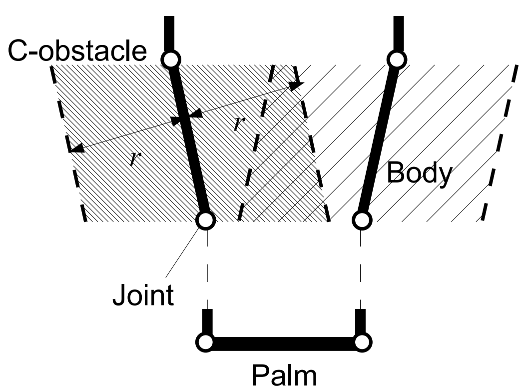

Fig. 4. C-obstacles of finger bodies in the case of caging a sphere

The face formed among the palm and the first bodies of two adjacent fingers is also shaped into a symmetric trapezoid as well. However, the height of the trapezoid is shortened by r_{sphere} because of $\mathcal{C}_{\text{plm}}$ (Fig. 6). Thus, the desired condition is written as follows:

$$r'_1 = \frac{r_{\text{sphere}}}{\sqrt{1-\left(\frac{d_1(\bar{\boldsymbol{\theta}})-d_2(\bar{\boldsymbol{\theta}})}{2l_1}\right)^2}} > \max\left(\frac{d'_1(\bar{\boldsymbol{\theta}})}{2}, \frac{d_2(\bar{\boldsymbol{\theta}})}{2}\right), \qquad (12)$$

where $d'_1(\bar{\boldsymbol{\theta}})$ is the distance between the intersection points of $\mathcal{C}_{\text{plm}}$ with the first bodies, and written as follows:

$$d'_1(\bar{\boldsymbol{\theta}}) = \frac{l - r_{\text{sphere}}}{l}|d_1(\bar{\boldsymbol{\theta}}) - d_2(\bar{\boldsymbol{\theta}})| + \min(d_1(\bar{\boldsymbol{\theta}}), d_2(\bar{\boldsymbol{\theta}})). \qquad (13)$$

The face whose vertices are all the fingertips is shaped into a regular N-gon. Thus, the target sphere cannot escape from the face when its circumradius: $r_c(\bar{\boldsymbol{\theta}})$, which is equal to the distance between the fingertip of the ith finger and the vertical center axis of the palm, is shorter than r_{sphere}. Therefore,

$$r_c(\bar{\boldsymbol{\theta}}) < r_{\text{sphere}}. \qquad (14)$$

Consequently, a sufficient condition for caging a sphere can be written as follows:

1) (11), (12) and (14) are satisfied.
2) (7) is also satisfied.

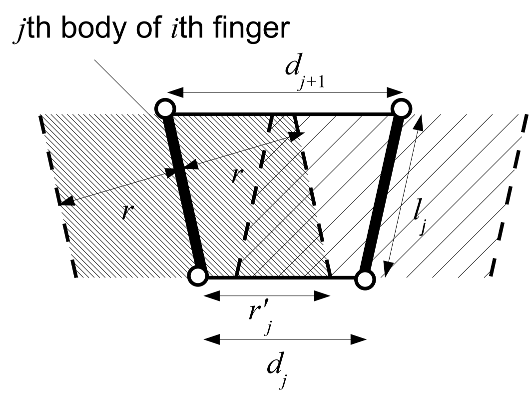

Fig. 5. C-obstacles cover completely the face formed between both finger bodies

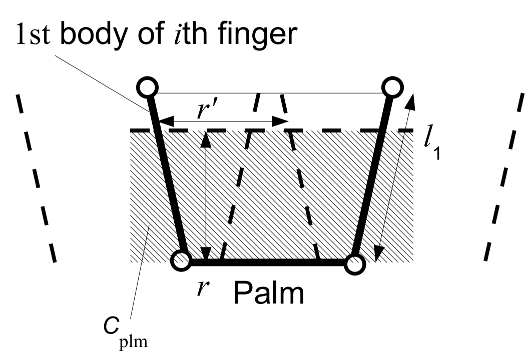

Fig. 6. $\mathcal{C}_{\text{plm}}$ covers a part of the face formed among the palm and both first bodies

C. Caging a Disk

Let us derive a sufficient condition for caging a disk (Fig. 7). The radius of the disk is $r_{\rm disk}$. We assume that $N \geq 3$.

