A Mechanical Model of Elasticity Attributed to Flexor Digitorum

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This paper addresses a hypothesis of estimating elasticity of flexor digitorum muscle and tendon by measuring angle of MP joint. For ease of measurement and estimation, we propose a simplified musculoskeletal model of stretching of muscle-tendon complex (MTC), and formulate relationship between joint angle and resultant fingertip force attributed to MTC stretching. In addition, relationship between joint angle and angular velocity of released finger is also approximated. These hypothesis and approximation are verified by measurement experiments.

1 Introduction

In this paper, we propose a hypothesis and a mechanical model of muscle-tendon complex (MTC) in the finger based on musculoskeletal mechanism to observe its behavior by easy measurement of joint angles and fingertip forces, instead of using ultrasonography or MRI scanning. The mechanical model may be constructed by any mechanical and electric components to improve performances of robots.

Muscle-tendon complexes behave as springs in the human body, particularly tendons, which are seldom stretched by usual muscular strength, contribute to improve sports performances. There are two typical method to analyze tendons behavior: measuring viscoelasticity of anatomically-isolated tendons in mechanical way [1]; using ultrasonography to observe MTC in vivo [2]. In our previous work [3], we have investigated estimation of elastic and dynamic properties of fingers based on a consideration of musculoskeletal mechanism. These parameters can be estimated as a function of joint angles and fingertip forces, which both can be easily measured. This paper addresses a mechanical model applied to the estimation.

2 Mechanical Modeling

We introduce a simple hypothesis that a MTC of a finger composed of flexor digitorum muscle and its tendon can be stretched during dorsiflexion by pushing corresponding fingertip with external forces (Fig. 1). The stretched MTC of flexor digitorum exerts resultant force according to its elasticity, and the force works to pull the fingertip toward palmarflexion. Thus elasticity of flexor digitorum muscle and tendon can be approximated by a relationship between joint angle and fingertip force. We express the relationship by an exponential curve as [1], which reported mechanical property of an isolated tendon.

$$F_{\rm tip} = ae^{b\theta} + c, \tag{1}$$

where F_{tip} and θ denote fingertip force and angle of MP joint respectively. *a*, *b* and *c* are arbitrary values.

From Fig. 1, mechanical properties of digitorum muscle and tendon: changed length of MTC during stretching, x and



Fig. 1 Simplified model of stretching of flexor digitorum muscle and tendon

resultant force caused by stretching, F_{mtc} , can be approximated by following equations.

$$x = r\theta, \tag{2}$$

$$rF_{\rm mtc} = lF_{\rm tip},\tag{3}$$

where r and l denote distance from the center of MP joint to the tendon and to the force applying point on the fingertip respectively. Although r may change according to hand posture, it assumes to be negligible.

Elastic energy accumulated in the MTC during x changes from 0 to x_a , E_e , can be calculated:

$$E_e = \int_0^{x_a} F_{\rm mtc} dx = \frac{l}{r} \left(\frac{ar}{b} e^{\frac{b}{r} x_a} + cx_a - \frac{ar}{b} \right). \tag{4}$$

When the external force applied to the fingertip is instantly removed, the movement of finger is induced by the MTC shortening with energy transduction, where elastic energy shifts resultant kinetic energy of finger, E_k , and any losses, E_1 :

$$E_e = E_k + E_1 = \frac{1}{2}m\dot{x_a}^2 + E_1, \qquad (5)$$

where x_a denotes changed length of the MTC before the external force is removed; \dot{x}_a denotes induced shortening velocity of the MTC; *m* denotes mass of finger driven by resultant force of the MTC.

From (2), the resultant angular velocity, $\dot{\theta}_a$ can be estimated:

$$\dot{\theta}_a = \frac{\dot{x}_a}{r}.$$
(6)

From (4), (5) and (6), the resultant angular velocity of MP joint can be estimated from the joint angle when the external force is removed:

$$\dot{\theta}_a = \frac{1}{r} \sqrt{\frac{2l}{mr} \left(\frac{ar}{b} e^{b\theta_a} + cr\theta_a - \frac{ar}{b} - \frac{r}{l} E_1\right)}.$$
 (7)

3 Measurement

This section presents two measurement results: relationship between dorsiflexion angle of MP joint and resultant fingertip force; relationship between dorsiflexion angle of MP joint when an external force is applied to the fingertip and resultant angular velocity of MP joint after the force is instantly removed. This results have already appeared in our previous work [3].

3.1 Procedure

Our measurement procedures for the above two relationship are as follows:

- **Step 1.** Fix a subject's hand on the measurement bed, and lift up the finger as high as possible by his/her own muscular strength. The angle is called as "initial angle".
- **Step 2.** Push up the fingertip until the MP joint shifts a certain angle, and measure the angle and fingertip force by a goniometer and a force sensor respectively. When the limitation angle that the finger cannot move no longer occurs, the measurement is suspended. This process from the initial to the limitation angle is called "loading".
- **Step 3.** After the limitation angle, we bring the finger down and similarly measure the two parameters until the initial angle. The downward process is "unloading".

As to measuring angular velocity, we use the same experimental equipment.

- **Step 1.** Once we lift the finger up at a certain angle by pushing its fingertip, the applied force is instantly removed to release the finger. The angular velocity can be estimated from several time-shift images captured by a high speed camera.
- **Step 2.** In the unloading process, we lift the finger up to the limitation angle once and bring it down to a certain angle, and release it.

3.2 Results

Fig. 2 shows that fingertip force in dorsiflexion is exponentially proportional to angle of MP joint in loading process. We can approximate the plots with an exponential function: $F_{\rm tip} = 0.000017e^{0.14\theta} + 1.41$. This trend is similar to a relationship between tension and stretched length of isolated tendon reported in previous works [1, 4]. On the other hand, in the unloading process, larger hysteresis with sudden reduction of exerted force is observed. The loss of the force may caused by any loss of muscular strength, although the factor has not yet been proved. Some physiological approaches and investigation using ultrasonography or MRI as many previous works on musculoskeletal are considered.

As for measured angular velocity of finger, the relationship between angular velocity and the angle of MP joint where the finger is released can be expressed as an approximation written in (7): $\dot{\theta}_a = \sqrt{3.18e^{4.17\theta_a} + 31.9\theta_a}$ (Fig. 3). Note that it also has large hysteresis in its trend, which is probably caused by the same factor mentioned above.

4 Conclusion

In this paper, we addressed a hypothesis of estimating elasticity of flexor digitorum muscle and tendon by measuring angle of MP joint. The hypothesis is based on a simplified musculoskeletal model, where only a pair of muscle and tendon is stretching and shortening. Approximation with exponential function can be fit to express relationship between



Fig. 2 Relationship between angle of MP joint and resultant fingertip force



Fig. 3 Relationship between angle of MP joint and angular velocity of released finger motion

angle of MP joint and resultant fingertip force. Moreover a relationship between angle of MP joint and angular velocity of released finger motion can be also approximated by suitable function.

In future works, the musculoskeletal model will be composed with mechanical and electrical components to improve performance of robotic arm/hand systems.

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