In this case, it is necessary that the target object cannot escape among the fingers of the hand. Then, all the fingertips should be in contact with each other because the disk, whose thickness is zero, may escape from the infinitesimal gap between the fingertips. Therefore, the distance between the fingertip of the finger and the vertical center axis of the palm: $r_c(\bar{\boldsymbol{\theta}})$ should be zero:

$$r_c(\bar{\boldsymbol{\theta}}) = 0. \tag{15}$$

If every distance between the joints or the fingertips is shorter than $2r_{\rm disk}$, it is impossible for the disk to escape between the finger bodies. Therefore, the distance between the jth joint of the ith finger and the kth joint of the ith finger: d_{ijik} (Fig. 8) should satisfy the following conditions:

$$d_{ijik}(\bar{\boldsymbol{\theta}}) < 2r_{\rm disk} \quad (k \neq j) \tag{16}$$

$$d_{ij(i+1)k}(\bar{\boldsymbol{\theta}}) < 2r_{\rm disk}. \tag{17}$$

$$(i = 1, \ldots, N) \quad (j = 1, \ldots, \bar{L}) \quad (k = 1, \ldots, \bar{L}+1)$$

Consequently, a sufficient condition for caging a disk can be written as follows:

1) (15), (16) and (17) are satisfied.
2) (7) is also satisfied.

D. Caging a Ring-like Object

Let us derive a sufficient condition for caging a ring-like object, such as a torus (Fig. 9). We assume that $N = 2$. Then, the palm of the hand is approximated by a line segment. As an example of ring-like object, we give a sweeping volume that a circle moves along a closed curve, keeping vertical to the closed curve. The diameter of the circle is $d_{\rm ring}$.

When the two fingertips of the hand approach to each other at the hole area of the ring-like object, and the distance between both fingertips: $d_{L+1}(\bar{\boldsymbol{\theta}})$ is shorter than $d_{\rm ring}$, the hand can capture the object, that is, caging a ring-like object is achieved. Therefore, $d_{L+1}(\bar{\boldsymbol{\theta}})$ should satisfy the condition written as follows:

$$d_{L+1}(\bar{\boldsymbol{\theta}}) < d_{\rm ring}. \tag{18}$$

Then, the closed curve that is composed of the hand and the line segment connecting both fingertips of the hand, and the ring-like object make *Hopf link*.

E. Caging complex-shaped objects

Let us consider caging complex-shaped objects by a multi-fingered hand. We can represent the complex-shaped objects approximately by using simple shapes such as spheres and disks as shape primitives (Fig. 10). Then the sufficient conditions for caging the simple shapes can be used as a sufficient condition for caging the complex-shaped objects. For example, we may deal with caging a cuboid as the problem of caging an inscribed sphere.

V. Motion Planning of Robot Fingers

In Section IV, we derived sufficient conditions for caging in some cases: a sphere, a disk and a ring-like object. Next, finger configurations that satisfy the derived conditions should be determined to plan robotic caging. In this paper, we use *Rapidly-exploring Random Trees (RRT)* [6] for motion planning. RRT is a path planner, which generates path

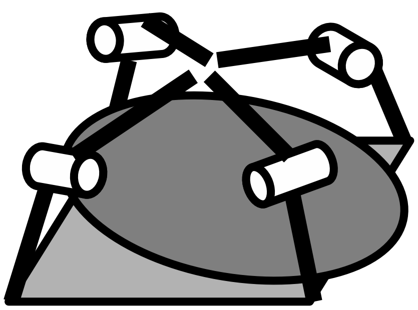

Fig. 7. Caging a disk

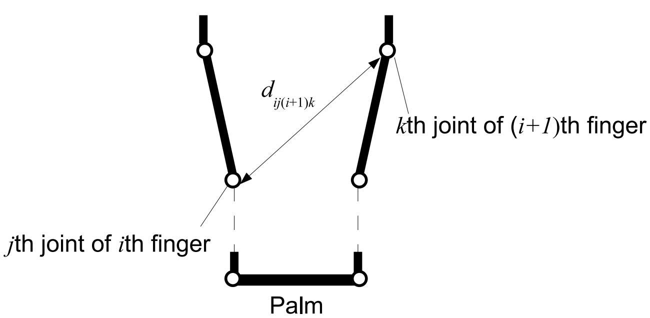

Fig. 8. The distance between joints

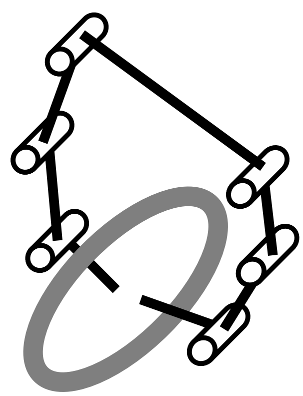

Fig. 9. Caging a ring-like object

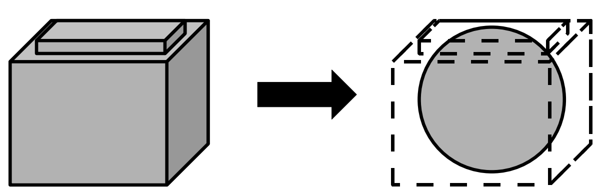

Fig. 10. Using shape primitives for a complex-shaped object

branches randomly from an initial state, finally to a goal state. By using RRT for motion planning of robot finger configurations in caging, we can obtain a configuration path to a goal configuration that satisfies the sufficient condition for caging.

We have to detect collisions between the objects and the robot to check the feasibility of configurations. We use PQP – A Proximity Query Package [7] in motion planning. Our proposed procedure of caging motion planning is described as follows:

1) Set an initial finger configuration: $\boldsymbol{\theta}_{\mathrm{ini}}$ as a seed of a configuration path branch.
2) Generate a random configuration: $\boldsymbol{\theta}_{\mathrm{rand}}$.
3) Find the nearest configuration: $\boldsymbol{\theta}_{\mathrm{near}}$ in the current configuration path branches.
4) Generate a candidate of new configuration: $\boldsymbol{\theta}_{\mathrm{cand}}$, which is located between $\boldsymbol{\theta}_{\mathrm{rand}}$ and $\boldsymbol{\theta}_{\mathrm{near}}$.
5) Examine whether the robot collides with the target object between $\boldsymbol{\theta}_{\mathrm{rand}}$ and $\boldsymbol{\theta}_{\mathrm{near}}$.
6) When no collision is detected, the candidate configuration becomes a new configuration: $\boldsymbol{\theta}_{\mathrm{new}}$, and added to the configuration path branches.
7) Repeat the steps from 2) to 6) until $\boldsymbol{\theta}_{\mathrm{new}}$ satisfies a sufficient condition for caging.

In 3), the nearest state can be found by minimum norm calculation. Note that this motion planning deals with only finger configurations. In other words, the relative position and orientation of the object and the palm are fixed.

VI. Results of Caging Planning

According to the proposed procedure of caging planning in Section V, we can obtain finger configurations that satisfy the sufficient conditions for caging in some cases. We show some examples of caging planning: a sphere, a disk and a ring-like object, for which sufficient conditions are derived in Section IV.

The motion planning in this paper is calculated on a Linux PC whose CPU is Pentium4 running at 3.2GHz.

A. Result: Caging a Sphere

Let us consider caging a sphere (Fig. 3). For caging planning in this case, parameters are as follows:

- $r_{\mathrm{sphere}} = 0.1$.
- $N = 4$.
- $\bar{L} = 3$.
- $l_j = 0.1 \quad (j = 1, \ldots, \bar{L})$.
- The palm of the robot hand is a square 0.1412 on a side.
- The bodies of the fingers are cylinders with radius 0.01.

In collision detection by PQP, the sphere is approximated by a polyhedron with 64 surfaces.

Fig. 11 shows an example of calculation results to obtain finger configurations for caging a sphere. The joint angles in this result are as follows:

$$\bar{\boldsymbol{\theta}} = [1.063, 0.994, 0.402]^T (\mathrm{rad}).$$

The calculation time of the planning was 10.9 CPU seconds on average.

B. Result: Caging a Disk

Let us consider caging a disk (Fig. 7). In this case, parameters are as follows:

- $r_{\mathrm{disk}} = 0.1$.
- $N = 4$.
- $\bar{L} = 3$.
- $l_j = 0.08 \quad (j = 1, \ldots, \bar{L})$.
- The palm of the robot hand is a square 0.113 on a side.
- The bodies of the fingers are cylinders with radius 0.01.

In collision detection by PQP, the disk is approximated by a regular dodecagon.

Fig. 12 shows an example of calculation results to obtain finger configurations for caging a disk. The joint angles in this result are as follows:

$$\bar{\boldsymbol{\theta}} = [0.991, 1.240, 1.276]^T (\mathrm{rad}).$$

The calculation time of the planning was 13.3 CPU seconds on average.

C. Result: Caging a Ring-like Object

Let us consider caging a ring-like object (Fig. 9). The target ring-like object is a torus. In this case, parameters are

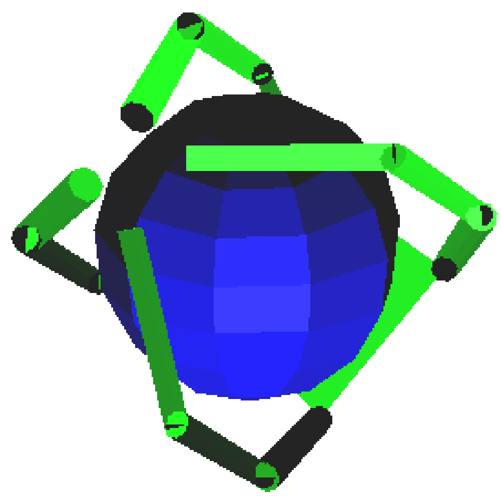

Fig. 11. Result: caging a sphere

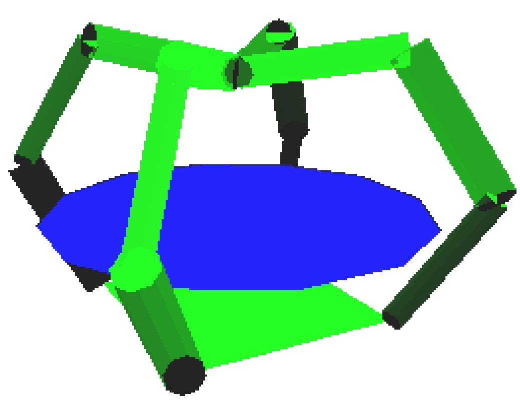

Fig. 12. Result: caging a disk

as follows:

- The external diameter of the torus is 0.3.
- The internal diameter of the torus is 0.1.
- $N = 2$.
- $\bar{L} = 3$.
- $l_j = 0.1(j = 1, \ldots, \bar{L})$.
- The palm of the robot hand is a line segment with length 0.2.
- The bodies of the fingers are cylinders with radius 0.01.

In collision detection by PQP, the torus is approximated by a polyhedron with 144 surfaces.

Fig. 13 and Fig. 14 show an example of calculation results to obtain finger configurations for caging a ring-like object (a torus). The palm is drawn as a square, but it is dealt with as a line segment in caging computation, as mentioned above. The joint angles in this result are as follows:

$$\bar{\boldsymbol{\theta}} = [0.914, 1.069, 0.669]^T (\text{rad}).$$

The computation time of the planning was 1.3 CPU seconds on average.

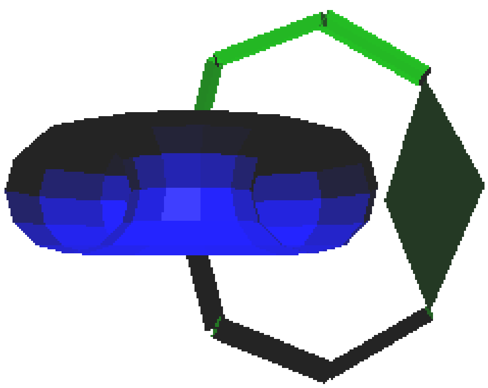

Fig. 13. Result (Front view): caging a ring-like object (a torus)

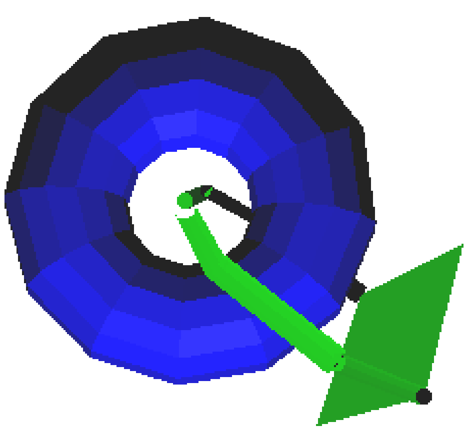

Fig. 14. Result (Side view): caging a ring-like object (a torus)

VII. Conclusions and Future Works

A. Conclusions

In this paper, three-dimensional caging by a multifingered hand (3D multifingered caging) was studied. To construct the theory of 3D multifingered caging, we formulated the condition for caging in an abstract form. Then, we derived sufficient conditions for caging in a concrete form in some cases. Finally, we construct an RRT-based motion planner of finger configuration for caging. Three planning examples of caging: a sphere, a disk and a ring-like object (a torus) were successfully demonstrated.

B. Future Works

In future work, it is necessary to plan not only finger configurations but also a palm configuration, so that we can obtain the approaching motions of robot hands for caging. It is also necessary to plan caging of complex-shaped objects by using shape primitives.

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